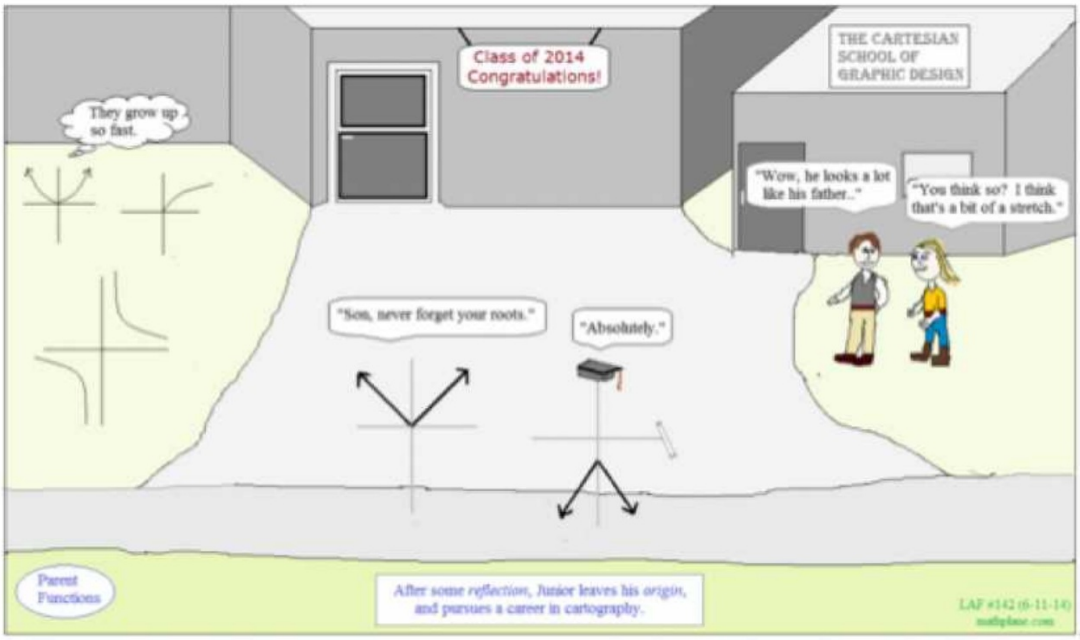
Tabular Data
5.7 Average Rate of Change



HW:

Average Rate of Change (ARC) slope

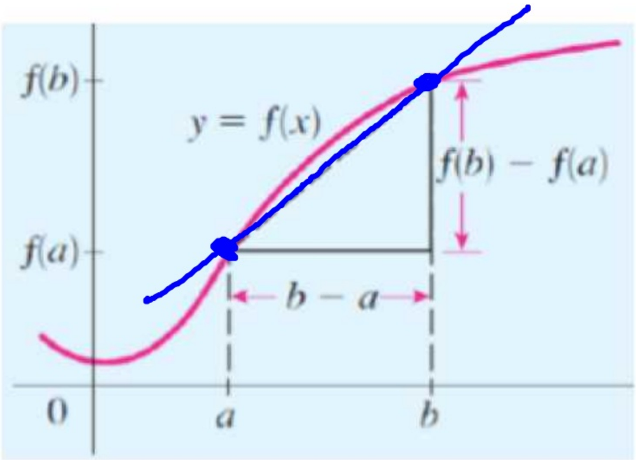
The average rate of change of the function $f(x)$ on the interval $x = a$ to $x = b$ is

Average Rate of Change: $\dfrac{f(b) - f(a)}{b - a}$

$$\frac{y_2 - y_1}{x_2 - x_1}$$

Graphical Perspective

The average rate of change of the function $f(x)$ on the interval $x = a$ to $x = b$ is the slope of the secant line between $x = a$ and $x = b$, that is, the line that passes through the points $(a, f(a))$ and $(b, f(b))$.



ex: Find the average rate of change over the indicated interval.

$$\frac{f(b)-f(a)}{b-a}$$

a) $f(x)=\dfrac{x-1}{x+2}$, $\overset{a\ \ b}{[0,3]}$

$$\frac{f(3)-f(0)}{3-0} = \frac{\frac{2}{5}-\left(-\frac{1}{2}\right)}{3-0} = \frac{\frac{2}{5}+\frac{1}{2}}{3}$$

$f(3)=\frac{2}{5}$

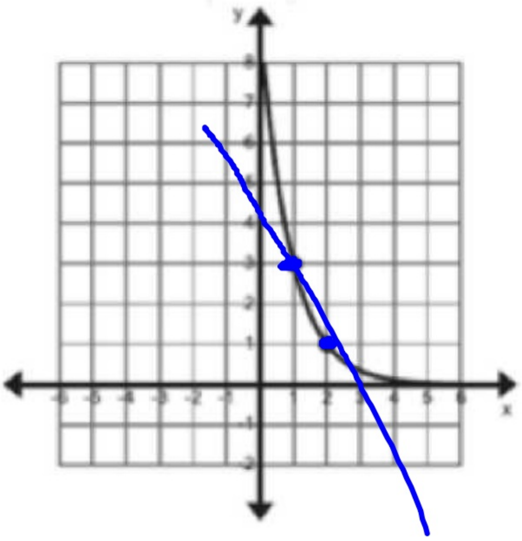
$f(0)=-\frac{1}{2}$

$$\frac{\frac{9}{10}}{3} = \frac{9}{10}\cdot\frac{1}{3} = \frac{9}{30}$$

$$=\frac{3}{10}$$

ex: Find the average rate of change over the indicated interval.

b) [1,2]



-2

ex: Find the average rate of change over the indicated interval.

c) $3<t<5$

Time (years)	1	2	3	4	5
Height(in.)	27	35	37	42	45

$(3, 37)$ $(5, 45)$

$$\frac{45-37}{5-3} = 4$$

ex: On which interval does the function $f(x) = \frac{1}{8}x^3 - x^2$ have an average rate of change of 0.5?

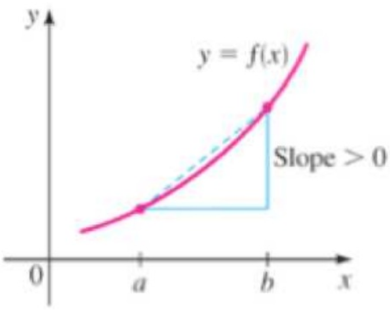
$-2 < x < 2$
$0 < x < 4$
$-3 < x < 2$
$-4 < x < 1$

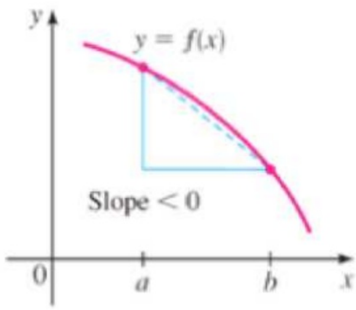
- If $f(x)$ is strictly increasing on the interval [a, b], then average rate of change of $f(x)$ is

  positive on the interval [a, b].

- If $f(x)$ is strictly decreasing on the interval [a, b], then average rate of change of $f(x)$ is

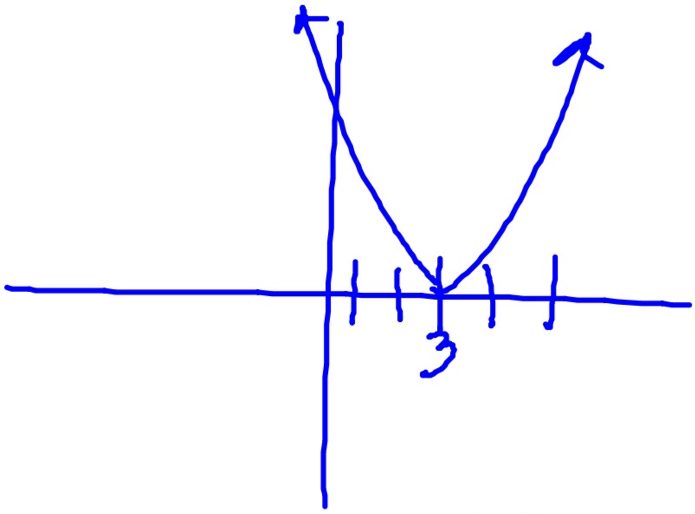
  negative on the interval [a, b].





ex:

a) Sketch: $f(x)=(x-3)^2$



b) State an interval on which the average rate of $f(x)$ is positive. How do you know?

$[3, 5]$

c) State an interval on which the average rate of $f(x)$ is negative. How do you know?

$[2, 3]$

ex: The graph below shows the United States population from 1900 to 2010, as recorded by the U.S. Census Bureau.

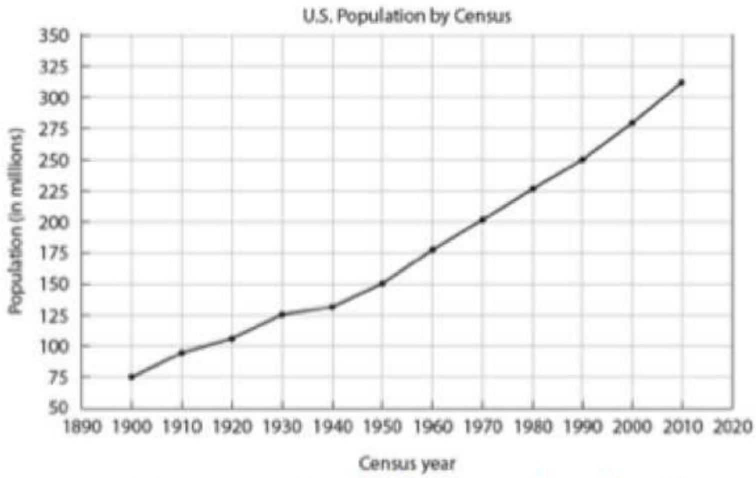
3.7 millions/year

a) What was the rate of change in the populationfrom 1900 to 2000? Is this greater or less than the rate of change in the population from 2000 to 2010?

1900-2000

(1900, 75)
(2000, 280)

$= \frac{205}{100} = 2.05$ millions/year ex: The graph below shows the United States population from 1900 to 2010, as recorded by the U.S. Census Bureau.



b) Which 10-year time periods have the highest and lowest rates of change? How did you know?

2000 - 2010

1930 - 1940

Tabular Data

1. Complete the following tables and answer the questions.

a)

x	$y = 3x$	1st Diff
-3	-9	+3
-2	-6	+3
-1	-3	+3
0	0	⋮
1	3	
2	6	
3	9	

This function is ❐ linear ❐ quadratic ❐ exponential ❐ logarithmic

How do you know?

1st difference is constant
(slope of 3)

1. Complete the following tables and answer the questions.

b)

x	$y = x^2$	1st Diff	2nd Diff
0	0		
1	1	1	2
2	4	3	2
3	9	5	2
4	16	7	2
5	25	9	2
6	36	11	

This function is. ❑ linear ❑ quadratic ❑ exponential ❑ logarithmic

How do you know?

2nd difference is constant (2)

1. Complete the following tables and answer the questions.

c)

x	$y = 3^x$
0	1
1	3
2	9
3	27
4	81
5	243
6	729

This function is. ❑ linear ❑ quadratic ❑ exponential ❑ logarithmic

How do you know?

common ratio of 3

1. Complete the following tables and answer the questions.

d)

x	$y = \log_3 x$
1	0
3	1
9	2
27	3
81	4
243	5
729	6

This function is. ❒ linear ❒ quadratic ❒ exponential ❒ logarithmic

How do you know?

The y's go up by 1

The x's go up exponentially.

2. Use differences to identify the type of function represented by the table of values. Then label which type of function each table of values models.

x	y
-4	5
-3	8
-2	13
-1	20
0	29
1	40

Quad.
2nd diff.
2

x	y
-5	32
-4	16
-3	8
-2	4
-1	2
0	1

exp.
(decay)

x	y
0.5	0.9
0.75	1.1
1	1.3
1.25	1.5
1.5	1.7
1.75	1.9

common
diff
.2
linear

x	y
-2	1/16
-1	$\frac{1}{4}$
0	1
1	4
2	16
3	64

exp.
(growth)

3.

Rebecca records the amount of money, $f(x)$, in her bank account each month, x, as shown in the table.

Bank Account

Month (x)	Amount of Money $f(x)$
0	$ 6
1	$12
2	$24
3	$48
4	$96

Create a function that models this relationship.

$\cancel{f(x) = 2^x}$

$f(x) = 6 \cdot 2^x$

4.

A scientist studies several colonies of bacteria. She records the number of cells in the colony every hour. Several tables containing the data are shown.

Click on the table or tables that represent exponential growth.

Colony 1

Hours	Cells
0	10
1	25
2	40
3	[illegible]

linear

Colony 2

Hours	Cells
0	3
1	12
2	48
3	192

Colony 3

Hours	Cells
0	4.0
1	4.5
2	5.0
3	5.[illegible]

linear

Colony 4

Hours	Cells
0	8
1	12
2	18
3	27

Colony 5

Hours	Cells
0	200
1	100
2	[illegible]
[illegible]	25

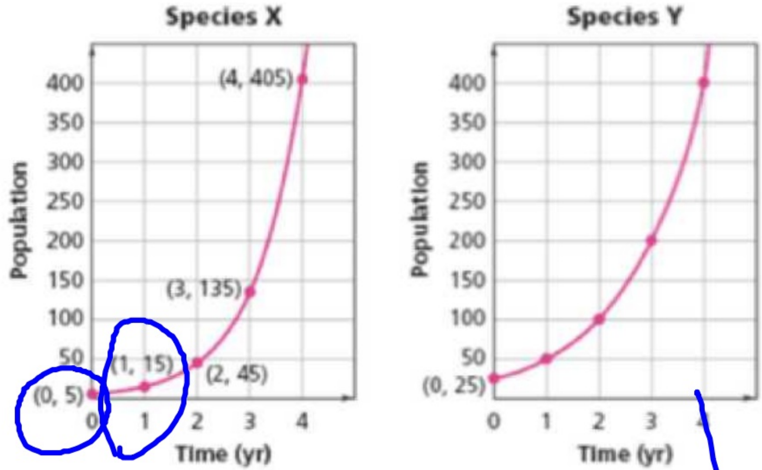
exp. decay

$$8 \cdot \frac{3}{2} = 12$$

$$12 \cdot \frac{3}{2} = 18$$

5a)

The following graphs show the population growth for two species.



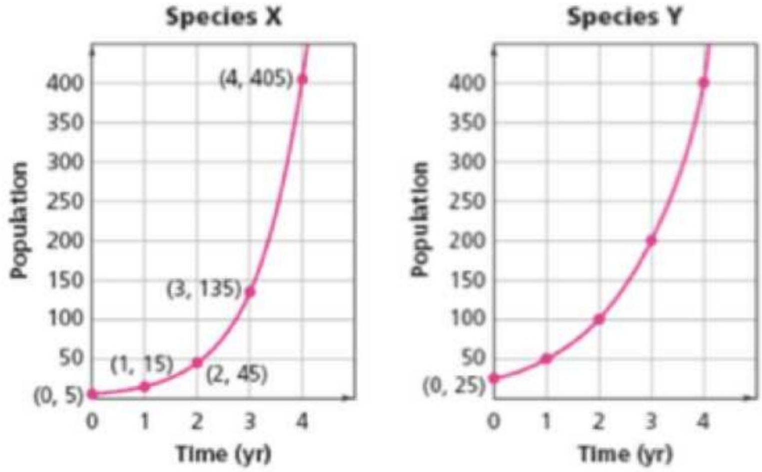
Show both graphs are exponential. $y = ab^x$

$y = 5 \cdot 3^x$

$y = 25 \cdot 2^x$

5b)

The following graphs show the population growth for two species.



Write the exponential function $y = ab^x$ for species X and Y. What will be the population at time t=6 for both species?