Symmetry & Even/Odd Functions

ex 1) Evaluate.

$$(-x)^2 = x^2 \qquad\qquad (-x)^4 = x^4$$
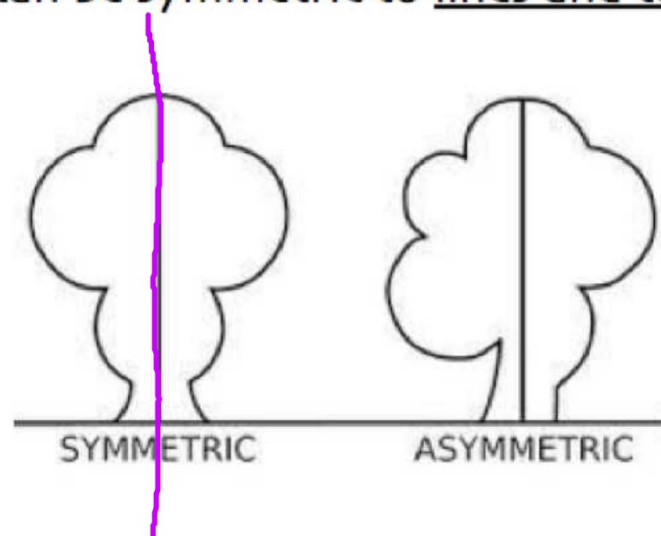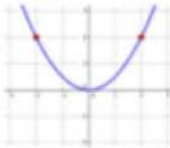
$$(-x)^3 = -x^3 \qquad\qquad -3(-x)^3 = 3x^3$$
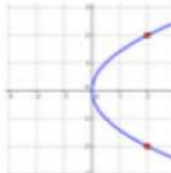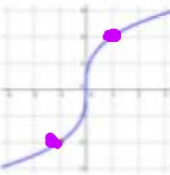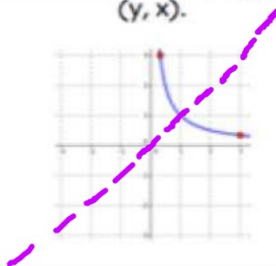
What is Symmetry?

"Symmetry is a recognition of the "matching-ness" of the parts of a shape. In other words, if a graph is reflected across an axis and the graph looks exactly the same as the original, it means that the graph is symmetric with respect to that axis. Graphs can be symmetric to lines and to points.

SYMMETRIC   ASYMMETRIC

There are 4 types of symmetry we are concerned with...

y-axis	x-axis
A graph has y-axis symmetry if it contains to points (x, y) and (-x, y).	A graph has x-axis symmetry if it contains to points (x, y) and (x, -y).

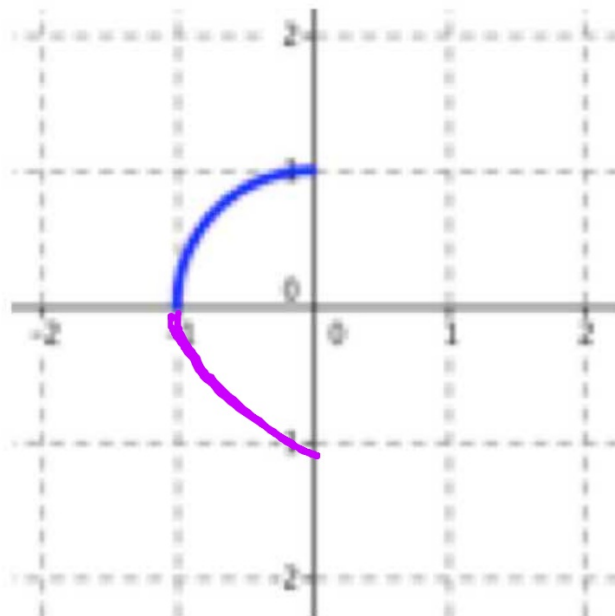| Origin	y=x
A graph has origin symmetry if it contains to points (x, y) and (-x, -y).	A graph has y=x symmetry if it contains to points (x, y) and (y, x).

ex 2)

The graph of a curve contains the point (3, -4).

a) If the graph of the curve is symmetric about the y-axis, the graph will also contain the point $(-3, -4)$.	b) If the graph of the curve is symmetric about the x-axis, the graph will also contain the point $(3, 4)$.
c) If the graph of the curve is symmetric about the origin, the graph will also contain the point $(-3, 4)$.	d) If the graph of the curve is symmetric about the line y=x, the graph will also contain the point $(-4, 3)$.

ex 3) The graphs below are portions of complete graphs.
Sketch a complete graph for each of the following
symmetries: with respect to a) the x-axis, b) the y-axis, c)
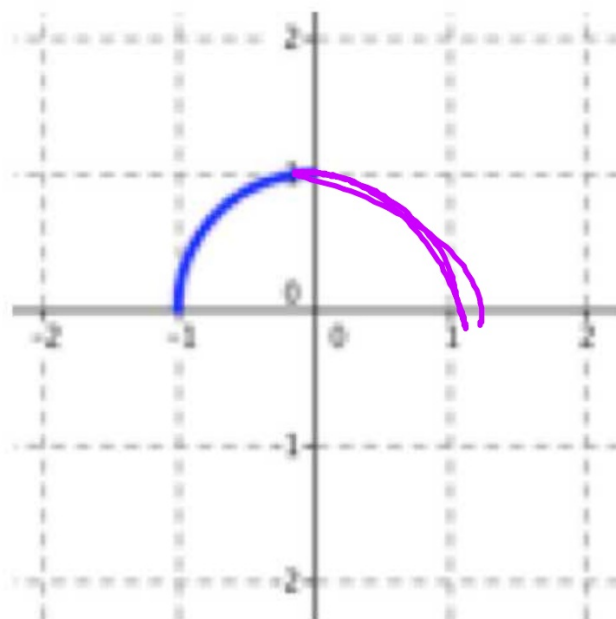the origin and d) the line y=x.

a)

ex 3) The graphs below are portions of complete graphs.
Sketch a complete graph for each of the following
symmetries: with respect to a) the x-axis, b) the y-axis, c)
the origin and d) the line y=x.

b)

ex 3) The graphs below are portions of complete graphs. Sketch a complete graph for each of the following symmetries: with respect to a) the x-axis, b) the y-axis, c) the origin and d) the line y=x.
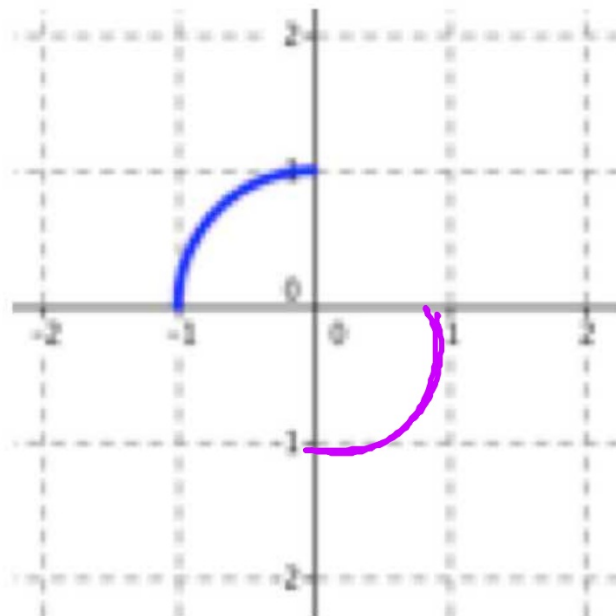
c)

ex 3) The graphs below are portions of complete graphs. Sketch a complete graph for each of the following symmetries: with respect to a) the x-axis, b) the y-axis, c) the origin and d) the line y=x.
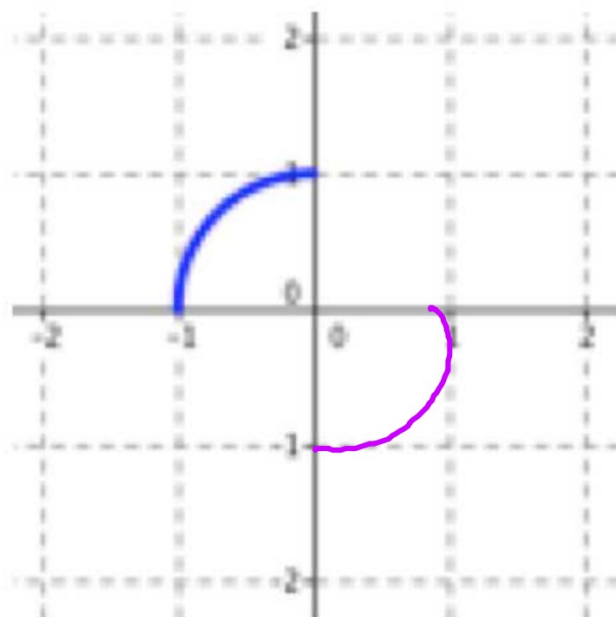
d)

ex 4) The graphs below are portions of complete graphs. Sketch a complete graph for each of the following symmetries: with respect to a) the x-axis, b) the y-axis, c) the origin and d) the line y=x.
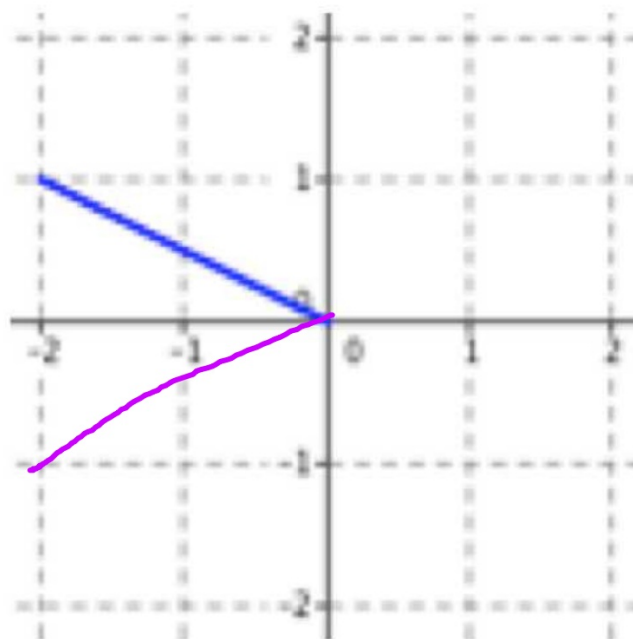
a)

ex 4) The graphs below are portions of complete graphs.
Sketch a complete graph for each of the following
symmetries: with respect to a) the x-axis, b) the y-axis, c)
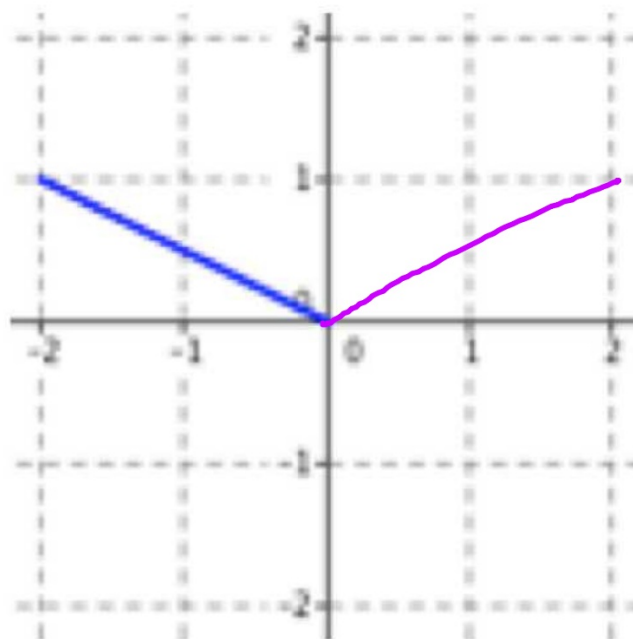the origin and d) the line y=x.

b)

ex 4) The graphs below are portions of complete graphs.
Sketch a complete graph for each of the following
symmetries: with respect to a) the x-axis, b) the y-axis, c)
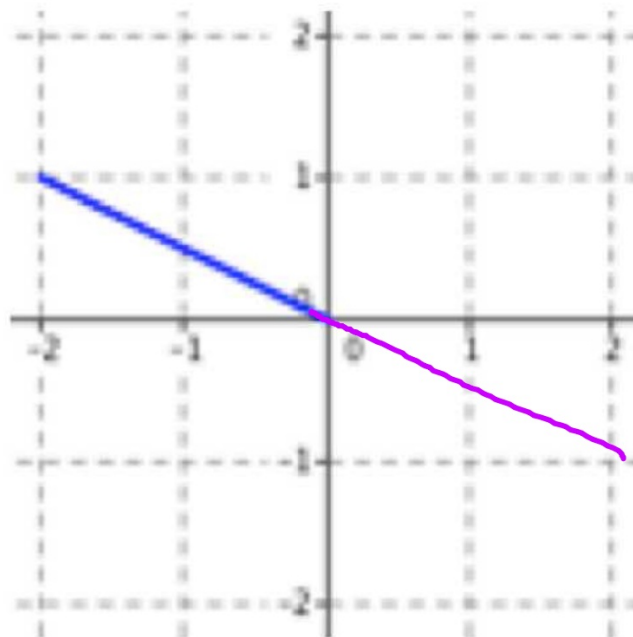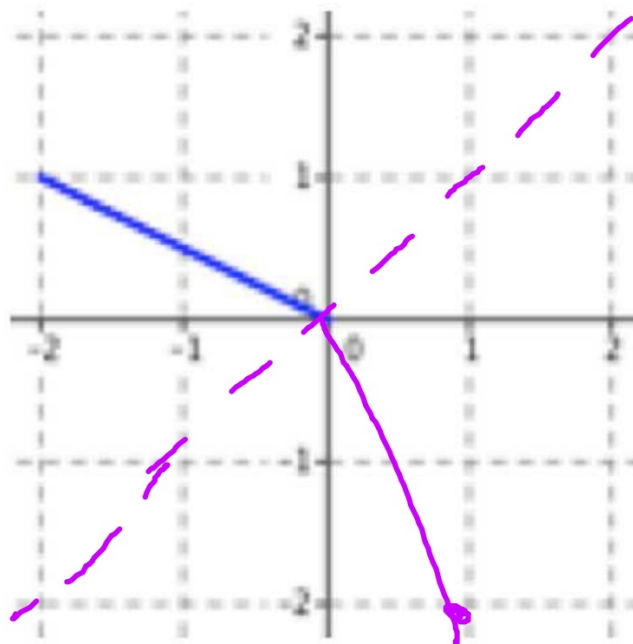the origin and d) the line y=x.

c)

ex 4) The graphs below are portions of complete graphs.
Sketch a complete graph for each of the following
symmetries: with respect to a) the x-axis, b) the y-axis, c)
the origin and d) the line y=x.

d)

(-2,1)

ex 5) Which library functions have...

a) y-axis symmetry?

    constant
    abs value
    quadratic
    rec of square
    semi circle

ex 5) Which library functions have...

b) x-axis symmetry?

none!!

ex 5) Which library functions have...

c) y=x symmetry?

identity
reciprocal

ex 5) Which library functions have...

d) origin symmetry?

cubic
identity
reciprocal
cube root

Algebraic Tests for Symmetry:

- y-axis: replacing x with −x produces an equivalent equation

- x-axis: replacing y with −y produces an equivalent equation

- origin: replacing x with −x and y with −y produces produces an equivalent equation

- y=x: replacing x with y produces an equivalent equation

ex 6) Determine algebraically if the functions below have y-axis, x-axis, origin or y=x symmetry.

a) $y = 2x^3 - x$

y-axis	x-axis
$y = 2(-x)^3 - (-x)$	$-y = 2x^3 - x$
$y = -2x^3 + x$	$y = -2x^3 + x$
NO	NO

Origin	y=x
$-y = 2(-x)^3 - (-x)$	$x = 2y^3 - y$
$-y = -2x^3 + x$	
$y = 2x^3 - x$ yes	NO

b.)

d.)

e.) change!

$$4x^2 + y^2 = 2$$

ex 6) Determine algebraically if the functions below have y-axis, x-axis, origin or y=x symmetry.

b) $y = 4x^4 - x^2 + 5$

y-axis

$$y = 4(-x)^4 - (-x)^2 + 5$$
$$y = 4x^4 - x^2 + 5$$
yes

x-axis

$$-y = 4x^4 - x^2 + 5$$
$$y = -4x^4 + x^2 - 5$$
NO

origin

$$-y = 4(-x)^4 - (-x)^2 + 5$$
$$-y = 4x^4 - x^2 + 5$$
No

y=x

$$x = 4y^4 - y^2 + 5$$
NO

ex 6) Determine algebraically if the functions below have y-axis, x-axis, origin or y=x symmetry.

c) $f(x) = x^3 - 2x^2 + x - 1$

ex 6) Determine algebraically if the functions below have y-axis, x-axis, origin or y=x symmetry.

d) $y = \dfrac{1}{x}$

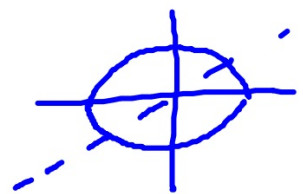| y-axis	x-axis
$y = \dfrac{1}{-x}$ NO	$-y = \dfrac{1}{x}$ NO
origin $-y = \dfrac{1}{-x}$ $y = \dfrac{1}{x}$ yes	**y=x** $y \cdot x = \dfrac{1}{y} \cdot y$ $yx = 1$ $y = \dfrac{1}{x}$ yes

ex 6) Determine algebraically if the functions below have y-axis, x-axis, origin or y=x symmetry.

e) $f(x) = \dfrac{5x^3 - x}{x^4 + 4}$ $4x^2 + y^2 = 2$

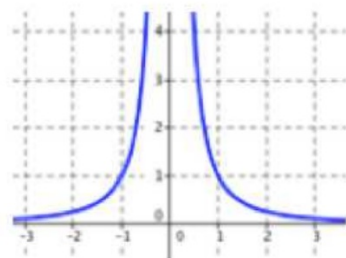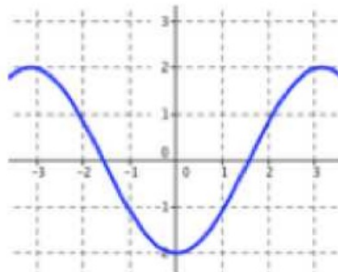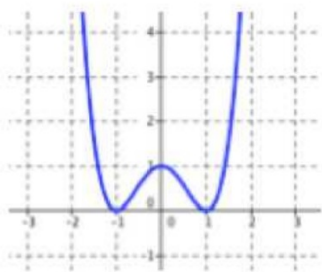| yaxis	xaxis
$4(-x)^2 + y^2 = 2$ $4x^2 + y^2 = 2$ yes	$4x^2 + (-y)^2 = 2$ $4x^2 + y^2 = 2$ yes
origin $4(-x)^2 + (-y)^2 = 2$ $4x^2 + y^2 = 2$ yes	y=x $4y^2 + x^2 = 2$ No

ex 6) Determine algebraically if the functions below have y-axis, x-axis, origin or y=x symmetry.

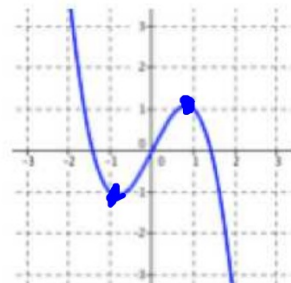f) $y = x\sqrt{x^2 - 9}$
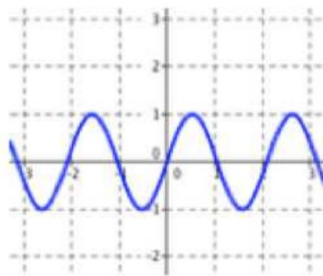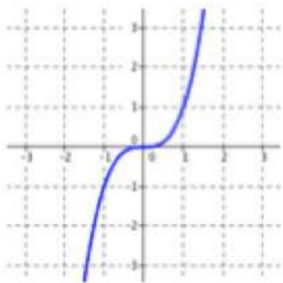
Examples of Even Functions

What do these functions have in common?
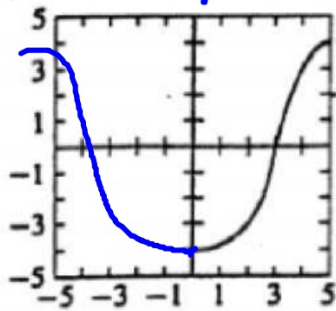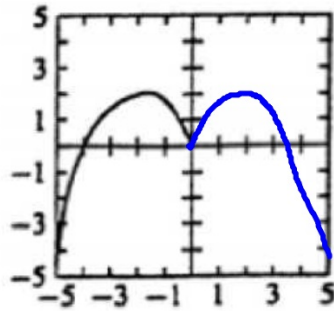
y-axis symmetry

Examples of Odd Functions

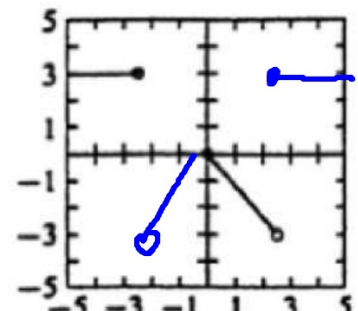## What do these functions have in common?

Origin symmetry

ex 7) Each of the graphs below shows a portion of a graph of an <u>even</u> function over the interval [-5,5]. Complete these graphs.
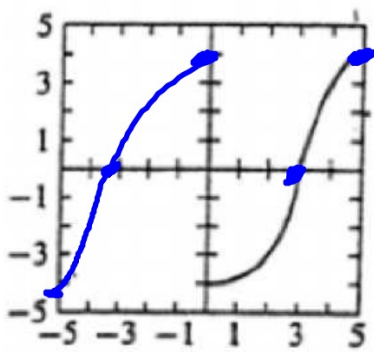
Graph (a)

Graph (b)

Graph (c)

ex 8) Which graph(s) below could be a portion of an <u>odd</u> function over the interval over the interval [-5,5]? Complete these graphs.
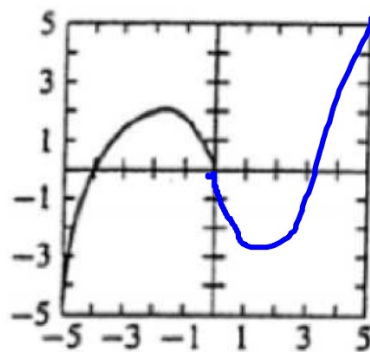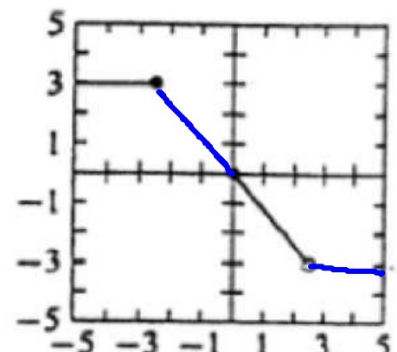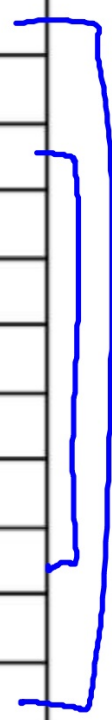
Graph (a)

Graph (b)

Graph (c)

ex 9) Table A represents an <u>even</u> function and Table B epresents an <u>odd</u> function. Complete these tables.

Table A	
-5	8
-4	6
-3	4
-2	0
-1	-1
0	you pick
1	-1
2	0
3	4
4	6
5	8

ex 9) Table A represents an <u>even</u> function and Table B represents an <u>odd</u> function. Complete these tables.

Table B	
-5	-6
-4	3
-3	-2
-2	-5
-1	-3
0	0
1	3
2	5
3	2
4	-3
5	6

ex 10) Which library functions are...

a) even functions?

Constant
abs value
quadratic
rec. of square
semi-circle

ex 10) Which library functions are...

b) odd functions?

identity
reciprocal
cube root
cubic

Algebraic Tests for Even and Odd Functions:

- A function is <u>even</u> if $f(-x) = f(x)$

- A function is <u>odd</u> if $f(-x) = -f(x)$

ex 11) Determine algebraically if the given functions are even, odd or neither.

a) $f(x) = 4x^4 - 3x^2 + 2$

$f(-x) = 4(-x)^4 - 3(-x)^2 + 2$

$\quad\quad = 4x^4 - 3x^2 + 2$

$f(-x) = f(x)$

$\quad\quad$ even

$(1, 3)$

$(-1, 3)$

#test

ex 11) Determine algebraically if the given functions are even, odd or neither.

b) $f(x) = 3x^2 - 2x + 1$

$$f(-x) = 3(-x)^2 - 2(-x) + 1$$
$$= 3x^2 + 2x + 1$$

$$f(-x) \neq f(x)$$
$$f(-x) \neq -f(x)$$

neither

ex 11) Determine algebraically if the given functions are
even, odd or neither.

c) $g(x) = \dfrac{3x}{x^2 + 7}$

$g(-x) = \dfrac{3(-x)}{(-x)^2 + 7}$

$\quad = \dfrac{-3x}{x^2 + 7}$

$\quad = -\left(\dfrac{3x}{x^2 + 7}\right)$

odd because

$g(-x) = -g(x)$

test

$\left(1, \ 3/8\right)$

$\left(-1, \ -3/8\right)$

b

    d   e

f

ex 11) Determine algebraically if the given functions are even, odd or neither.

d) $g(x) = \dfrac{4x^3}{x^3 - 2}$

ex 11) Determine algebraically if the given functions are even, odd or neither.

e) $h(x) = \dfrac{4x^3}{x^3 - 2x}$

$h(-x) = \dfrac{-4x^3}{-x^3 + 2x}$

$= \dfrac{+4x^3}{+(x^3 - 2x)}$

even

$h(-x) = h(x)$

ex 11) Determine algebraically if the given functions are even, odd or neither.

f) $G(x) = x\sqrt{x^2 - 1}$

$$G(-x) = -x\sqrt{x^2 - 1}$$

$$\text{odd}$$

$$G(-x) = -G(x)$$