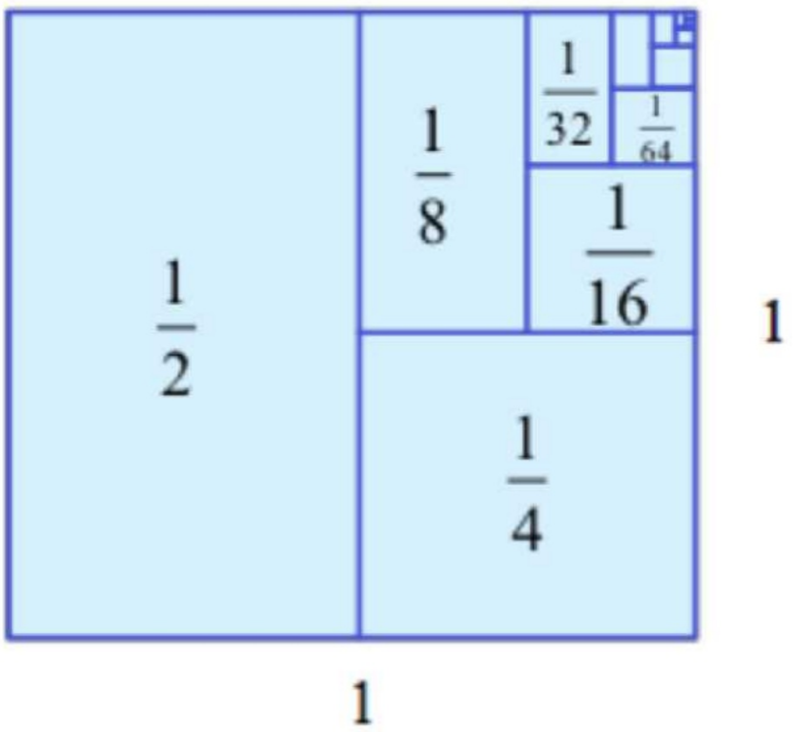
7.3/7.4 Geometric Sequences and Series



Geometric Sequences

In a **geometric sequence**, the ratio of any term to the previous term is constant. This constant ratio is called the **common ratio** and is denoted by *r*.

ex: Determine if the sequences is geometric. If so, identify the common ratio.

a) 10, 20, 30, 40, 50 . . . no

b) 3, 6, 12, 24, 48 . . . yes; $r = 2$

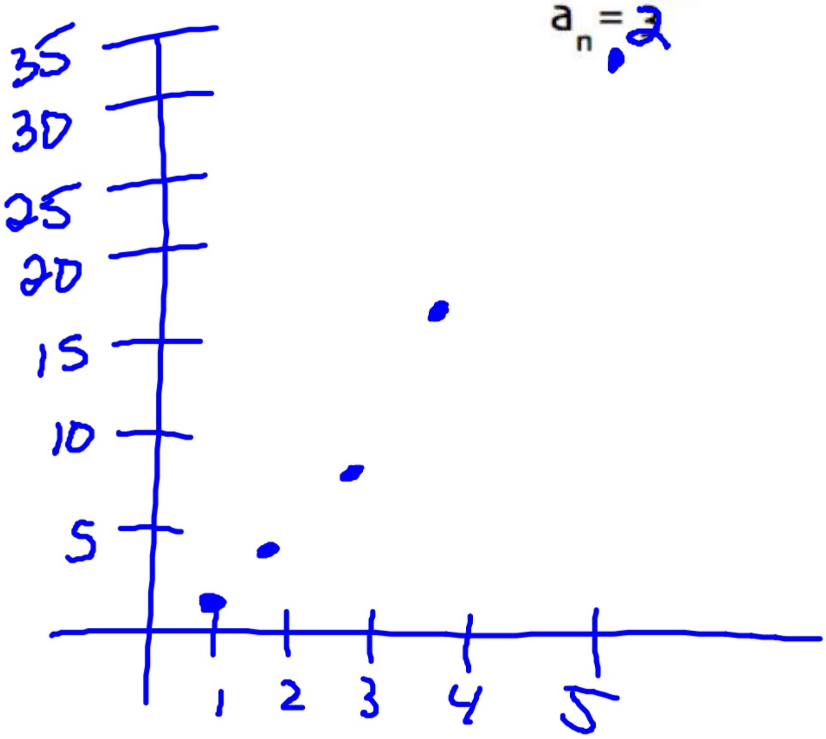
$\frac{6}{3}$ $\frac{12}{6}$ $\frac{24}{12}$

c) 16, 12, 9, 27/4 . . .

$\frac{12}{16} = \frac{3}{4}$ yes; $r = 3/4$

ex: Write the 1st 5 terms of the sequence and sketch the graph.

$a_n = 2^n$



Writing Explicit Rules for Geometric Sequences/Series

*Since geometric sequences have an exponential pattern, the explicit rule is exponential!

Recall Exponential Functions: $y = ab^x$

$$\boxed{\text{Explicit Rule: } a_n = a_1 r^{n-1}}$$

Where:

a_1 1st term

r common ratio

ex: Write an explicit rule for the geometric sequence.

a) 4, 20, 100, 500 . . .

$a_n = 4 \cdot 5^{n-1}$

$4^1 \cdot 4^{n-1}$

4^n

b) $a_2 = 3$, $r = 1/4$

$\underline{12}, 3, \frac{3}{4}, \ldots$

$a_n = 12 \cdot \left(\frac{1}{4}\right)^{n-1}$

ex: Write an explicit rule for the geometric sequence.

c) $a_3 = 10, a_6 = 270$

$\frac{10}{9}, \frac{10}{3}, \underset{a_3}{10}, \_\_, \_\_, 270$

$10 \cdot r^3 = 270$

$r^3 = 27$

$r = 3$

$a_n = \frac{10}{9} \cdot 3^{n-1}$

Writing Recursive Rules for Geometric Sequences/Series

ex: Write a recursive rule for the geometric sequence.

a) 4, 20, 100, 500 . . . $r=5$

$$\boxed{\begin{aligned} a_1 &= 4 \\ a_n &= a_{n-1} \cdot 5 \end{aligned}}$$

b) $a_2 = 3$, $r = 1/4$

$$\boxed{\begin{aligned} a_1 &= 12 \\ a_n &= a_{n-1} \cdot \frac{1}{4} \end{aligned}}$$

The Sum of a FINITE Geometric Sequence/Series

The Sum of a Finite Geometric Series

The sum of the first n terms of a geometric series with common ratio $r \neq 1$ is:

$$S_n = a_1\left(\frac{1 - r^n}{1 - r}\right)$$

S_n	sum of the 1st n terms
n	number of terms in the sum
a_1	1st term in the sequence
r	common ratio

The Sum of an INFINITE Geometric Sequence/Series

The Sum of an Infinite Geometric Series

The sum of an infinite geometric series with first term a_1 and common ratio r is given by

$$S = \frac{a_1}{1 - r}$$

provided $|r| < 1$. If $|r| \geq 1$, the series has no sum.

series diverges

S	sum of ALL terms
a_1	1^{st} term in the sequence/series
r	common ratio

ex: Find the indicated sum, if possible.

a) 1, 2, 4, 8, ... $S_9 = ?$

$r = 2$
$n = 9$
$a_1 = 1$

$$S_9 = \frac{1(1-2^9)}{1-2}$$

$$= \frac{1-512}{-1} = 511$$

ex: Find the indicated sum, if possible.

b) $\sum_{n=1}^{8} 6\left(-\frac{1}{2}\right)^{n-1}$

$n = 8$

$a_1 = 6$

$r = -\frac{1}{2}$

$$S_8 = \frac{6\left(1-\left(-\frac{1}{2}\right)^8\right)}{1-\left(-\frac{1}{2}\right)}$$

$$= \frac{6\left(1-\frac{1}{256}\right)}{\frac{3}{2}}$$

$$= 4\left(1-\frac{1}{256}\right)$$

$$= 4\left(\frac{255}{256}\right) = \frac{255}{64}$$

ex: Find the indicated sum, if possible.

c) $\sum_{n=1}^{\infty} 6\left(-\frac{1}{2}\right)^{n-1}$

geo. infinite

$|r| < 1$

$r = -\frac{1}{2}$ ✓

$a_1 = 6$

$$S = \frac{6}{1 - -\frac{1}{2}}$$

$$= \frac{6}{\frac{3}{2}}$$

$$= 4$$

ex: Find the indicated sum, if possible.

d) 4 - 2 + 1 - 0.5 + ...

geo. infinite

$r = \frac{-1}{2}$

$$S = \frac{4}{1-\left(-\frac{1}{2}\right)}$$

$$S = \frac{8}{3}$$

ex: Find the indicated sum, if possible.

e) $9 + 6 + 3 + 0 - 3 - \ldots$

arithmetic (infinite)

no sum

(diverges)

ex: Find the indicated sum, if possible.

f) $\sum_{n=0}^{6} n^2$ neither

$0+1+4+9+16+25+36=91$

ex: Express the series using summation notation. Then find the sum or explain why there is no sum.

a) $5 + 15 + 45 + 135 + \ldots$

$$\sum_{n=1}^{\infty} 5 \cdot 3^{n-1}$$

infinite geo.

$|r| < 1$

$|3| < 1$

no sum

ex: Express the series using summation notation. Then find the sum or explain why there is no sum.

b) 5 + 15 + 45 + 135

$$\sum_{n=1}^{4} 5 \cdot 3^{n-1} = 200$$

ex: Express the series using summation notation. Then find the sum or explain why there is no sum.

c) 100 + 20 + 4 + 4/5 + . . .

infinite geo

$$\sum_{n=1}^{\infty} 100\left(\frac{1}{5}\right)^{n-1}$$

$$S = \frac{100}{1-\frac{1}{5}}$$

$$= 125$$

ex: Solve for x.

a) $\sum_{i=1}^{x} 5-5i=-50$

$S=\frac{n}{2}(a_1+a_n)$

$-50=\frac{x}{2}(0+5-5x)$

$-100=5x-5x^2$

$5x^2-5x-100=0$

$5(x^2-x-20)=0 \quad 5(x-5)\cancel{(x+4)}=0$

$x=5$

ex: Solve for x.

b) $\sum_{n=0}^{\infty} 3\left(\frac{x}{2}\right)^n = 7$

$$S = \frac{a_1}{1-r}$$

$$7 = \frac{3}{1-\frac{x}{2}}$$

$$7 - \frac{7}{2}x = 3$$

$$x = \frac{8}{7}$$

ex: Find the explicit rule for. . .

$$\log x,\ \log\sqrt{x},\ \log\sqrt[4]{x}\ \ldots$$

$$\log x,\ \tfrac{1}{2}\log x,\ \tfrac{1}{4}\log x$$

geo.

$$a_n = \log x \cdot \left(\tfrac{1}{2}\right)^{n-1}$$

ex: Find the sum of the first 15 three digit whole numbers ending in 5.

$105 + 115 + 125 + \ldots + \underline{\qquad}$

Aritmetic

$S = \frac{n}{2}(a_1 + a_n)$

$S_{15} = \frac{15}{2}(105 + \underline{245})$

$a_n = 10\overset{(15)}{n} + 95$

$= 150 + \cancel{95}$

ex: Find the missing terms of the arithmetic sequence.

. . . _______, 33, _______, 25 . . .

ex: Find the missing terms of the geometric sequence.

$$\ldots 48, \underline{\ 16\ }, \underline{\ \tfrac{16}{3}\ }, \cancel{48/9} \ldots$$

$48/27$

$$48 \cdot r^3 = \frac{48}{27}$$

$$r^3 = \frac{1}{27}$$

$$r = \frac{1}{3}$$