9) $3^{2x} - 3^{x} - 2$

$u = 3^{x}$

$u^{2} = 3^{2x}$

$u^{2} - u - 2$

$(u-2)(u+1)$

$(3^{x}-2)(3^{x}+1)$

5.) $1-x^4$

$(1+x^2)(1-x^2)$

$(1+x^2)(1+x)(1-x)$

or $(1+x^2)(x+1)(-1)(x-1)$

or

$-(x^2+1)(x+1)(x-1)$

14.) $(x^2-5)^2-8(x^2-5)^1+16$

$u=(x^2-5)$

$u^2=(x^2-5)^2$

$u^2-8u+16$

$(u-4)^2$

$((x^2-5)-4)^2$

$(x^2-9)^2$

$(x+3)^2(x-3)^2$

20) $(3x+1)^{+2/3} - 9(3x+1)^{-1/3}$

$$(3x+1)^{-1/3}\left(3x+1-9\right)$$

$$\frac{3x-8}{(3x+1)^{1/3}} \text{ or } (3x+1)^{-1/3}(3x-8)$$

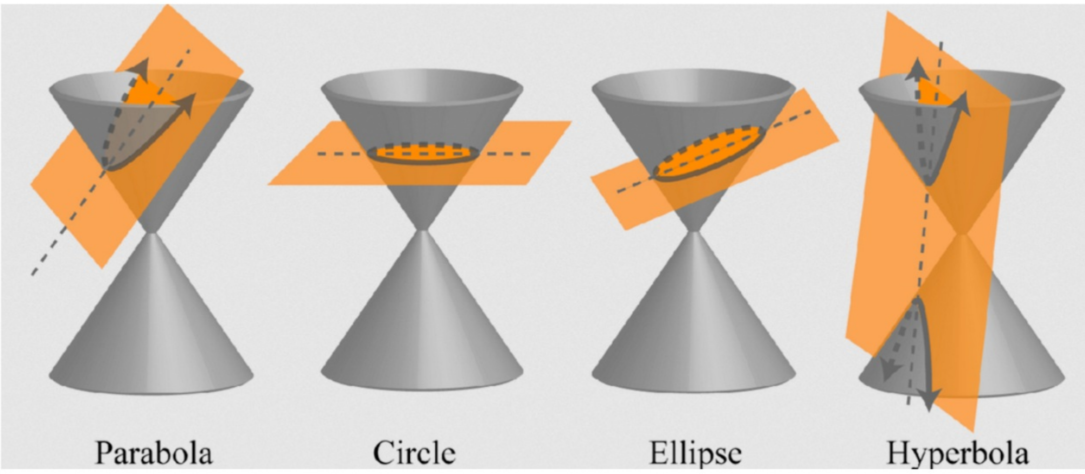
22) $5(x+1)^{-3} - 10(x+1)^{-2}$

$5(x+1)^{-3}\left(1 - 2(x+1)^{1}\right)$

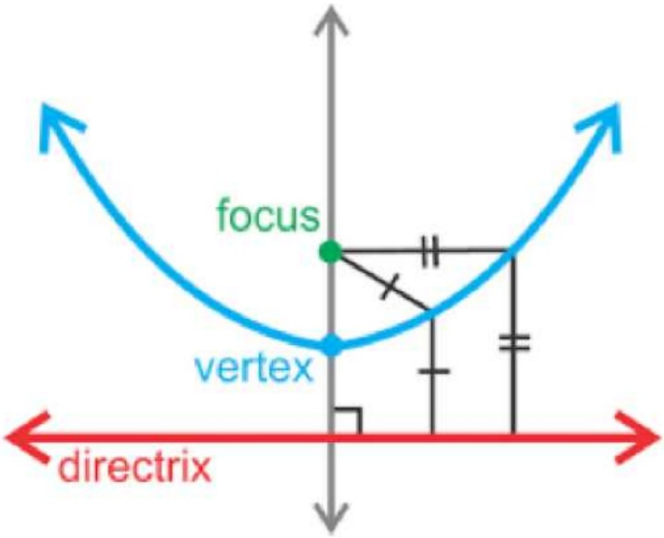
$1 - 2x - 2$

$$\frac{5(-2x-1)}{(x+1)^3} = \frac{-5(2x+1)}{(x+1)^3}$$

Conic Sections



parabola - locus of points equidistant from a focus and directrix



*The focus and directrix are not the actual graph. They are "graphing aids" that define the points on the parabola.

Standard Form

Opens: UP/DOWN

$$\boxed{(x-h)^2 = 4p(y-k)}$$

Opens: RIGHT/LEFT

$$\boxed{(y-k)^2 = 4p(x-h)}$$

Where:

vertex: (h,k)

p>0: Opens up or right

p<0: Opens down or left

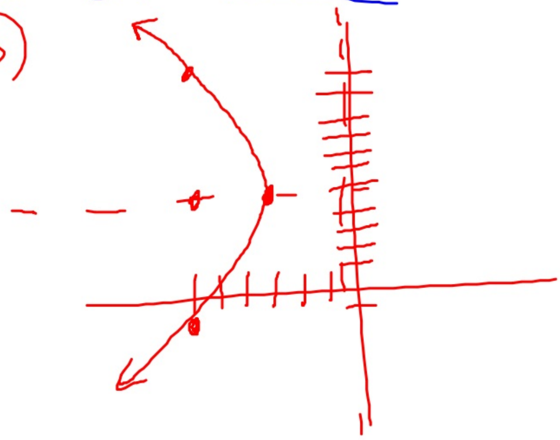
$|p|$: Distance from the focus to vertex and the distance from vertex to directrix

Write the equation of the parabola in standard form. Then sketch and state the vertex, focus, directrix, and axis of symmetry.

$$y^2 - 10y + 12x + 61 = 0$$

$$y^2 - 10y + \underline{25} = -12x - 61 + \underline{25}$$

$$(y-5)^2 = -12(x+3)$$



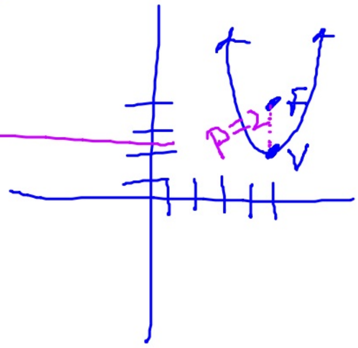
Vertex: (-3, 5)

Focus (-6, 5)

Directrix: x = 0

AOS: y = 5

Write an equation in standard form for the given characteristics of a parabola.



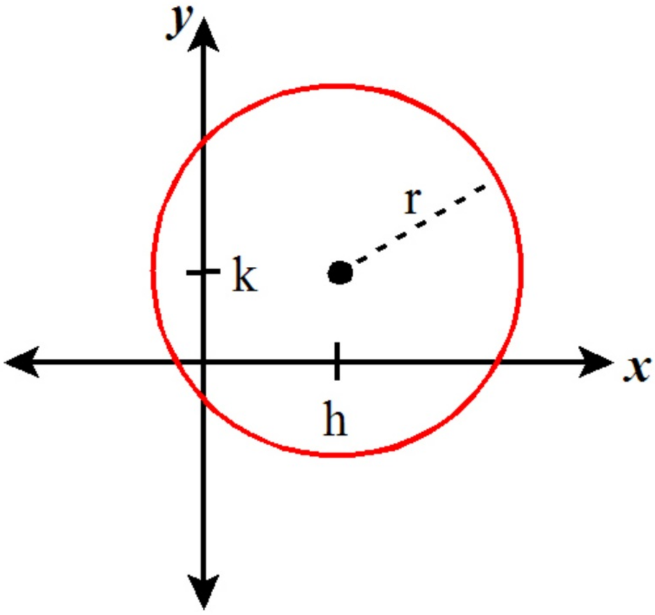
Vertex: (5, 2)

Focus: (5,4)

$$(x-5)^2 = 4p(y-2)$$

$$\boxed{(x-5)^2 = 8(y-2)}$$

***Circle:* Set of all points (x, y) in a plane equidistant from a fixed point called the center.**



$$(x - h)^2 + (y - k)^2 = r^2$$

(h,k): center

r: radius

Write the equation of the circle in standard form. Sketch and state the center and radius.

$x^2 + y^2 - 2x + 4y + 1 = 0$

$x^2 - 2x \underline{+1} + y^2 + 4y + \underline{4} = -1 + \underline{1} + \underline{4}$

$(x-1)^2 + (y+2)^2 = 4$

center $(1, -2)$

radius: 2

Write the equation of a circle given the following characteristics.

Endpoints of the diameter: (2, -5) and (6, -11)

(2, -5) center (6, -11)

midpoint

$(4, -8)$

$(x-4)^2+(y+8)^2=13$

radius

(4, -8) (6, -11)

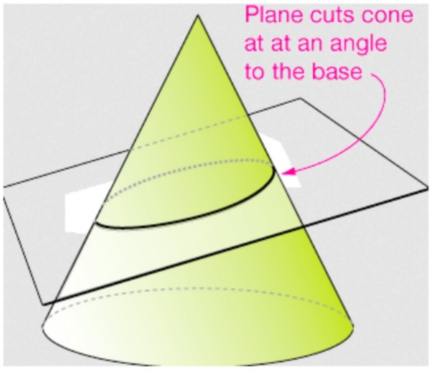
$d=\sqrt{(6-4)^2+(-11+8)^2}$

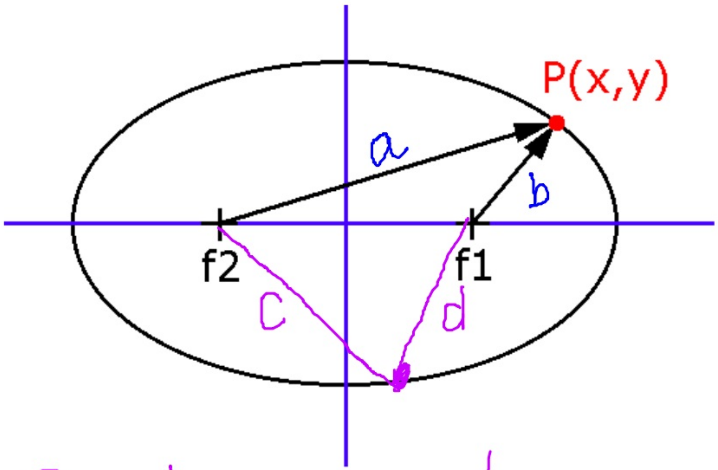
$d=\sqrt{4+9}=\sqrt{13}$

$r=\sqrt{13}$

$r^2=13$

***Ellipse:* Set of all points in a plane such that the sum of the distance between P and two fixed points, called the foci, is a constant.**

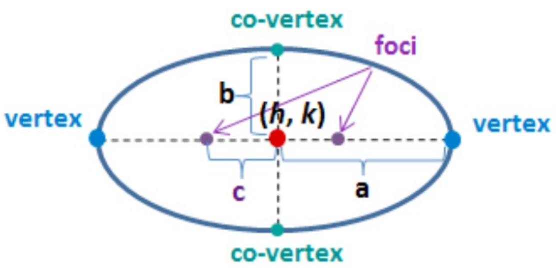


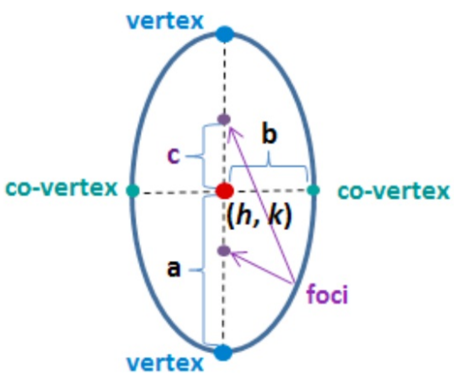


$a+b=c+d$

Standard equation of an ellipse

Horizontal Ellipse	**Vertical Ellipse**
At (0, 0): $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$	At (0, 0): $\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$
General: $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$	**General:** $\frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1$
$a^2-b^2=c^2$	$a^2-b^2=c^2$
Center: (h,k) **Foci:** $(h\pm c,k)$	**Center:** (h,k) **Foci:** $(h,k\pm c)$
Vertices: $(h\pm a,k)$ **Co-Vertices:** $(h,k\pm b)$	**Vertices:** $(h,k\pm a)$ **Co-Vertices:** $(h\pm b,k)$





c: Distance from a foci to a vertex

Major axis: Distance from vertices

Minor axis: Distance from co-vertices

Determine the center, vertices, covertices and foci. Then sketch the ellipse.

$$\frac{x^2}{16}+\frac{(y-3)^2}{25}=1$$

vertical ellipse

$a=5$

$b=4$

Center $(0,3)$

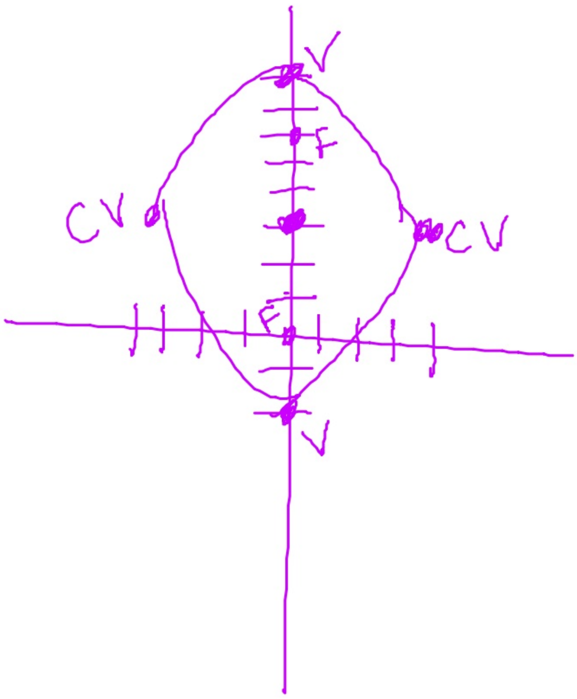
Vertices $(0,8)(0,-2)$

Covertices $(-4,3)(4,3)$

Foci $(0,0)(0,6)$

$c^2=a^2-b^2$

$c^2=25-16=9;\ c=3$



Write the equation of the ellipse in standard form. Then determine the vertices, covertices and foci.

$$2x^2 + y^2 + 8x - 16y + 52 = 0$$

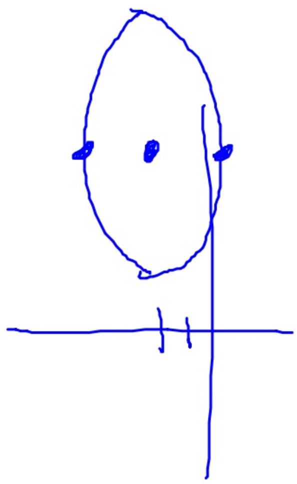
$$2x^2 + 8x + y^2 - 16y = -52$$

$$2(x^2 + 4x + \underline{4}) + (y^2 - 16y + \underline{64}) = -52 + \underline{8} + \underline{64}$$

$$2(x+2)^2 + (y-8)^2 = 20$$

$$\frac{(x+2)^2}{10} + \frac{(y-8)^2}{20} = 1$$

$$\frac{(x+2)^2}{10} + \frac{(y-8)^2}{20} = 1$$



Center: $(-2, 8)$

Vertices: $(-2, 8 \pm \sqrt{20})$

$(-2, 8+\sqrt{20})$ $(-2, 8-\sqrt{20})$

Covertices:

$(-2 \pm \sqrt{10}, 8)$

$(-2+\sqrt{10}, 8)$ $(-2-\sqrt{10}, 8)$

Foci:

$(-2, 8 \pm \sqrt{10})$

$(-2, 8+\sqrt{10})$ $(-2, 8-\sqrt{10})$

$c^2 = a^2 - b^2$

$c^2 = 20 - 10$

$c = \sqrt{10}$

Write an equation of an ellipse in standard form given the following characteristics.

Foci: (7, 9) and (-1, 9)
Covertices: (3, 12) and (3, 6)

Foci

center $(3,9)$

$(-1,9)$ $c=4$ $(7,9)$

$$\frac{(x-3)^2}{25}+\frac{(y-9)^2}{9}=1$$

$c^2=a^2-b^2$

$16=a^2-9$

$25=a^2$

CV

$(3,12)$

$b=3$ $(3,9)$

$b^2=9$

$(3,6)$

Write an equation of an ellipse in standard form given the following characteristics.

Endpoints of major axis: (4, 18) and (4, -4)
Endpoints of minor axis: (12, 7) and (-4, 7)

major axis

(4,18)

$a = 11$

(4,7)

$a^2 = 121$

(4,-4)

center (4,7)

$$\frac{(x-4)^2}{64} + \frac{(y-7)^2}{121} = 1$$

minor axis

$b = 8$ $b^2 = 64$

(-4,7) (12,7)