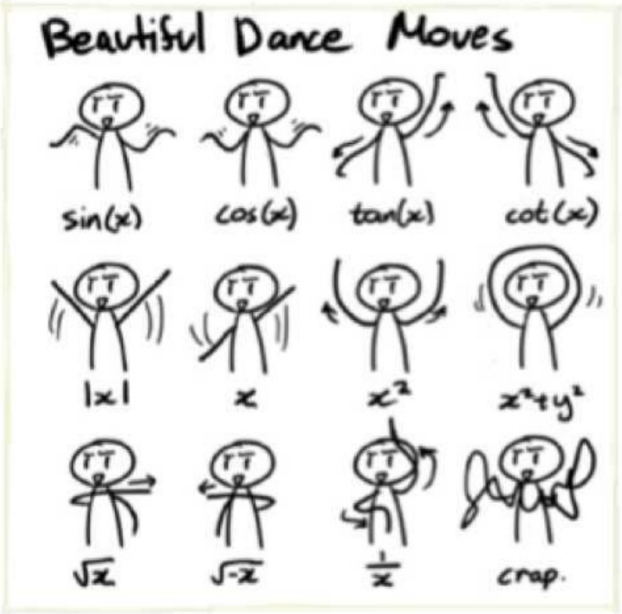
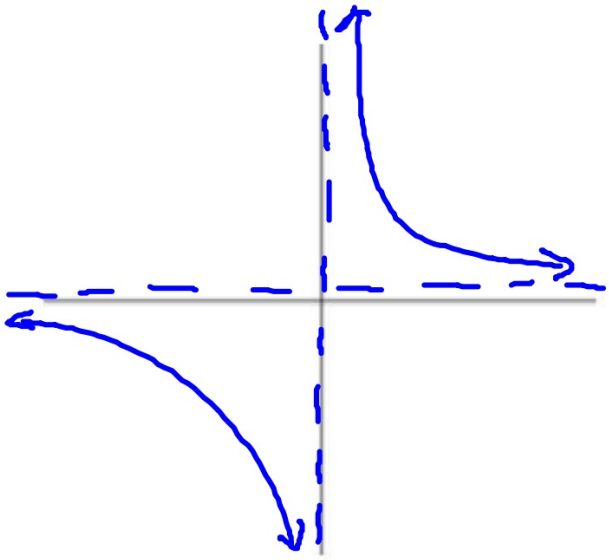
Library of Functions & Transformations



Check your LOF answers online!

6. Reciprocal

$f(x) = \frac{1}{x}$



Set

D: $\{x \mid x \neq 0\}$

R: $\{y \mid y \neq 0\}$

Interval

D: $(-\infty, 0) \cup (0, \infty)$

R: $(-\infty, 0) \cup (0, \infty)$

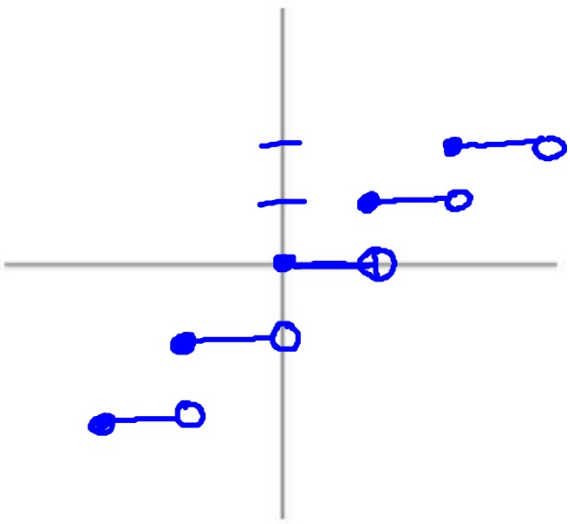
End Behavior

$x \to -\infty \quad f(x) \to 0$

$x \to \infty \quad f(x) \to 0$

9. Greatest Integer

$f(x) = \underline{[x]}$



Set

D: $\{x \mid x \in R\}$

R: $\{y \mid y \in Z\}$

Interval

D: $(-\infty, \infty)$

R: ~~_____~~

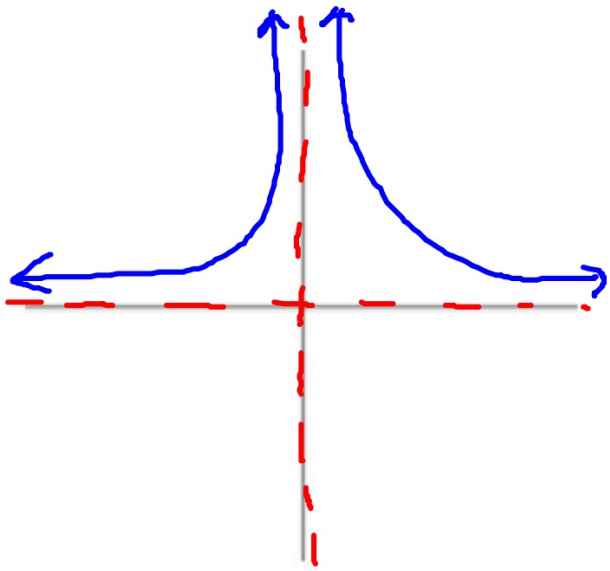
End Behavior

$x \to -\infty \quad f(x) \to \underline{-\infty}$

$x \to \infty \quad f(x) \to \underline{\infty}$

10. Reciprocal of a Square

$$f(x) = \frac{1}{x^2}$$



Set

D: $\{x \mid x \neq 0\}$

R: $\{y \mid y > 0\}$

Interval

D: $(-\infty, 0) \cup (0, \infty)$

R: $(0, \infty)$

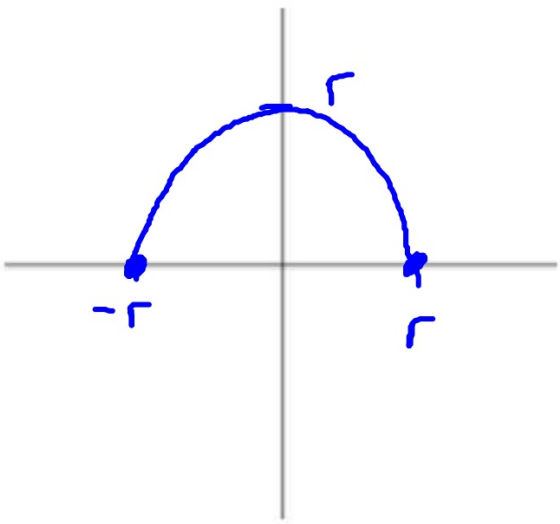
End Behavior

$x \to -\infty \quad f(x) \to 0$

$x \to \infty \quad f(x) \to 0$

11. Semicircle

$$f(x) = \sqrt{r^2 - x^2}$$



Set

D: $\{x \mid -r \le x \le r\}$

R: $\{y \mid 0 \le y \le r\}$

Interval

D: $[-r, r]$

R: $[0, r]$

End Behavior

$x \to -\infty \quad f(x) \to$ ~~____~~

$x \to \infty \quad f(x) \to$ ~~____~~

Function Transformations

$y = 3(x-1)^2 - 3$

$$\boxed{y = af\left(b\left(x-h\right)\right)+k}$$

Types of Transformations

- Shifts (vertical and horizontal)
- Dilations (vertical and horizontal)
- Reflections (about the x-axis, y-axis and origin)

*See printout.

Shifts

$$\boxed{y = af\left(b\left(x-h\right)\right)+k}$$

$(x - -2)$

$(x+2)$

Vertical Shifts

Consider: k

k>o up

k<o down

$y = (x-1)^2 + 3$

$(1, 3)$

Horizontal Shifts

Consider: h

h>o right

h<o left

ex: Describe the transformations.

$(-7, 4)$

$$f(x) = |x + 7| + 4$$

Translated ___7___ ***units*** ___left___

Translated ___4___ ***units*** ___up___

Dilations

$$\boxed{y = af\left(b\left(x-h\right)\right)+k}$$

Vertical

Consider: a

$|a| > 1$ vertical stretch

$|a| < 1$ vertical shrink

Horizontal

Consider: b

$|b| > 1$ horizontal shrink

$|b| < 1$ horizontal stretch

ex: Describe the transformations.

$$f(x) = \frac{1}{3}\sqrt{2x-5}+1$$

$$f(x) = \frac{1}{3}\sqrt{2\left(x-\frac{5}{2}\right)} + 1$$

Translated 5/2 *units* right

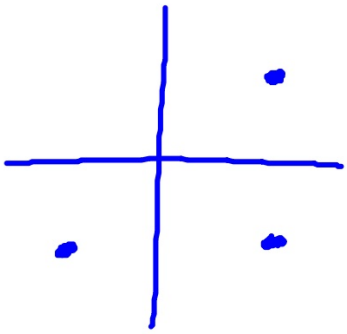
Translated 1 *units* up

Vertical shrink *by a factor of* 1/3

Horizontal shrink *by a factor of* 2

Reflections

$$\boxed{y = af\left(b\left(x-h\right)\right)+k}$$



<u>About the x-axis</u>

$a<0$

<u>About the y-axis</u>

$b<0$

<u>About the origin</u>

$a<0$ & $b<0$

ex: Describe the transformations.

a) $y = -3(x+1)^2$

Translated 1 *units* left

Reflected over the x-axis

vertical stretch (factor of 3)

b) $y = \frac{1}{2}[3-2x]+1$

$y = \frac{1}{2}\left[-2\left(x-\frac{3}{2}\right)\right]+1$

translated 3/2 *units* right
translated 1 *units* up
reflected over the y-axis

vertical shrink *by a factor of* $\frac{1}{2}$
horizontal shrink *by a factor of* 2

c) $y = 4-\sqrt{-5x+10}$

$y = -\sqrt{-5(x-2)}+4$

translated 2 *units* right
translated 4 *units* up
reflected over the x and y axis

vertical _____ *by a factor of* _____
horizontal shrink *by a factor of* 5

Sketching Graphs with "Key Points"

- Absolute Value
- Quadratic
- Square Root
- Cubic
- Cube Root

Process

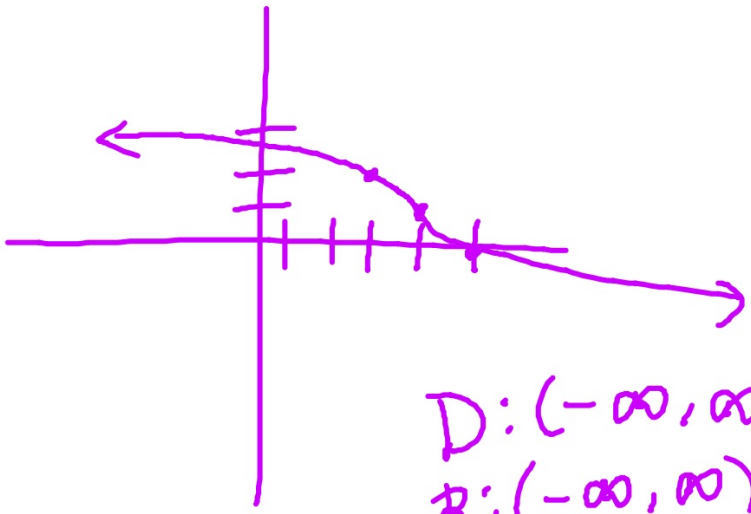
1. Plot the key point.
2. Make a table of values.

ex: Sketch and state the D/R.

a) $y = -\sqrt[3]{x-4} + 1$

Key point (4, 1)

x	y
3	2
4	1
5	0



D: $(-\infty, \infty)$

R: $(-\infty, \infty)$

Sketching Graphs with Asymptotes

- Reciprocal
- Reciprocal of a Square
- Exponential Growth/Decay
- Natural Logarithm

Process

1. Find the asymptote(s).
2. ~~Plot the key point.~~
3. Make a table of values.

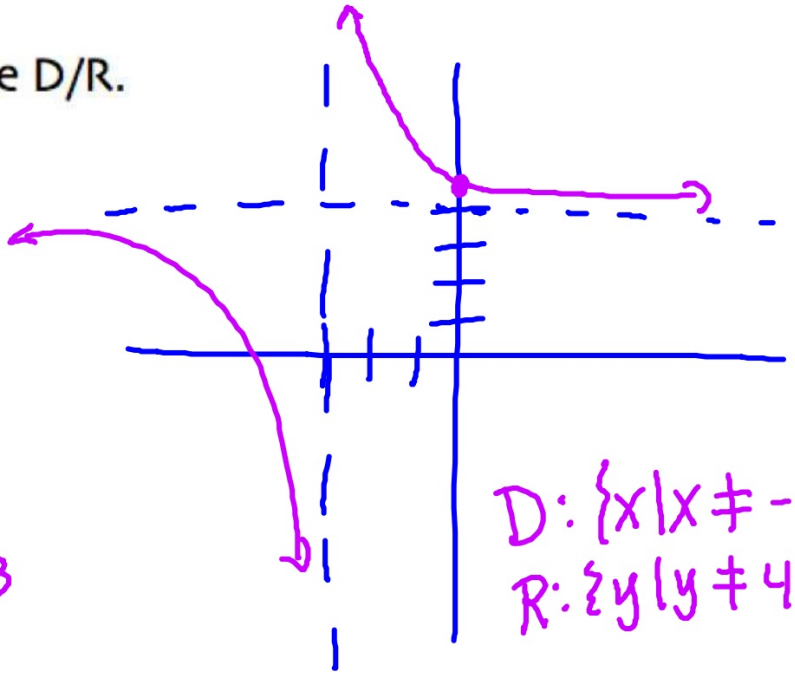
ex: Sketch and state the D/R.

b) $y = \dfrac{2}{x+3} + 4$

VA: $x = -3$

HA: $y = 4$

x	y
0	4 2/3



D: $\{x \mid x \neq -3\}$

R: $\{y \mid y \neq 4\}$

ex: Sketch and state the D/R.

c) $y = \dfrac{-5}{(x-1)^2} - 2$

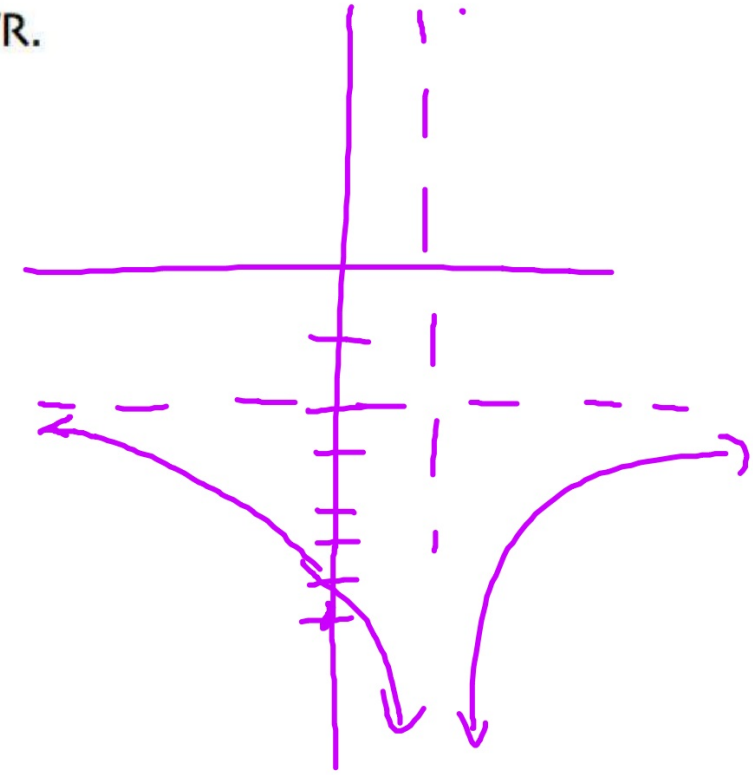
VA: $x = 1$

HA: $y = -2$

D: $\{x \mid x \neq 1\}$

R: $\{y \mid y < -2\}$

x	y
0	-7



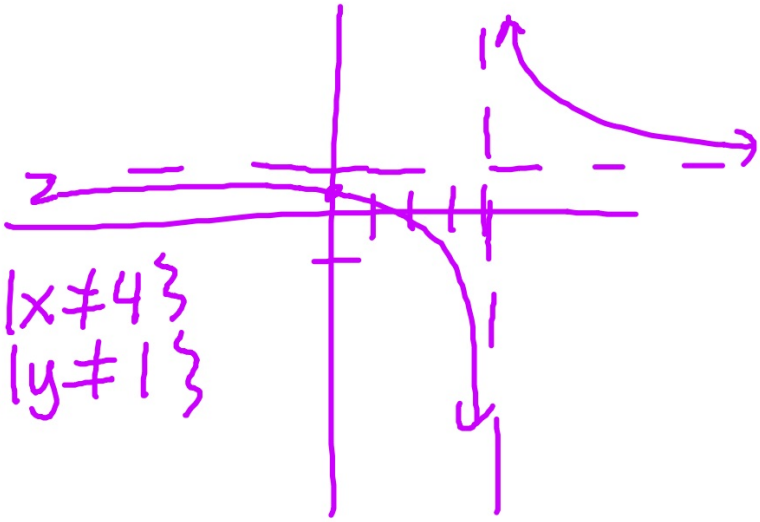
ex: Sketch and state the D/R.

$$y = \frac{1 + x - 4}{x - 4} = \frac{x - 3}{x - 4}$$

d) $y = \dfrac{1}{x-4} + 1$

VA: $x = 4$

HA: $y = 1$



D: $\{x \mid x \neq 4\}$

R: $\{y \mid y \neq 1\}$

ex: Sketch and state the D/R.

e) $y = 3^{x-1} - 2$

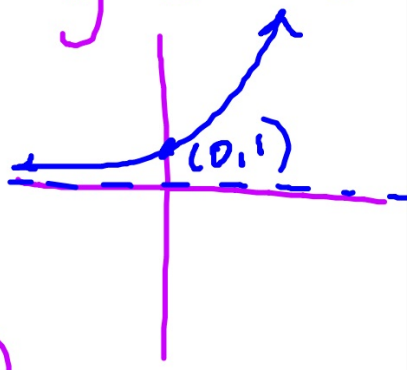
HA: $y = -2$

x	y
0	$\frac{1}{3} - 2$

D: $(-\infty, \infty)$

R: $(-2, \infty)$

$y = 3^x + 0$



Sketching Semicircles

$$y = \sqrt{r^2 - (x-h)^2} + k$$

Process

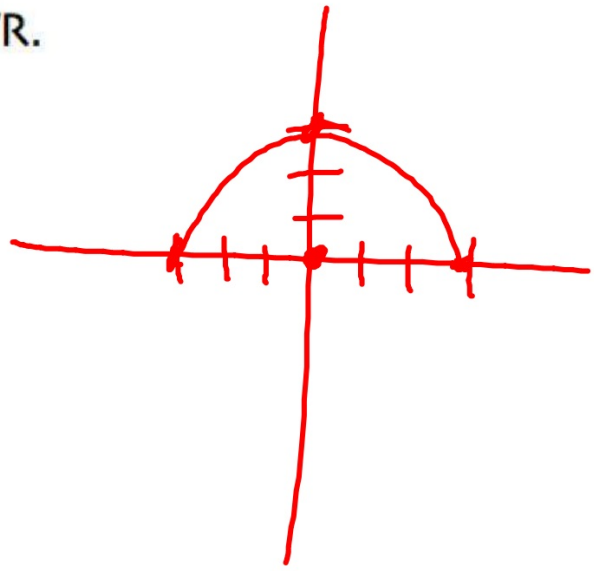
1. Identify the radius: r
2. Identify the center: (h, k)
3. Is there a reflection? look at "a"

ex: Sketch and state the D/R.

f) $y = \sqrt{9 - x^2}$

$r = 3$

center $(0, 0)$

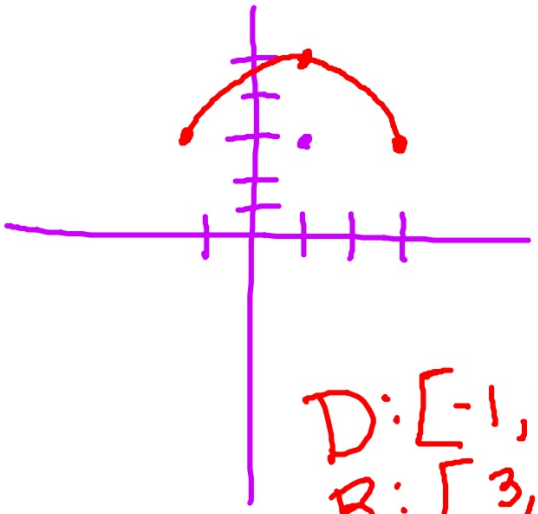


ex: Sketch and state the D/R.

g) $y = \sqrt{4-(x-1)^2} + 3$

$r = 2$

center: $(1, 3)$



D: $[-1, 3]$

R: $[3, 5]$

ex: Sketch and state the D/R.

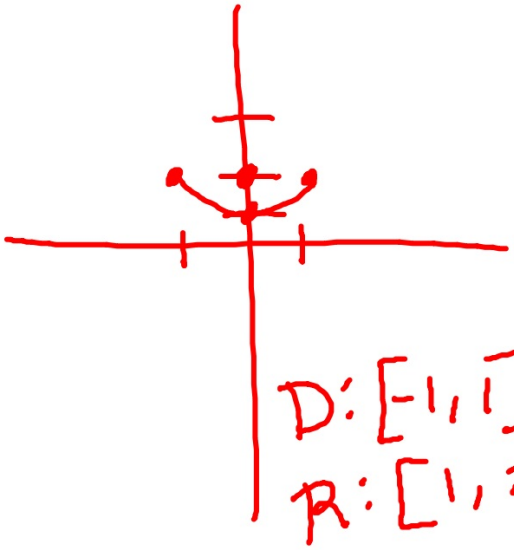
h) $y = 2 - \sqrt{1 - x^2}$

$y = -\sqrt{1 - x^2} + 2$

$r = 1$

center $(0, 2)$

reflect: x-axis



D: $[-1, 1]$

R: $[1, 2]$

Sketching Greatest Integer

$$y = a\left[b\left(x-h\right)\right]+k$$

Process

1. Identify the key point: (h, k)
2. Identify the bar length: $\frac{1}{b}$
3. Identify the vertical distance: a
4. Is there a reflection?

ex: Sketch and state the D/R.

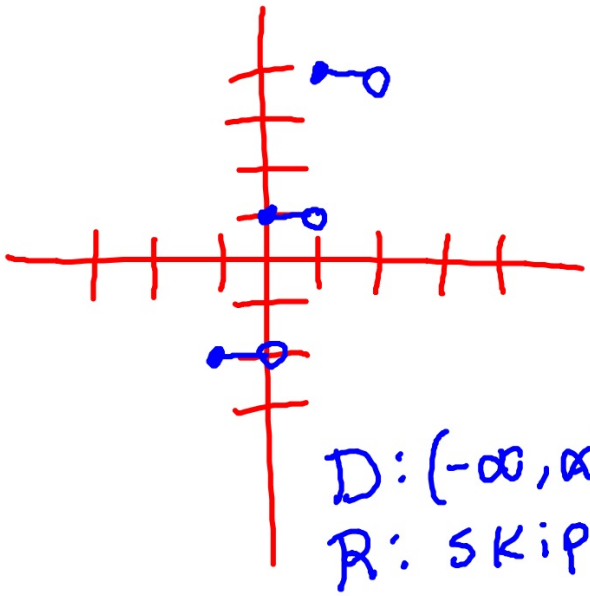
i) $y = 3[x] + 1$

Key point : $(0, 1)$

Bar length : $\frac{1}{1} = 1$

$b = 1$

Vert. Dist : 3



D: $(-\infty, \infty)$

R: skip.

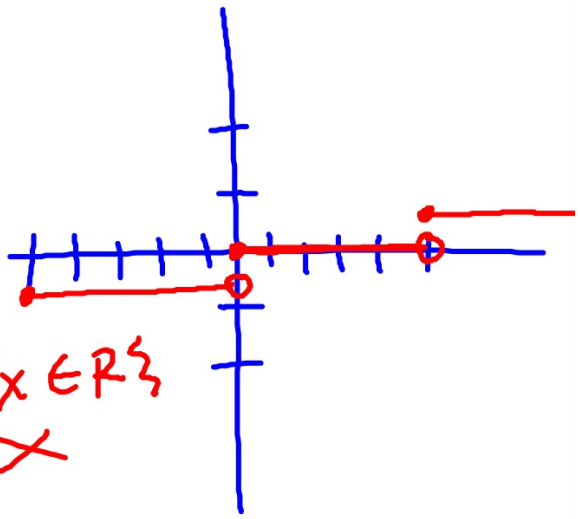
ex: Sketch and state the D/R.

j) $y = \frac{1}{2}\left[\frac{x}{5}\right] = \frac{1}{2}\left[\frac{1}{5}x\right]$

Key point: $(0,0)$

Bar length: $\frac{1}{b} = \frac{1}{\frac{1}{5}} = 5$

Vertical dist: $\frac{1}{2}$



D: $\{x \mid x \in R\}$

R: $\times$

ex: Sketch and state the D/R.

k) $y = 2[x+4]-1$

Key Point : $(-4,-1)$

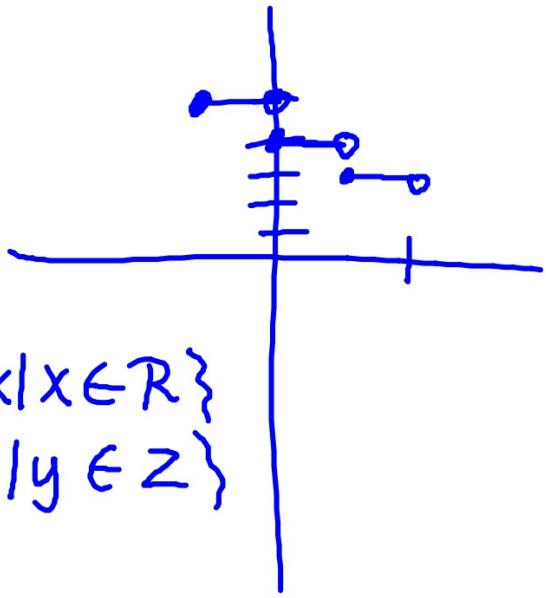
Bar length : 1

Vert. dist : 2

ex: Sketch and state the D/R.

1) $y = 4 - [2x]$

$y = -[2x] + 4$



Key point : $(0, 4)$

Bar length : $\frac{1}{2}$

Vert. dist. 1

D: $\{x \mid x \in R\}$

R: $\{y \mid y \in Z\}$