

p. 99

6. not poly.

8. not poly.

10. 76

p. 107

56.

a. 4

b. 4

c. 6

$$49.) V = \pi r^2 h$$

$$V = \pi (x-4)^2 (2x+3)$$

$$= \pi (x^2 - 8x + 16) (2x + 3)$$

$$= \pi (2x^3 - 13x^2 + 8x + 48)$$

$$= 2\pi x^3 - 13\pi x^2 + 8\pi x + 48\pi$$

$$\begin{array}{r|l} & x^2 \quad -8x \quad 16 \\ 2x & 2x^3 - 16x^2 \quad 32x \\ 3 & 3x^2 - 24x \quad 48 \end{array}$$

$$(2x-7)^3 \neq (2x)^3 - 7^3$$

$$(2x-7)(2x-7)(2x-7)$$

$$(4x^2 - 28x + 49)(2x-7) \quad (2x \cdot 7)^3$$

$$(x+2)^2 \neq x^2 + 4$$

21.)

$$\begin{array}{r|l} & a^2 \quad -10a \quad -2 \\ 2a & 2a^3 \quad -20a^2 \quad -4a \\ -3 & -3a^2 \quad 30a \quad 6 \end{array}$$

$$2a^3 - 23a^2 + 26a + 6$$

$$(x-1) - (3x-5)$$

$$x-1-3x+5 = -2x+4$$

$$\begin{array}{r|rrrr} 17.) & -6 & 1 & 8 & -7 & 35 \\ & & \downarrow & -6 & -12 & 114 \\ \hline & & 1 & 2 & -19 & 149 \end{array}$$

19.)

$$\begin{array}{r|rrrrr} 2 & -2 & 3 & 0 & -8 & 13 \\ & & -4 & -2 & -4 & -24 \\ \hline & -2 & -1 & -2 & -12 & (-11) \end{array}$$

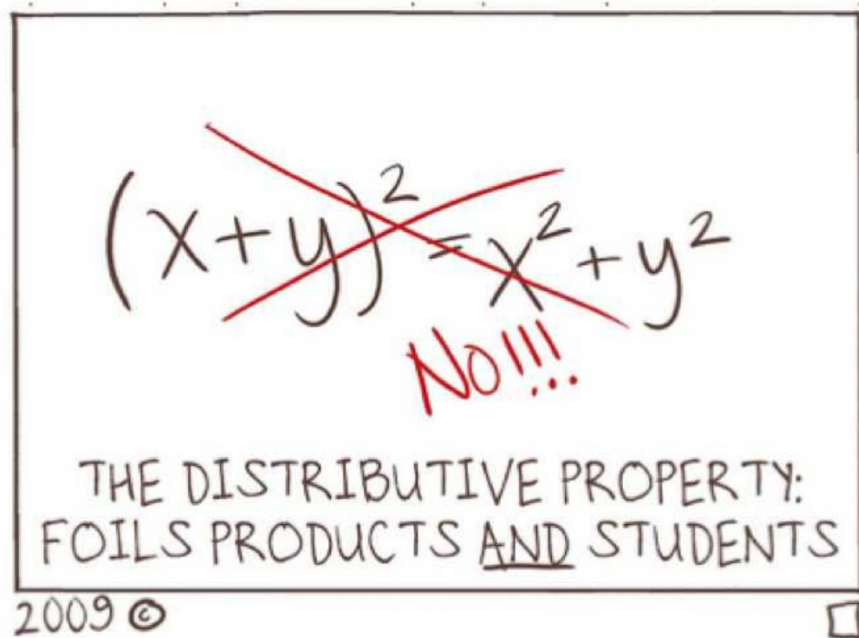
21.)

$$\begin{array}{r|rrrr} 3 & -7 & 11 & 4 & 0 \\ & & -21 & -30 & -78 \\ \hline & -7 & -10 & -26 & -78 \end{array}$$

$$\begin{aligned} f(2) &= 8(2) + 5(2)^4 - 3(2)^2 - (2)^3 \\ &= 16 + 80 - 12 - 8 \\ &= 96 - 20 \\ &= 76 \end{aligned}$$

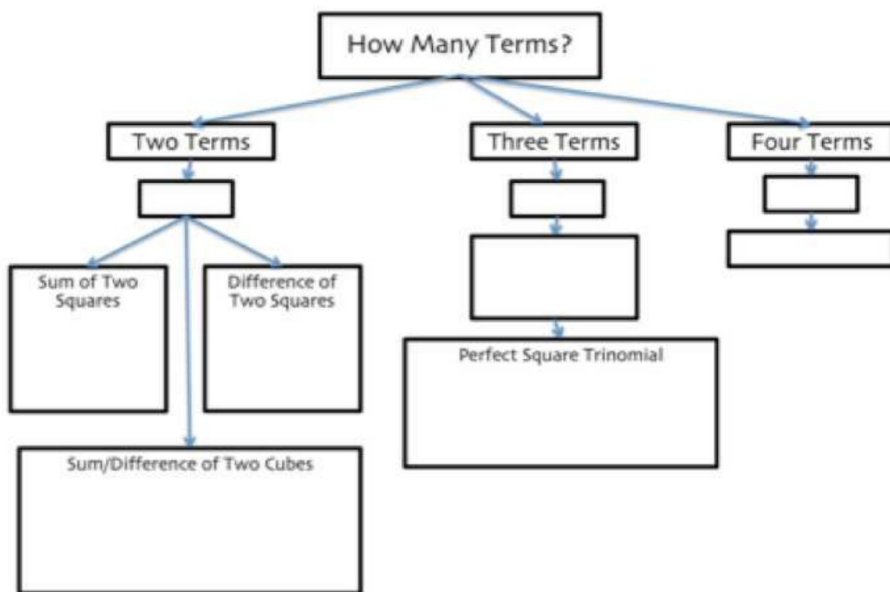
Factoring Bootcamp 2

2.4 Factor and Solve Polynomial Equations



HW:

*Grab your factoring flowchart!



GCF -
SOAP -
Prime -

REVIEW:

ex: Factor completely.

a) $x^2 - 9$

b) $2x^2 - 3x - 14$

c) $9x^2 + 12x + 4$

REVIEW:

ex: Factor completely.

d) $2x^2 + 162$

e) $x^3 + 3x^2 - 2x - 6$

f) $x^3 - 3x^2 - 16x + 48$

Perfect Cubes

$$1^3 = 1$$

$$2^3 = 8$$

$$3^3 = 27$$

$$4^3 = 64$$

$$5^3 = 125$$

$$6^3 = 216$$

$$10^3 = 1000$$

Sum/Difference of Cubes

$$x^3 + 27 \quad a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

SDAP

1

$$\frac{(3x^4y^{-5})^{-2}z^0}{51x^{-3}y^5z^6} = \frac{3^{-2}x^{-8}y^{10}}{51x^{-3}y^5z^6}$$

$$\frac{y^{10}3}{x^8}$$

$$51 \cdot 3^2 x^8 y^5 z^6$$

$$y^5$$

$$459 x^5 z^6$$

$$(x-2)(x^2+2x+4) = x^3-8$$

$$\begin{array}{r} x^2 \quad 2x \quad 4 \\ x \overline{) x^3 \quad 2x^2 \quad 4x} \\ -2 \overline{) -2x^2 \quad -4x \quad -8} \end{array}$$

Remembering the SIGNS in the Cubes Formula

SOAP

S - same sign

O - opposite sign

AP - always positive

ex: Factor completely.

$$\text{a) } x^3 - 27 = (x - 3)(x^2 + 3x + 9)$$

$$a = x$$

$$b = 3$$

$$\text{b) } x^3 + 216 = (x + 6)(x^2 - 6x + 36)$$

$$a = x$$

$$b = 6$$

ex: Factor completely.

c) $8x^3 + 1$

$$(2x+1)(4x^2 - 2x + 1)$$

$$a = 2x \quad a^2 = 4x^2$$

$$b = 1$$

$$\rightarrow -7(x^3 - 1000)$$

d) $7000 - 7x^3 = 7(1000 - x^3)$

$$a = 10 \\ b = x$$

$$\stackrel{\text{or}}{=} 7(10 - x)(100 + 10x + x^2)$$

$$\stackrel{\leftarrow}{=} -7(x - 10)(x^2 + 10x + 100)$$

ex: Factor completely.

e) $x^6 - 1$

Quadratic Form

a, b, e, f

$$ax^{2n} + \underline{bx^n} + c$$

$$x^2 + 7x + 12$$
$$(x+4)(x+3)$$

$$x^4 + 7x^2 + 12$$
$$(x^2+4)(x^2+3)$$

ex: Factor completely.

a) $x^2 - \underline{3x} - 4$
 $(x-4)(x+1)$

b) $x^{10} - \underline{3x^5} - 4$
 $(x^5-4)(x^5+1)$

ex: Factor completely.

c) $x^{\frac{2}{13}} - 3x^{\frac{1}{13}} - 4$

d) $x^{2n} - 3x^n - 4$

ex: Factor completely.

e) $x^4 - 3x^2 - 4$

$$(x^2 - 4)(x^2 + 1)$$
$$(x+2)(x-2)(x^2+1)$$

$$(x^2+1) \neq (x+1)^2$$

f) $x^6 - 3x^3 - 4$

$$(x^3 - 4)(x^3 + 1)$$
$$(x^3 - 4)(x+1)(x^2 - x + 1)$$

ex: Factor completely.

g) $2x^4 + 7x^2 + 6$

$$(2x^2 + 3)(x^2 + 2)$$

h) $2x^{11} - 9x^6 + 10x$

$$x(2x^{10} - \underline{9}x^5 + 10)$$

$$x(2x^5 - 5)(x^5 - 2)$$

ex: Factor completely.

$$8x^2 - 7x - 1$$

i) $8x^6 - 7x^3 - 1$

$$(8x^3 + 1)(x^3 - 1)$$

$$(2x+1)(4x^2-2x+1)(x-1)(x^2+x+1)$$

ii) $16x^4 - 24x^2 + 9$

$$(4x^2 - 3)^2$$

ex: Factor completely.

$$k) x^5 - x^3 + 64x^2 - 64$$

$$\underline{x^3(x^2 - 1) + 64(x^2 - 1)}$$

$$(x^2 - 1)(x^3 + 64)$$

$$(x+1)(x-1)(x+4)(x^2 - 4x + 16)$$

Theorem:

A polynomial equation with degree n has n solutions.

Vocabulary:

solutions/roots - answers to an equation

zeros - quantities that make a function equal to zero

ex: Solve by factoring.

a) $x^2 - 8x + 15 = 0$

$(x-5)(x-3) = 0$

$x = 5, 3$

ex: Solve by factoring.

4 solutions

b) $2x^4 + 7x^2 - 15 = 0$

$$(2x^2 - 3)(x^2 + 5) = 0$$

$$2x^2 - 3 = 0$$
$$\sqrt{x^2} = \sqrt{\frac{3}{2}} \frac{\sqrt{2}}{\sqrt{2}}$$

$$|x| = \frac{\sqrt{6}}{2}$$

$$x = \pm \frac{\sqrt{6}}{2}$$

$$x^2 + 5 = 0$$
$$\sqrt{x^2} = \sqrt{-5}$$

$$|x| = i\sqrt{5}$$

$$x = \pm i\sqrt{5}$$

$$x^5 + 6x^4 = 0$$
$$x^4(x+6) = 0$$



ex: Solve by factoring.

$$c) 24x^4 + 3x = 0$$

$$3x(8x^3 + 1) = 0$$

$$3x(2x+1)(4x^2 - 2x + 1) = 0$$

$$\downarrow \quad \downarrow$$
$$x=0 \quad -\frac{1}{2}$$

$$x = \frac{2 \pm \sqrt{4 - 4(4)(1)}}{8}$$

$$x = \frac{2 \pm \sqrt{-12}}{8} = \frac{2 \pm 2i\sqrt{3}}{8}$$

$$\frac{1 \pm i\sqrt{3}}{4}$$

ex: Solve by factoring.

d) $x^3 - 5x^2 - 9x + 45 = 0$

$$(x^2 - 9)(x - 5) = 0$$

$$\boxed{\pm 3, 5}$$

ex: Solve by factoring.

e) $x^4 + 2x^2 + 1 = 0$

$$(x^2 + 1)^2 = 0$$

$$x^2 + 1 = 0$$

$$x = \pm i, \text{ mult. } 2$$

i mult of 2
 $-i$ mult of 2

ex: Solve by factoring.

f) $x^7 - 64x^5 = 0$

$$x^5(x^2 - 64) = 0$$



0

mult
5



± 8

ex: Solve by factoring.

$$g) -2x^7(x^2 - 2)^2(3x + 4) = 0$$

↓

$$x = 0$$

mult
7

↓

$$x^2 - 2 = 0$$
$$x^2 = 2$$
$$x = \pm\sqrt{2}$$

mult
2

↘

$$x = -\frac{4}{3}$$

ex: Write a polynomial equation in standard form with integral coefficients and the given solutions.

a) 5, $-\frac{2}{3}$, 0

$$\begin{array}{c} \downarrow \\ (x-5)(3x+2)x = 0 \\ \downarrow \end{array}$$

ex: Write a polynomial equation in standard form with integral coefficients and the given solutions.

b) $2i, -2i, 0$ multiplicity 2

$$(x^2 + 4)x^2 = 0$$

$$x^4 + 4x^2 = 0$$

$$x^2 + 4 = 0$$

$$x^2 + 9 = 0$$

Review

ex: Simplify.

$$\frac{(3x^4y^{-5})^{-2}z^0}{51x^{-3}y^5z^6}$$

Review

ex: Evaluate using synthetic substitution.

$$f(x) = 5x^4 - x^3 + 7, \quad f(3) = ?$$

Review

ex: Perform the indicated operation.

$$(x^2 + x - 6) - (5x^2 - 2x + 3)$$