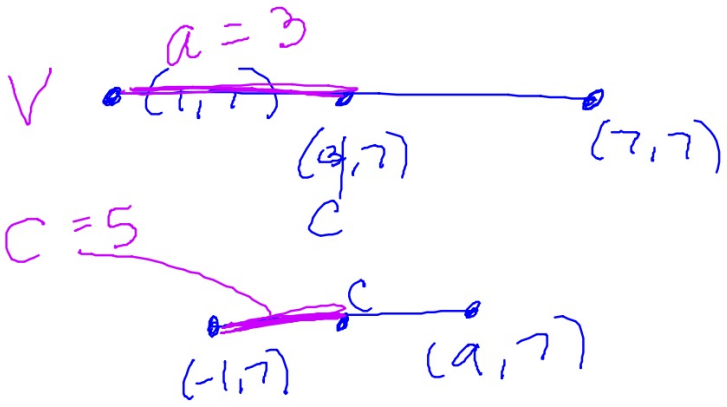
20.) Hyp.

$$\frac{(x-4)^2}{9} - \frac{(y-7)^2}{16} = 1$$



$$c^2 = a^2 + b^2$$

$$25 = 9 + b^2$$

$$16 = b^2$$

40.) $9y^2 - x^2 - 54y + 8x + 56 = 0$

$9y^2 - 54y - x^2 + 8x$

$9(y^2 - 6y + \underline{9}) - (x^2 - 8x + \underline{16})$

$\frac{9(y-3)^2 - (x-4)^2 = 9}{9}$

$\boxed{\frac{(y-3)^2}{1} - \frac{(x-4)^2}{9} = 1}$

Asymp.

$y - 3 = \pm\frac{1}{3}(x-4)$

$C(4,3)$

$c = \sqrt{10}$

$(4, 3+\sqrt{10})(4, 3-\sqrt{10})$

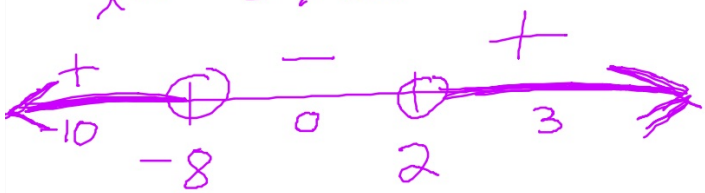
Polynomial and Rational Inequalities

Polynomial Inequalities: Solve. State the answer in interval notation.

1) $x^2 + 6x - 16 > 0$ (+)

$(x+8)(x-2) > 0$

$x = -8, 2$



$(-\infty, -8) \cup (2, \infty)$

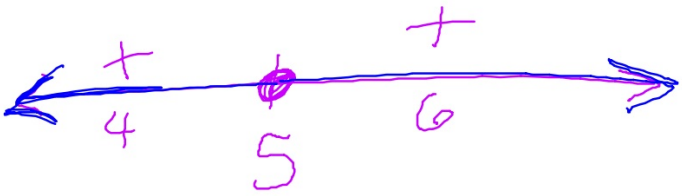
Steps

1. Must have zero on one side
2. Find the real zeros
3. Make a number line Test each region.
4. State the answer

2) $x^2 - 10x + 25 \geq 0$

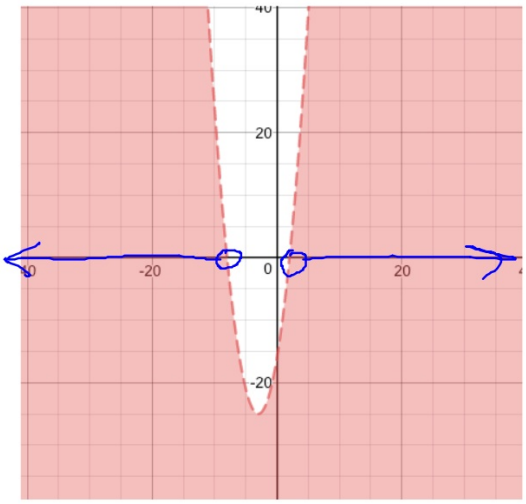
$(x-5)^2 \geq 0$

$x = 5$

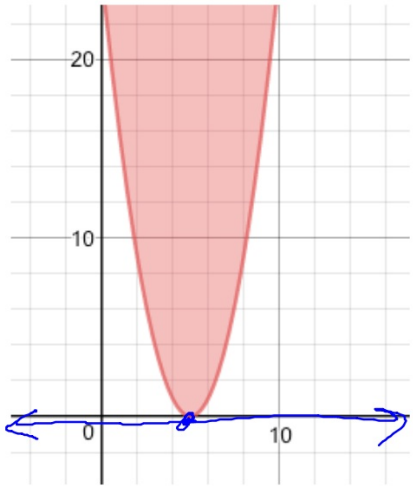


$(-\infty, \infty)$

Graph for Example 1



Graph for example 2

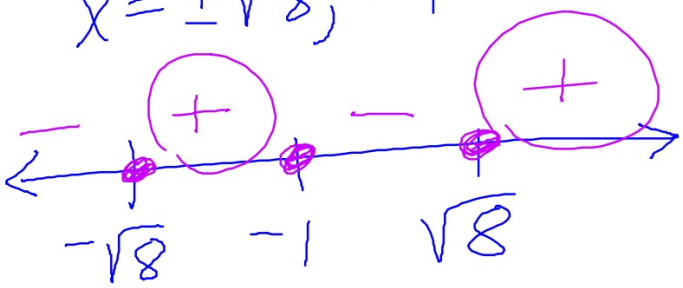


3) $(x^3 + x^2)(-8x - 8) \geq 0$

$x^2(x+1) - 8(x+1) \geq 0$

$(x^2 - 8)^1(x+1)^1 \geq 0$

$x = \pm\sqrt{8}, -1$

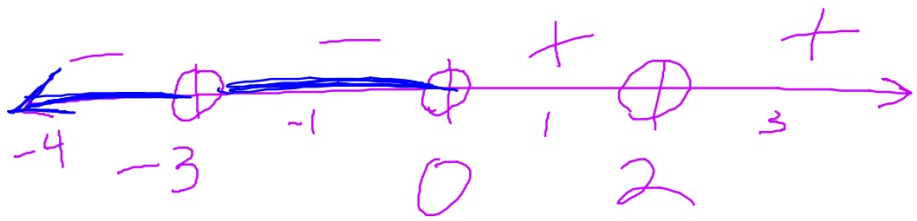


$[-\sqrt{8}, -1] \cup [\sqrt{8}, \infty)$

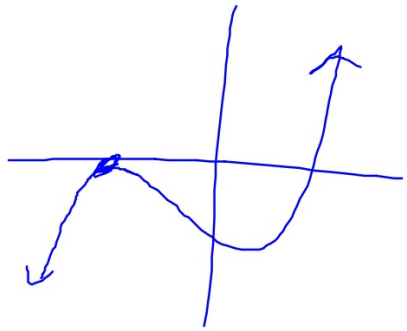
The factors have odd multiplicity.

At each zero, the signs will change because if this was a function, the graph would cross the x-axis.

4) $x(x-2)^2(x+3)^4 < 0$



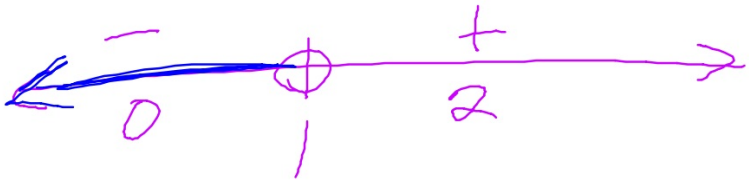
$(-\infty,-3)\cup(-3,0)$



5) $x^3 - 1 < 0$ negative imaginary

$(x-1)(x^2+x+1) < 0$

$x = 1$ (only real zero)



$(-\infty, 1)$

Solving Rational Inequalities

negative

1) $$\frac{x-1}{x+5} < 0$$

$x = 1, -5$

$(-5, 1)$

Look out for undefined values!

Steps

1) Need zero on one side and one term on the other side.

2) Find the critical numbers (real zeros of the numerator and den.)

3) Make a number line Test each region.

4) State the answer.

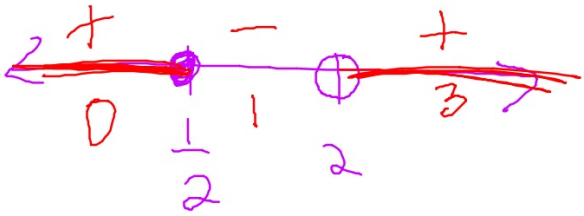
2) $\dfrac{6}{x-2} \geq -4$

positive

$$\frac{6}{x-2} + 4 \geq 0$$

$$\frac{6+4(x-2)}{x-2} \geq 0$$

$$\frac{4x-2}{x-2} \geq 0$$

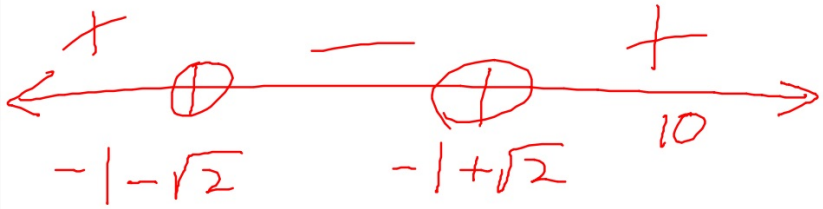


$$\left(-\infty, \tfrac{1}{2}\right] \cup (2, \infty)$$

3) $\dfrac{x^2+2x-1}{x^2+1} > 0$

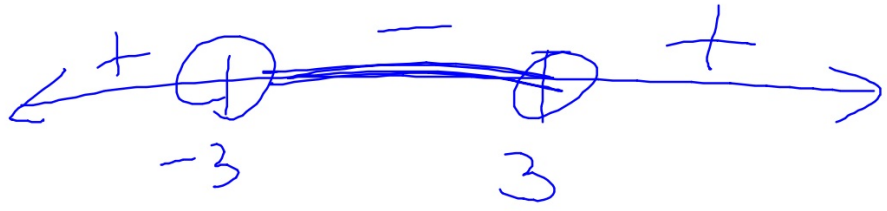
Quad formula

$-1 \pm \sqrt{2}$



$(-\infty, -1-\sqrt{2}) \cup (-1+\sqrt{2}, \infty)$

4) $$\frac{x^2+4}{x^2-9} < 0$$

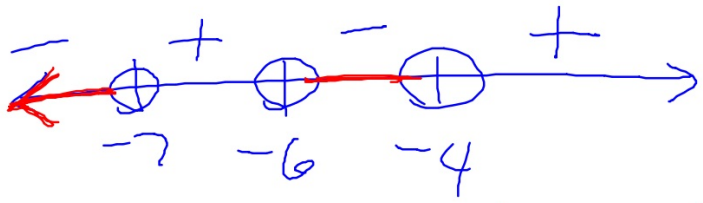


$(-3, 3)$

5) $\dfrac{3}{x+4} < \dfrac{1}{x+6}$

$$\frac{3}{x+4} - \frac{1}{x+6} < 0$$

$$\frac{2(x+7)}{(x+4)(x+6)} < 0$$



$(-\infty, -7) \cup (-6, -4)$

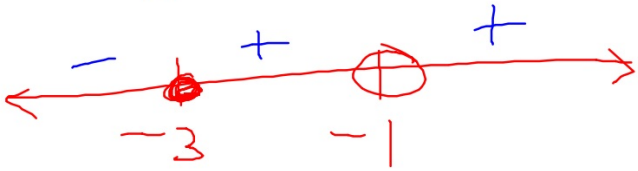
6) $\dfrac{x^2+4x+3}{x+1} \le 0$

$\dfrac{(x+3)\cancel{(x+1)}}{\cancel{x+1}} \le 0$

look for critical numbers before simplifying

$x = -3, -1$

$x + 3 \le 0$



$(-\infty, -3]$