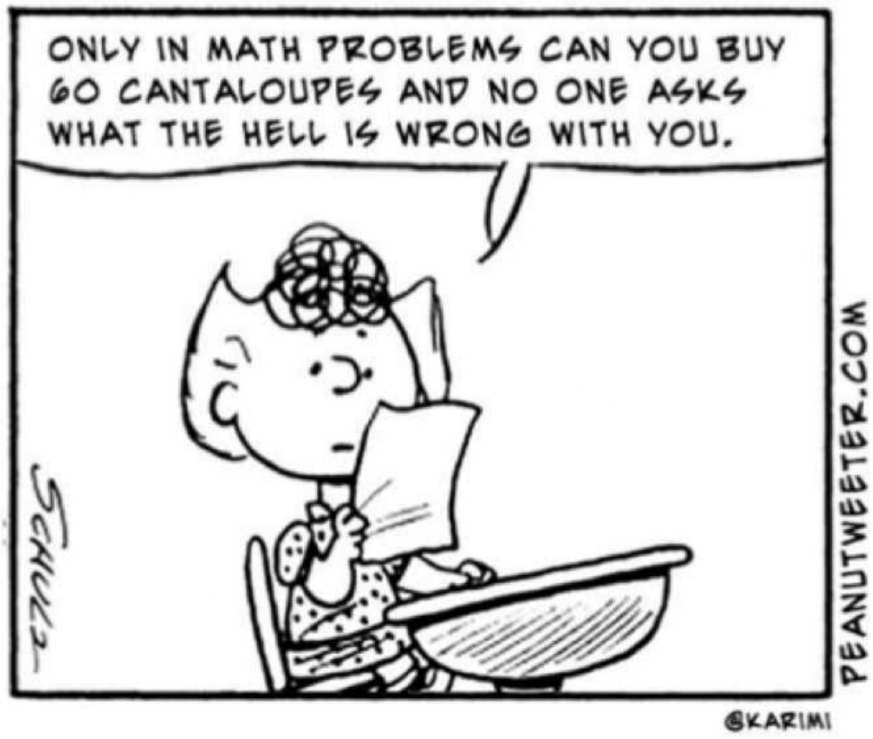
Quadratic Words Problems



40. 3/4 mult. of 2

42. $\frac{7 \pm \sqrt{41}}{4}$

42.) $4(x-1)^2 = 6x+2$

$4(x^2-2x+1) - 6x - 2 = 0$

$4x^2 - 14x + 2 = 0$

$\frac{7}{4} \pm \frac{\sqrt{41}}{4}$

$2(2x^2-7x+1) = 0$

$a = 2$

$b = -7$

$c = 1$

$x = \frac{7 \pm \sqrt{49-4(2)(1)}}{2(2)}$

$\frac{7 \pm \sqrt{41}}{4}$

40) $16t^2 - 24t + 9 = 0$

$(4t - 3)(4t - 3) = 0$

$t = 3/4$

mult of 2

33. $6r^2 + 6r + 12 = 0$

$6(r^2 + r + 2) = 0$

$a = 1$
$b = 1$
$c = 2$

$$x = \frac{-1 \pm \sqrt{1^2 - 4(1)(2)}}{2(1)}$$

$$= \frac{-1 \pm \sqrt{-7}}{2} = \frac{-1}{2} \pm \frac{i\sqrt{7}}{2}$$

30.) $A = \frac{1}{2}bh$

(37) 2 $40 = \frac{1}{2}(x)(x+4)$

$80 = x(x+4)$

$0 = x^2 + 4x - 80$

$0 = x^2 + 4x + 4 - 4 - 80$

$\sqrt{84} = \sqrt{(x+2)^2}$

$2\sqrt{21} = |x+2|$

$-2 \pm 2\sqrt{21} = x$

$-2 + 2\sqrt{21}$

~~$-2 - 2\sqrt{21}$~~

HW:

Quad word problems WKST : 2, 4, 6, 7, 8, 9, 10 AND Set C

Translating Words to Mathematical Expressions

Addition

plus	"a number plus 2"
and	"3 and a number"
added to	"8 added to a number"
greater than	"3 greater than a number"
more than	"3 more than a number"
increased by	"a number increased by 2"
sum of	"The sum of the length and width"

Translating Words to Mathematical Expressions

Subtraction

minus	"a number minus 2" $x-2$
difference between	"the difference between a number and 8"
from	"2 from a number"
less	"a number less 3"
less than	"3 less than a number" $x-3$
fewer than	"2 fewer than a number" $x-2$
decreased by	"a number decreased by 2" $x-2$
take away	"a number take away 2"

Translating Words to Mathematical Expressions

Multiplication	**times**	"5 times a number"
	product	"The product of 3 and a number"
	double, triple, etc.	"double a number"
	twice	"twice a number"
	of (fractions of)	"three-fourths of a number"

Translating Words to Mathematical Expressions

Division	**quotient of**	"The quotient of 5 and a number"	$\frac{5}{x}$
	Half of	"half of a number"	
	per	"The price is $8 per 50"	
Equals	**Is, is the same as, gives, will be, was, is equivalent to, result**		

ex: Express each statement as a mathematical expression.

a) 7 more than 3 times a number

$7+3x$

b) two less than three times a number

$3x-2$

c) the sum of a number and five times the same number

$x+5x$

ex: Express each statement as a mathematical expression.

d) the sum of a number and the square of the same number

$n + n^2$

e) the product of two consecutive integers

$n(n+1)$

"No Equation Given" Procedure

1) Read the problem carefully. Decide what unknown numbers are asked for and what facts are known. Making a sketch may help.

2) Choose a variable and use it with the given facts to represent the unknowns decribed in the problem.

3) Reread the problem and write an equation that represents relationships among the numbers in the problem.

4) Solve the equation and find the unknowns.

5) Reread the question and check your results with the words of the problem. State the answer(s). Attach units when necessary.

Set up a quadratic equation and solve algebraically.

n: number

1) The square of a number decreased by three times the number is 28. Find all possible values for the number.

$$n^2 - 3n = 28$$

$$n^2 - 3n - 28 = 0$$

$$(n-7)(n+4) = 0$$

$$\boxed{n = 7, -4}$$

Set up a quadratic equation and solve algebraically.

2) If a positive number is subtracted from its square, the result is 72. Find all possible values for the number.

less than

$n^2 - n = 72$

$n^2 - n - 72 = 0$

$(n-9)(n+8) = 0$

$n = 9, -8$

Set up a quadratic equation and solve algebraically.

3) The length of a rectangle is 7 less than its width. Find the perimeter if the area is 60 square units.

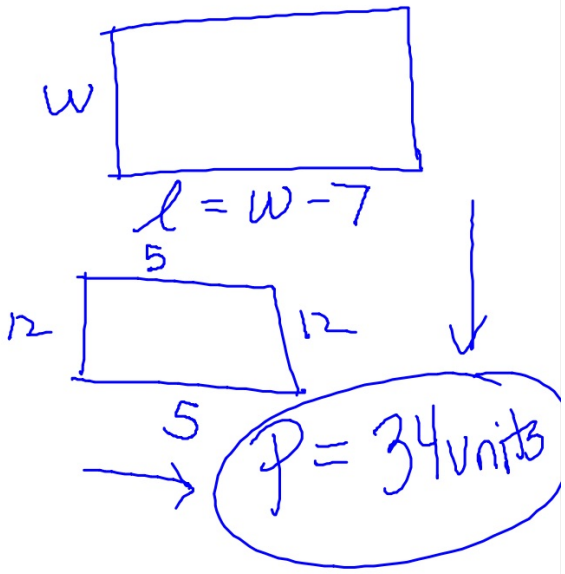
$$lw = 60$$

$$w(w-7) = 60$$

$$w^2 - 7w - 60 = 0$$

$$(w+5)(w-12) = 0$$

$w = 12$ units



$P = 34$ units

Set up a quadratic equation and solve algebraically.

4) The length of a rectangle is 6 cm more than three times its width. The area of the rectangle is 144 cm^2. Find the dimensions of the rectangle.

Consecutive integers	Consecutive odd	Consecutive even
n	n	n
$n+1$	~~$n+1$~~	~~$n+1$~~
$n+2$	$n+2$	$n+2$
$n+3$	~~$n+3$~~	~~$n+3$~~
$\vdots$	$n+4$	$n+4$

Set up a quadratic equation and solve algebraically.

5) Find two consecutive negative integers whose product is 90.

$n(n+1)=90$

$n^2+n-90=0$

$(n+10)(n-9)=0$

$n=-10, \cancel{9}$

$-10, -9$

Set up a quadratic equation and solve algebraically.

6) Two consecutive positive odd integers have a product of 99. Find the numbers.

Set up a quadratic equation and solve algebraically.

7) The product of two consecutive even integers is 48. What are the integers?

$$n(n+2) = 48$$

$$n^2 + 2n - 48 = 0$$

$$(n+8)(n-6) = 0$$

$-8, -6$ or $6, 8$

Set up a quadratic equation and solve algebraically.

8) A right triangle has sides that are consecutive even integers. Find the numbers.

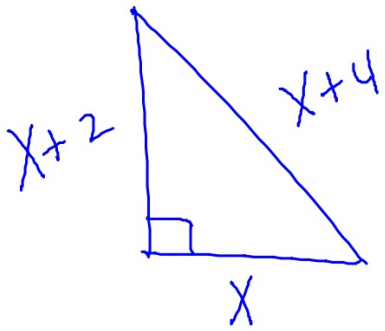
$x^2 + (x+2)^2 = (x+4)^2$

$x^2 + x^2 + 4x + 4 = x^2 + 8x + 16$

$x^2 - 4x - 12 = 0$

$(x-6)(x+2) = 0$

$6 \quad \cancel{-2}$



6, 8, 10

Set up a quadratic equation and solve algebraically.

9) Find three consecutive positive odd integers such that the product of the first and the third is 4 less than 7 times the second. Set up a quadratic equation and solve algebraically.

$n, n+2, n+4$

$n(n+4) = 7(n+2) - 4$

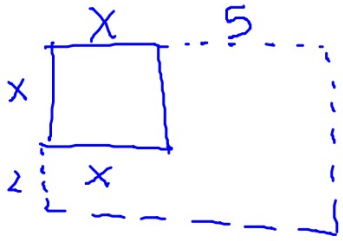
5, 7, 9

Set up a quadratic equation and solve algebraically.

10) A frame for a picture is 2.5 inches wide. The picture enclosed by the frame is 5 inches longer than it is wide. If the area of the picture itself is 300 square inches, determine the outer dimensions of the frame.

Set up a quadratic equation and solve algebraically.

11) A square field has 5 meters added to its length and 2 meters added to its width. The new field has an area of 130 square meters. Find the length of a side of the original field.



$(x+5)(x+2)=130$

$x^2+7x+10-130=0$

$x^2+7x-120=0$

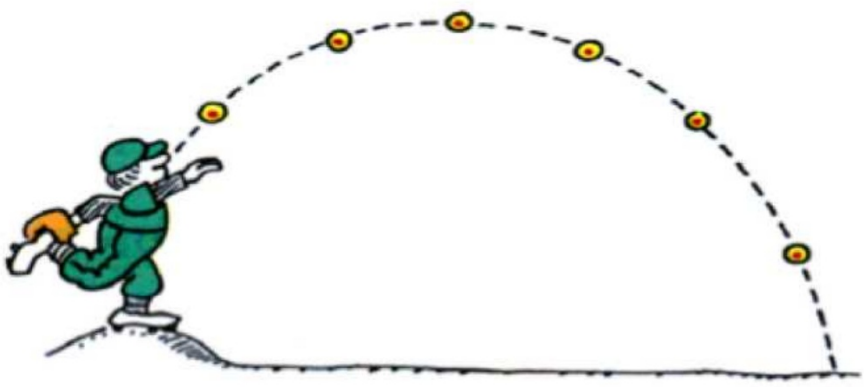
$(x+15)(x-8)=0$

~~-15~~ $8 m = x$

Set up a quadratic equation and solve algebraically.

12) A decorator plans to place a rug in a 9 m by 12 m room so that a uniform strip of flooring around the rug will remain uncovered. How wide will this strip be if the area of the rug is to be half the area of the room?

Falling Objects



Falling Objects - "things to consider..."

1) Are you finding a height or a time?

2) If you are finding a height plug in a time. If you are finding a time plug in a height.

3) The maximum height occurs at the vertex → (a, b). The maximum height is the y-coordinate of the vertex.

4) The word "initial" implies $t = 0$ (time = 0).

5) The word "ground" implies $h(t) = 0$ (height = 0).

1. The height of a rocket launched upward from a 160-foot cliff is modeled by $h(t) = -16t^2 + 48t + 160$ where h(t) is the height in feet and t is the time in seconds.

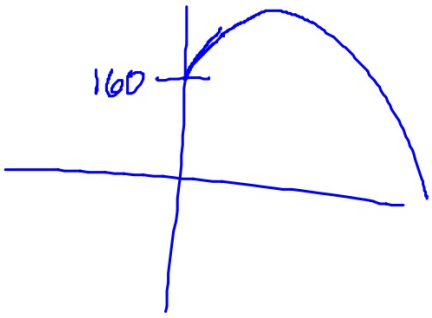
a) What is the initial height of the rocket? What is the height of the rocket after 1 sec?

$t = 0$

$h(0) = 160 \text{ ft}$

$t = 1$

$h(1) = -16 + 48 + 160 = 192 \text{ ft}$



1. The height of a rocket launched upward from a 160-foot cliff is modeled by $h(t) = -16t^2 + 48t + 160$ where h(t) is the height in feet and t is the time in seconds.

b) At what time does the rocket reach its maximum height?

$$t = \frac{-b}{2a} = \frac{-48}{2(-16)} = \frac{3}{2} \text{ sec}$$

1. The height of a rocket launched upward from a 160-foot cliff is modeled by $h(t) = -16t^2 + 48t + 160$ where h(t) is the height in feet and t is the time in seconds.

c) What is the maximum height?

$t = \frac{3}{2}$

$h\left(\frac{3}{2}\right) = -16\left(\frac{3}{2}\right)^2 + 48\left(\frac{3}{2}\right) + 160$

$= \frac{-16 \cdot 9}{4} + 72 + 160 = -36 + 232 = 196$ ft

1. The height of a rocket launched upward from a 160-foot cliff is modeled by $h(t) = -16t^2 + 48t + 160$ where h(t) is the height in feet and t is the time in seconds.

d) At what time does the rocket hit the ground?

$0 = -16t^2 + 48t + 160$

$0 = -16(t^2 - 3t - 10)$

$0 = -16(t-5)(t+2)$

$t = 5$ sec ~~$t = -2$~~

2. The height of a flare fired from the deck of a ship in distress can be modeled by $h = -16t^2 + 112t + 56$ water in feet and t is the time in seconds. At what time(s) will the flare be at a height of 56 feet?

$56 = -16t^2 + 112t + 56$

$0 = -16t^2 + 112t$

$0 = -16t(t - 7)$

$t = 0, 7 \text{ sec}$

3. During a game of golf Kayla hits her ball out of a sand trap. The height of the golf ball is modeledby the equation,

$$h = -16t^2 + 20t - 4,$$

where h is the height in feet and t is the time in seconds since the ball was hit. How long was the ball in the air?

4. The profits for Mr. Unlucky's company can be modeled by the equation $P = -3t^2 + 18t - 4$, where P is the amount of profit in thousands of dollars and t represents the number of years of operation. He realizes his company is on the downturn and wishes to sell before he ends up in debt.

a) When will Unlucky's business show a maximum profit?

4. The profits for Mr. Unlucky's company can be modeled by the equation $P = -3t^2 + 18t - 4$, where P is the amount of profit in thousands of dollars and t represents the number of years of operation. He realizes his company is on the downturn and wishes to sell before he ends up in debt.

b) What is the maximum profit?

4. The profits for Mr. Unlucky's company can be modeled by the equation $P = -3t^2 + 18t - 4$, where P is the amount of profit in thousands of dollars and t represents the number of years of operation. He realizes his company is on the downturn and wishes to sell before he ends up in debt.

c) At what time will it be too late to sell his business? (when will he start losing money?)

5. An object is moving according to the distance equation

$$d = 2t^2 + 6t,$$

where d is in meters and t is in seconds. How long will it take for the object to travel 108 meters?

6. Abigail tosses a coin off a bridge into a stream. The distance, in feet, the coin is above the water is modeled by the equation $y = -16x^2 + 96x + 112$ where x represents time in seconds.

a) What is the greatest height of the coin?

6. Abigail tosses a coin off a bridge into a stream. The distance, in feet, the coin is above the water is modeled by the equation $y = -16x^2 + 96x + 112$ where x represents time in seconds.

b) How much time will it take for the coin to hit the water?

Review

ex: Solve.

a) $7x^2 + 14x = 0$

Review

ex: Solve.

b) $10(x-3)^2-1=0$

Review

ex: Solve.

c) $11x^2 - 2x - 9 = 3\left(x^2 - 2\right)$

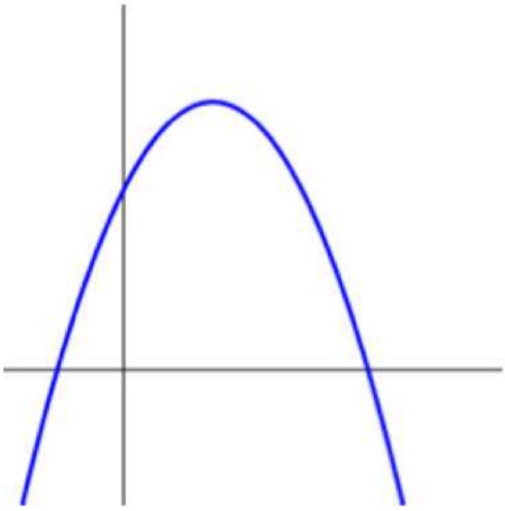
Review

ex: Solve.

d) $x(x+6)=-15$

Review

ex: Consider the quadration function, $y = ax^2 + bx + c$ pictured below. Determine whether each quantity is positive, negative or zero.



a) discriminant

b) a

c) c