7.) $y = \frac{3}{x^2+1}$ $\quad 3 \neq 0$

Domain $\{x \mid x \in \mathbb{R}\}$

Zeros none

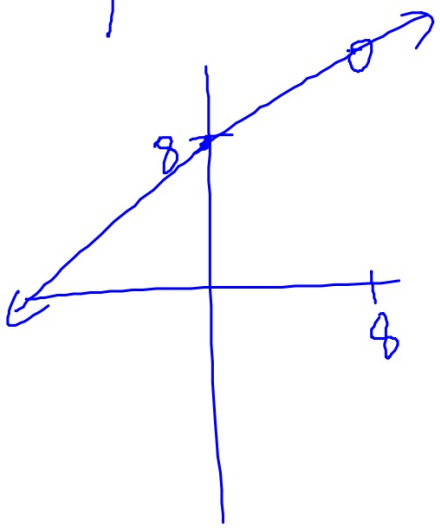
Yint $(0, 3)$

VA: none

HA: $y=0$

5e $g(x) = \frac{\cancel{(x-8)}(x+8)}{\cancel{x-8}} = x+8$

e.)



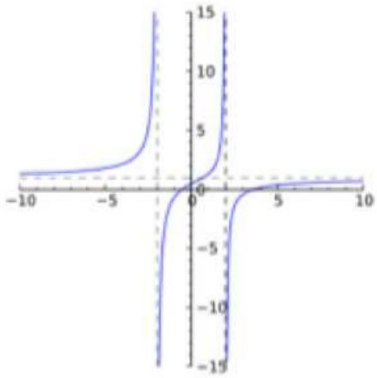
17.) $y = \frac{2(x-5)}{x+3}$

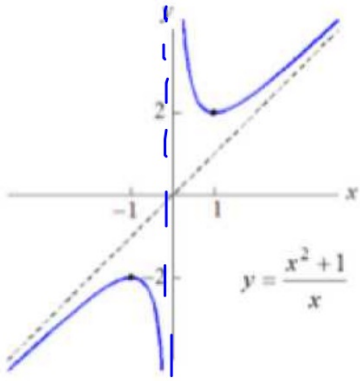
5.3 Graphs of Rational Functions - cont.

Sketching Requirements:

- x-intercepts
- y-intercept
- asymptotes
- holes
- plot at least one point per "piece"

Number of VA and the Number of "Pieces"





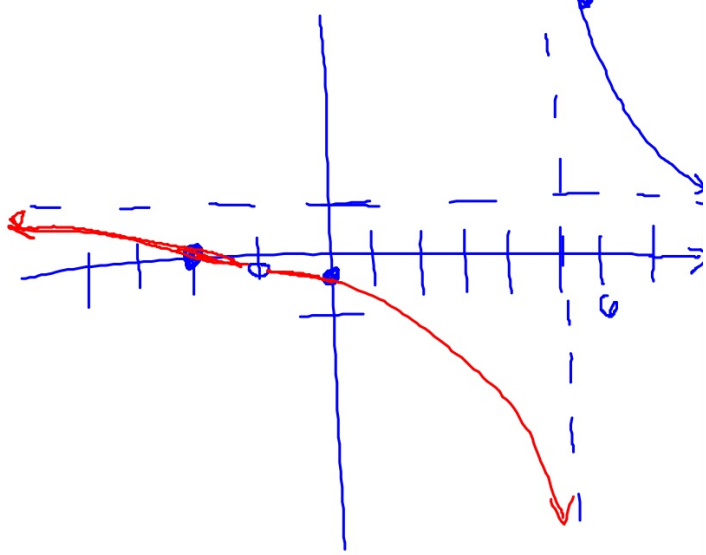
Number of Vertical Asymptotes	Number of "Pieces"
0	1
1	2
2	3
n	n + 1

ex: Sketch and state the domain and range.

a) $f(x) = \dfrac{x^2+3x+2}{x^2-4x-5} = \dfrac{(x+2)\cancel{(x+1)}}{(x-5)\cancel{(x+1)}} = \dfrac{x+2}{x-5}$

x	y
6	8

x-int	$(-2, 0)$
y-int	$(0, -2/5)$
VA	$x = 5$
HA	$y = 1$
SA	none
Holes	$(-1, -1/6)$
* Domain	$\{x \mid x \neq 5, -1\}$
Range	$\{y \mid y \neq 1, -1/6\}$

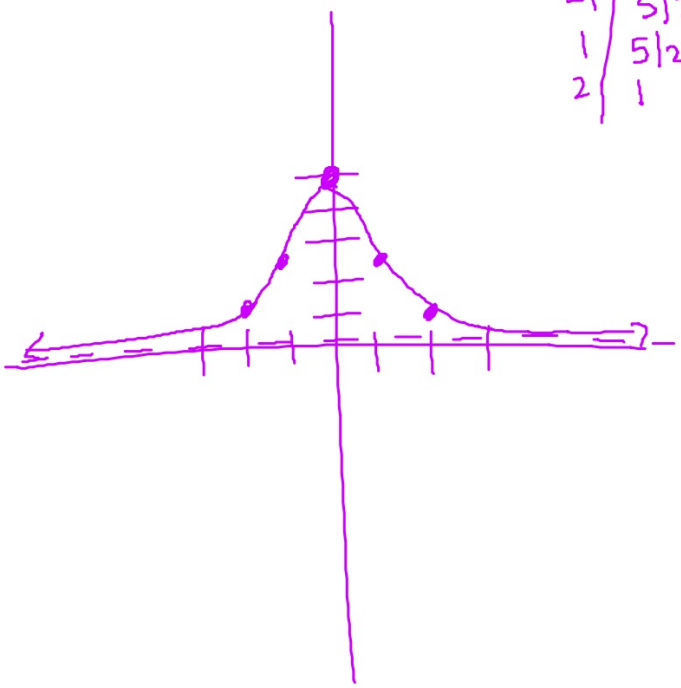


ex: Sketch.

b) $f(x) = \dfrac{5}{x^2+1}$

x-int	none
y-int	(0, 5)
VA	none
HA	y = 0
SA	none
Holes	none
Domain	$\{x \mid x \in R\}$
Range	$\{y \mid 0 < y \leq 5\}$

x	y
-2	1
-1	5/2
1	5/2
2	1



ex: Sketch.

c) $y = \dfrac{x^2}{x-2}$

2	1	0	0
		2	4
	1	2	✗

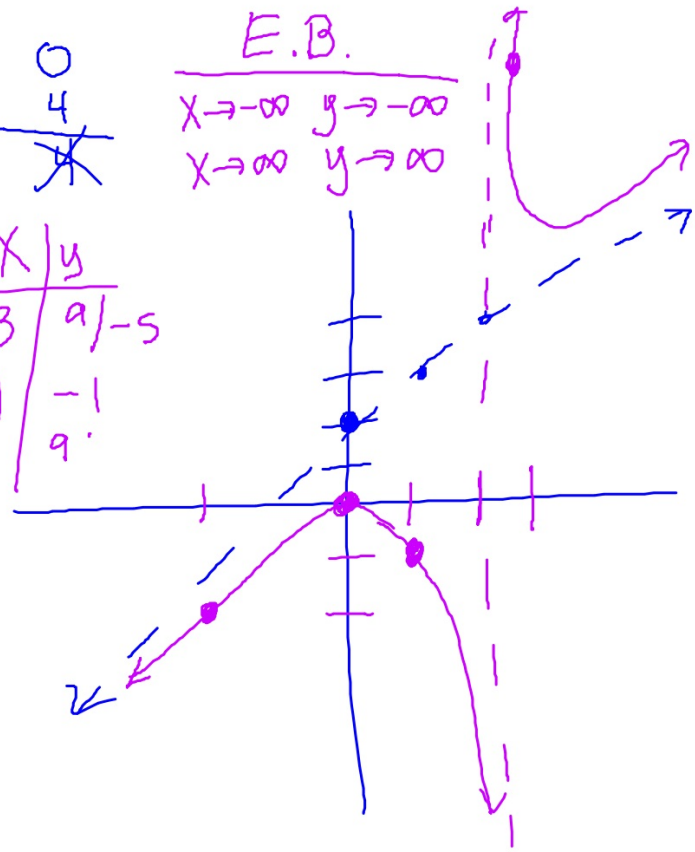
E.B.

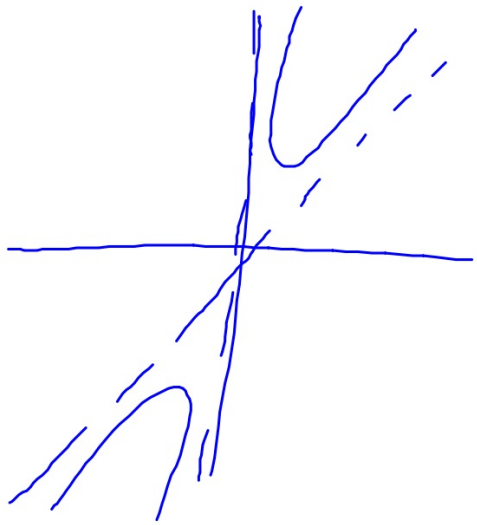
$x \to -\infty \quad y \to -\infty$

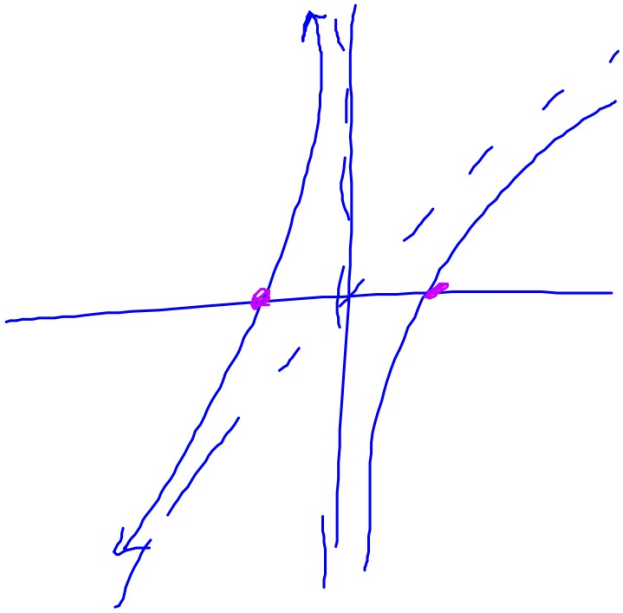
$x \to \infty \quad y \to \infty$

x	y
-3	9/-5
1	-1
3	9

x-int	(0, 0)
y-int	(0, 0)
VA	x = 2
HA	none
SA	y = x + 2
Holes	none
Domain	$\{x \mid x \neq 2\}$
Range	can't do







ex: Sketch.

Fun fact: VA's cannot be crossed
BUT HA's can be crossed

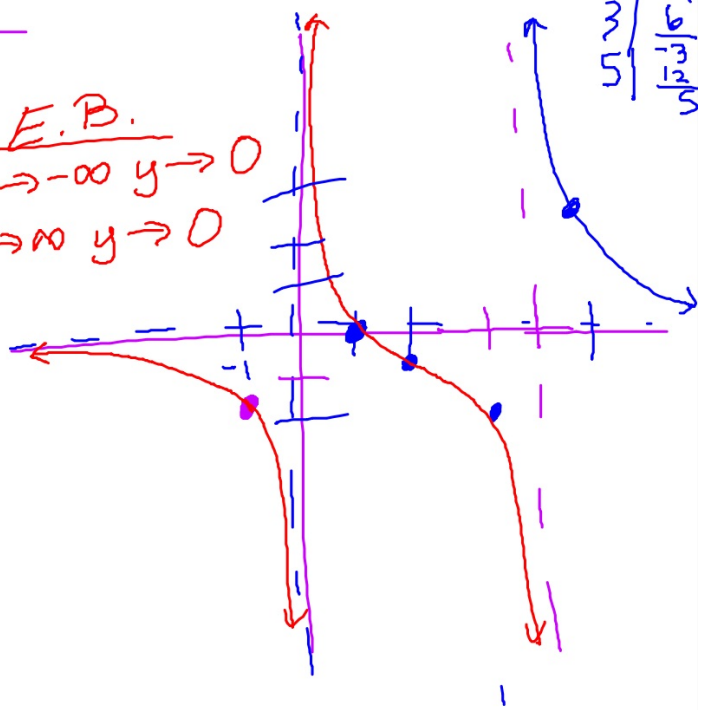
d) $y = \dfrac{3x-3}{x^2-4x} = \dfrac{3(x-1)}{x(x-4)}$

x-int	(1, 0)
y-int	none
VA	$x = 0, 4$
HA	$y = 0$
SA	none
Holes	none
Domain	$\{x \mid x \neq 0, 4\}$
Range	$\{y \mid y \in \mathbb{R}\}$

VA holes

E.B.
$x \to -\infty \quad y \to 0$
$x \to \infty \quad y \to 0$

x	y
-1	$\frac{-6}{5}$
2	$\frac{3}{-4}$
3	$\frac{6}{-3}$
5	$\frac{12}{5}$

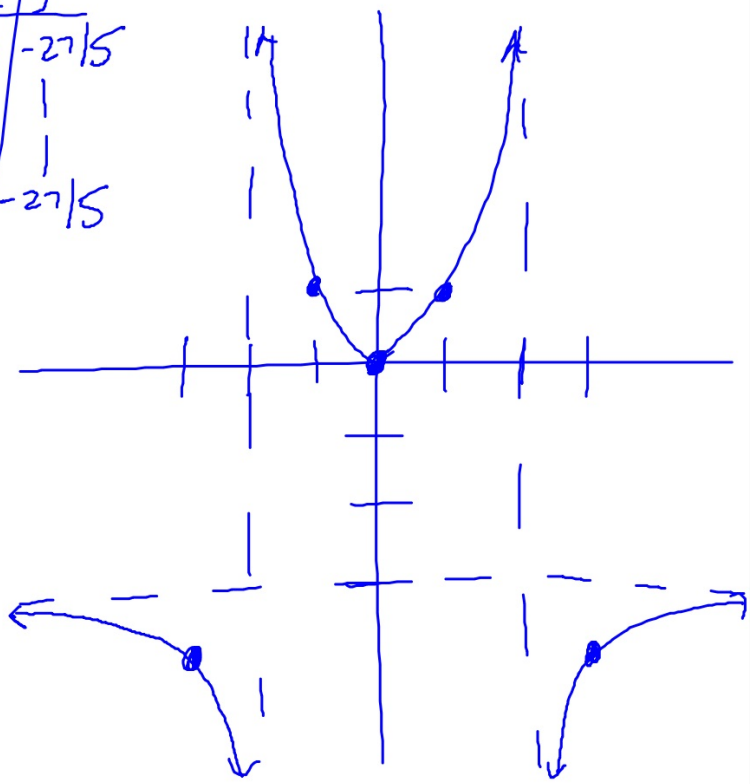


ex: Sketch.

e) $f(x) = \dfrac{-3x^2}{x^2 - 4}$

x	y
-3	-27/5
-1	1
1	1
3	-27/5

x-int	(0,0)
y-int	(0,0)
VA	$x = 2, -2$
HA	$y = -3$
SA	none
Holes	none
Domain	$\{x \mid x \neq 2, -2\}$
Range	$\{y \mid y < -3 \text{ or } y \geq 0\}$

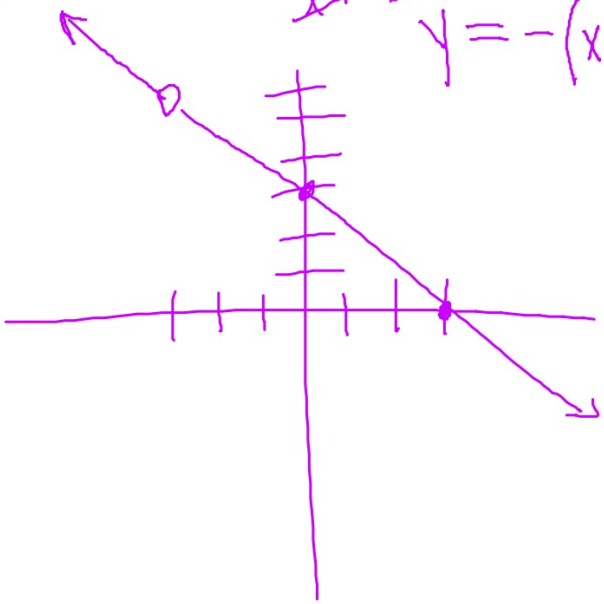


ex: Sketch.

f) $f(x) = \dfrac{9 - x^2}{x+3} = \dfrac{-(x^2-9)}{x+3} = \dfrac{-\cancel{(x+3)}(x-3)}{\cancel{x+3}}$

$y = -(x-3)$

x-int	(3, 0)
y-int	(0, 3)
VA	none
HA	none
SA	none
Holes	(-3, 6)
Domain	$\{x \mid x \neq -3\}$
Range	$\{y \mid y \neq 6\}$

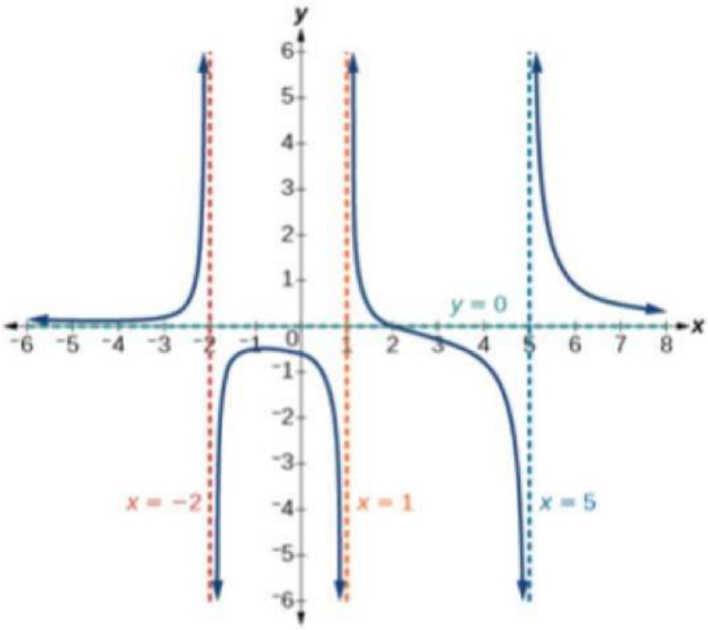


ex: State the end behavior.

a) $f(x) = \dfrac{x^2 + 3x + 2}{x^2 - 4x - 5}$

ex: State the end behavior.

b) $y = \dfrac{x^2}{x-2}$

ex: State the end behavior.

c)