Sketching Rational Functions

In order to graph, you will need:

x-intercepts (real zeros)
y-intercept
asymptotes
hole

Bobo Botn eats d.c.

Finding asymptotes

Horizontal

consider the degree of the numerator and denominator

	Degree	Degree	Asymptote	example
1	Num.	< Den.	$y = 0$	$f(x) = \frac{1}{x+2}$
2	Num	> Den.	none	$f(x) = \frac{x^2}{x-1}$
3	Num.	= Den.	y = ratio of leading coeff	$f(x) = \frac{1-3x}{5x+4}$

Find the HA.

1. $y = \dfrac{16x+1}{5x^2+4}$ $\qquad y = 0$

2. $y = \dfrac{(3x+1)^2}{6x^2-4}$ $\qquad y = \dfrac{3}{2}$

3. $y = \dfrac{1-3x^2}{x-4}$ $\qquad$ none

Vertical Asymptotes

to find:

simplify first
set the denominator = 0 (look for real values only)

Find the VA (or VA's)

3

$$f(x) = \frac{x^2-4x-5}{x^2-1}$$

$$f(x) = \frac{(x-5)\cancel{(x+1)}}{(x-1)\cancel{(x+1)}}$$

$$f(x) = \frac{x-5}{x-1}$$

VA: $x = 1$

4

$$f(x) = \frac{3}{x^2+8x+16}$$

$$f(x) = \frac{3}{(x+4)^2}$$

VA: $x = -4$

5

$$f(x) = \frac{5x+2}{x^2-3x-4}$$

$(x-4)(x+1)$

VA: $x = 4$

$x = -1$

6 $f(x) = \dfrac{x+1}{x^2+1}$

no VA

7 $f(x) = \dfrac{x-7}{x^3-8}$

$f(x) = \dfrac{x-7}{(x-2)(x^2+2x+4)}$

VA : $x = 2$

Slant asymptotes (oblique)

If the degree of the numerator is greater than the degree of the denominator, find the slant asymptote using long division

$$x-5+\frac{18}{x+3}$$

8 $g(x)=\dfrac{x^2-2x+3}{x+3}$

$$\begin{array}{r} x-5 \\ x+3\overline{)\,x^2-2x+3} \\ -x^2+3x \\ \hline -5x+3 \\ +5x+15 \\ \hline 18 \end{array}$$

SA: $y=x-5$

9 $y = \dfrac{x^2+4x+6}{x-1}$

$$
\begin{array}{r}
x+5 \\
x-1 \overline{\big) x^2+4x+6} \\
-\ x^2+x \\
\hline
5x+6 \\
-\ 5x+5 \\
\hline
11
\end{array}
$$

SA: $y = x+5$

$$y = \frac{x^2+4x-5}{x-1}$$

$$= \frac{(x+5)\cancel{(x-1)}}{\cancel{x-1}}$$

Holes

How to find:

factor completely

If a factor cancels, a hole exists

The hole will be written as a coordinate (x,y)

(use the simplified function to find the coordinate of the hole)

Find all holes

10

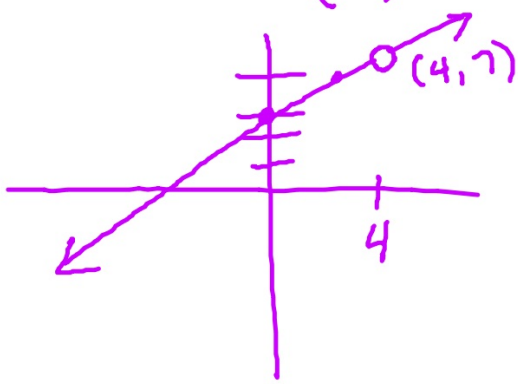
$$y = \frac{x^2 - x - 12}{x - 4}$$

$$y = \frac{\cancel{(x-4)}(x+3)}{\cancel{x-4}}$$

hole at $x = 4$

$(4, 7)$

$$y = x + 3$$



x-intercepts

f(x) = 0
simplify
set numerator = 0

11 $y = \frac{x^2-1}{x^3-1}$

$0 = \frac{(x+1)\cancel{(x-1)}}{\cancel{(x-1)}(x^2+x+1)}$

$x+1=0$

$x=-1$

$(-1,0)$

12 $y = \frac{7}{x-1}$

$0 = \frac{7}{x-1}$

$0 = 7$

no x-int.

y-intercept

let x = 0; plug in and simplify

13 $y = \dfrac{x+1}{x-4}$ $(0, -1/4)$

$y(0) = -1/4$

Average rate of change (find the slope)

$$f(x) = \frac{x+1}{x-1} \qquad [\underset{a}{2}, \underset{b}{4}]$$

$$\frac{f(b)-f(a)}{b-a} = \frac{f(4)-f(2)}{4-2}$$

$$= \frac{\frac{5}{3} - 3}{2} = \frac{\frac{-4}{3}}{2}$$

$$= \frac{-2}{3}$$

$$\frac{\frac{-4}{3}}{\frac{2}{1}} = \frac{-4}{3} \cdot \frac{1}{2}$$