Symmetry

Evaluate:

$$(-x)^2 = x^2$$

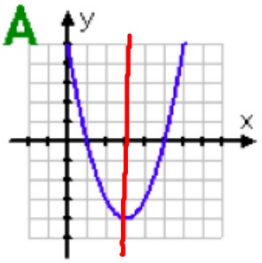
$$(-x)^3 = -x^3$$

$$(-x)^4 = x^4$$

$$-3(-x)^3 = 3x^3$$

Ex 2) For each of the following graphs, list any symmetries, and state whether the graph shows a function.

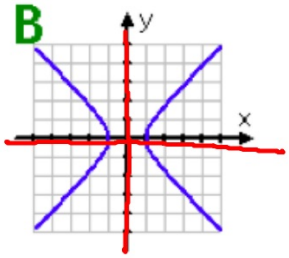
A



$x=3$

Yes

B

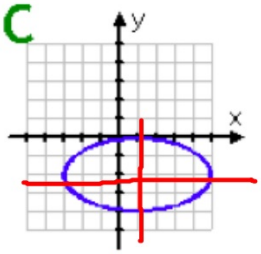


$x=0$

$y=0$

No

C

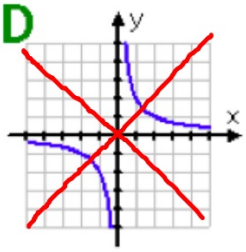


$x=1$

$y=-2$

No

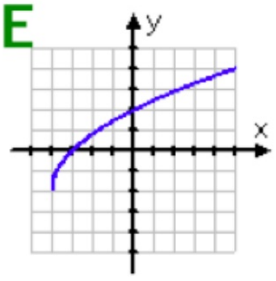
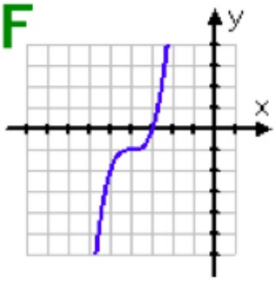
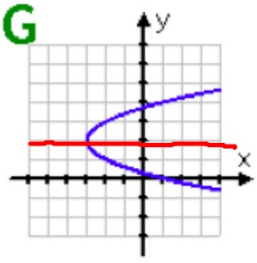
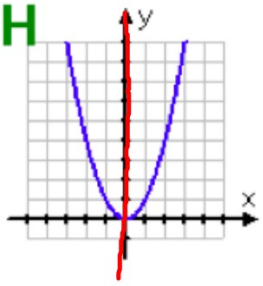
D

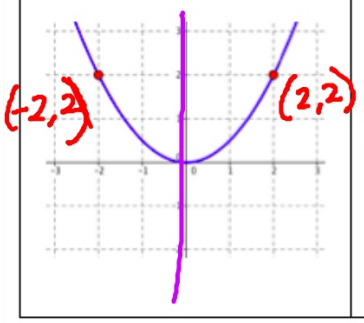
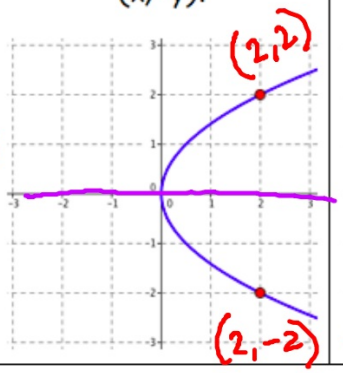
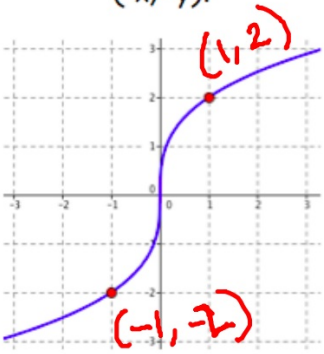
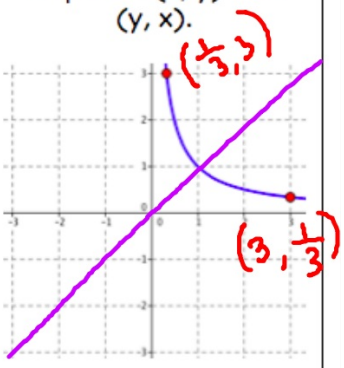


$y=x$

$y=-x$

Yes

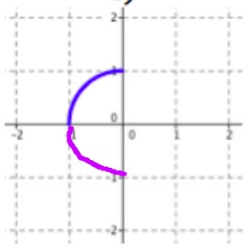
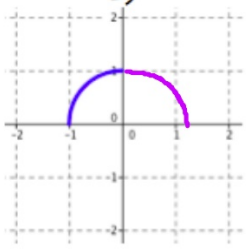
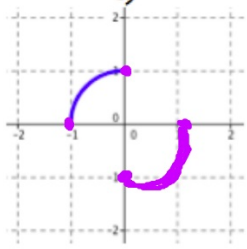
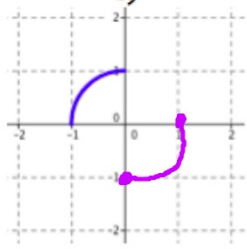
E  none Yes	**F**  none Yes	**G**  $y = 2$ No	**H**  $x = 0$ Yes

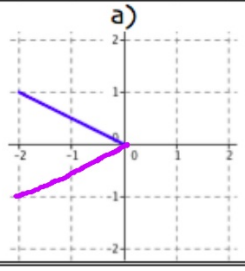
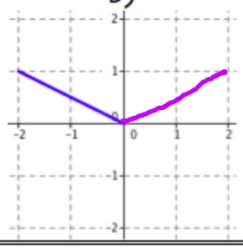
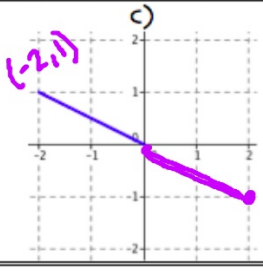
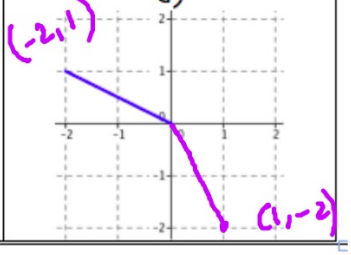
y-axis	x-axis	Origin	y=x
A graph has y-axis symmetry if it contains to points (x, y) and (-x, y).	A graph has x-axis symmetry if it contains to points (x, y) and (x, -y).	A graph has origin symmetry if it contains to points (x, y) and (-x, -y).	A graph has y=x symmetry if it contains to points (x, y) and (y, x).
			

Ex 3)

The graph of a curve contains the point (3, -4).

a) If the graph of the curve is symmetric about the y-axis, the graph will also contain the point (-3, -4).	b) If the graph of the curve is symmetric about the x-axis, the graph will also contain the point (3, 4).
c) If the graph of the curve is symmetric about the origin, the graph will also contain the point (-3, 4).	d) If the graph of the curve is symmetric about the line y=x, the graph will also contain the point (-4, 3).

x-axis	y-axis	origin	y = x
a) 	b) 	c) 	d) 

x-axis	y-axis	origin	y=x
a) 	b) 	c) 	d) 

These replacements will produce an equivalent function if the graph has that type of symmetry

y-axis: replacing x with -x

x-axis: replacing y with -y

origin: replacing x with -x and y with -y

y = x: replacing x with y

Ex 5) Determine algebraically if the functions below have y-axis, x-axis, origin or y=x symmetry.

a) $y = |x| - 2$

y-axis ✓: $y = |-x| - 2$

$y = |x| - 2$

x-axis: $-y = |x| - 2$

$y = -|x| + 2$

Origin: $-y = |-x| - 2$

$-y = |x| - 2$

$y = -|x| + 2$

y=x: $x = |y| - 2$

$|y| = x + 2$

b) $y = 2x^3 - x$

y-axis: $y = 2(-x)^3 - (-x)$

$y = -2x^3 + x$

x-axis: $-y = 2x^3 - x$

$y = -2x^3 + x$

Origin ✓: $-y = 2(-x)^3 - (-x)$

$y = 2x^3 - x$

y=x: $x = 2y^3 - y$

c) $y = x\sqrt{x^2 - 9}$

y-axis: $y = -x\sqrt{(x)^2 - 9}$

$y = -x\sqrt{x^2 - 9}$

x-axis: $-y = x\sqrt{x^2 - 9}$

$y = -x\sqrt{x^2 - 9}$

Origin ✓: $-y = -x\sqrt{(-x)^2 - 9}$

$-y = -x\sqrt{x^2 - 9}$

$y = x\sqrt{x^2 - 9}$

d) $y = 4x^4 - x^2 + 5$

y-axis ✓ $y = 4(-x)^4 - (-x)^2 + 5$

$y = 4x^4 - x^2 + 5$

e) $g(x) = \frac{5x^3 - x}{x^2 + 4}$

y-axis $y = \frac{5(-x)^3 - (-x)}{(-x)^2 + 4}$

$y = \frac{-5x^3 + x}{x^2 + 4}$

Origin ✓

$-y = \frac{5(-x)^3 - (-x)}{(-x)^2 + 4}$

$-y = \frac{-5x^3 + x}{x^2 + 4}$

$y = \frac{5x^3 - x}{x^2 + 4}$

f) $f(x) = x^3 - 2x^2 + x - 1$

y-axis $y = (-x)^3 - 2(-x)^2 + (-x) - 1$

$y = -x^3 - 2x^2 - x - 1$

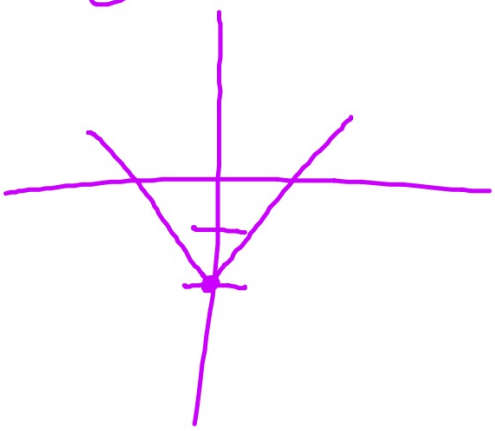
origin

$-y = (-x)^3 - 2(-x)^2 + -x - 1$

$y = x^3 - 2x^2 + x + 1$

none

$y = |x| - 2$



$y = x^2 - 2$