Empirical Rule
68%–95%–99.7%

If a data set has a bell shaped distribution then…

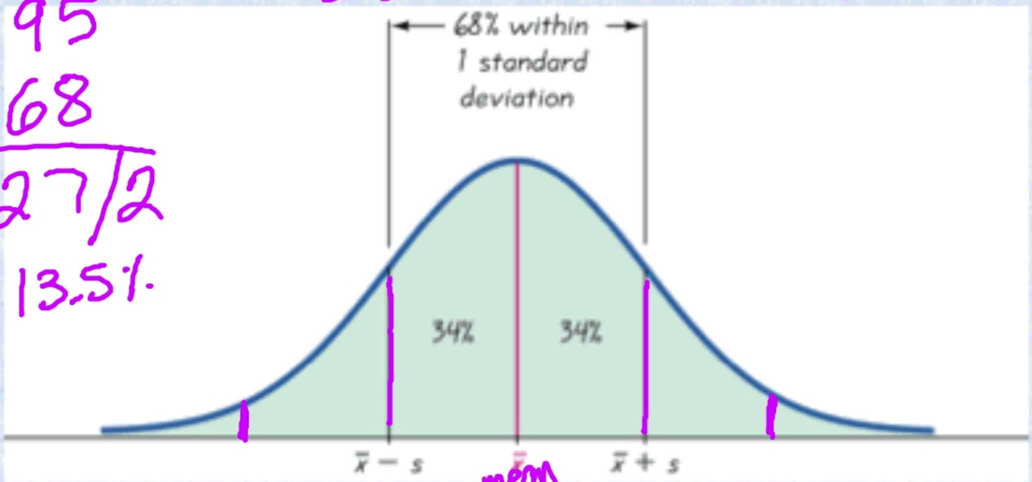
- ✓ About 68% of all values fall within 1 standard deviation of the mean
- ✓ About 95% of all values fall within 2 standard deviations of the mean
- ✓ About 99.7% of all values fall within 3 standard deviations of the mean

The Empirical Rule

68% - 95% - 99.7%

$$\begin{array}{r} 95 \\ -\ 68 \\ \hline 27/2 \end{array}$$

13.5%

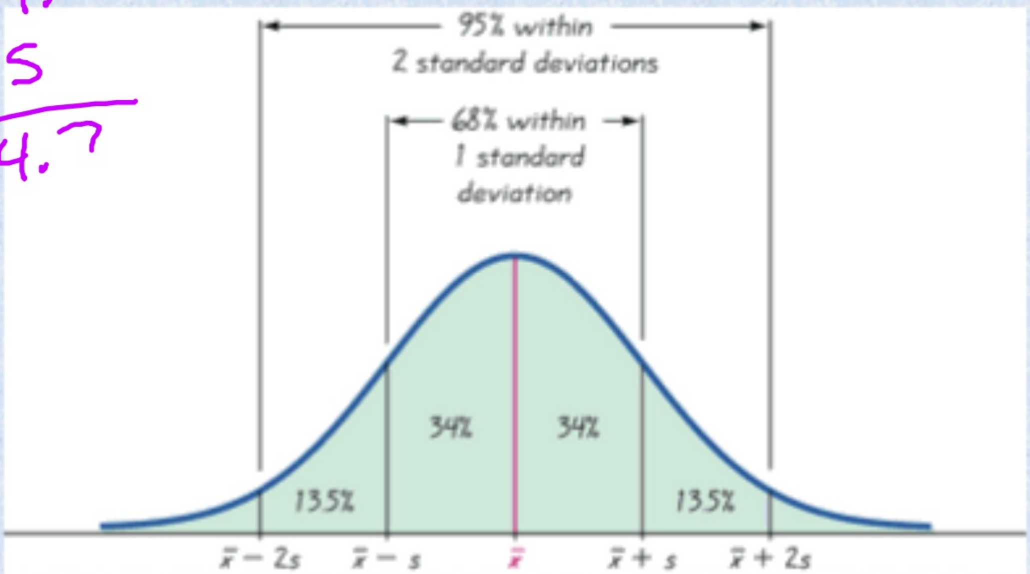


ONLY TRUE FOR BELL SHAPED DISTRIBUTIONS

(same as normal distributions)

The Empirical Rule

$$\begin{array}{r} 99.7 \\ -\ 95 \\ \hline 4.7 \end{array}$$



The Empirical Rule

$$\begin{array}{r} 100 \\ -\ 99.7 \\ \hline 0.3\% \end{array}$$

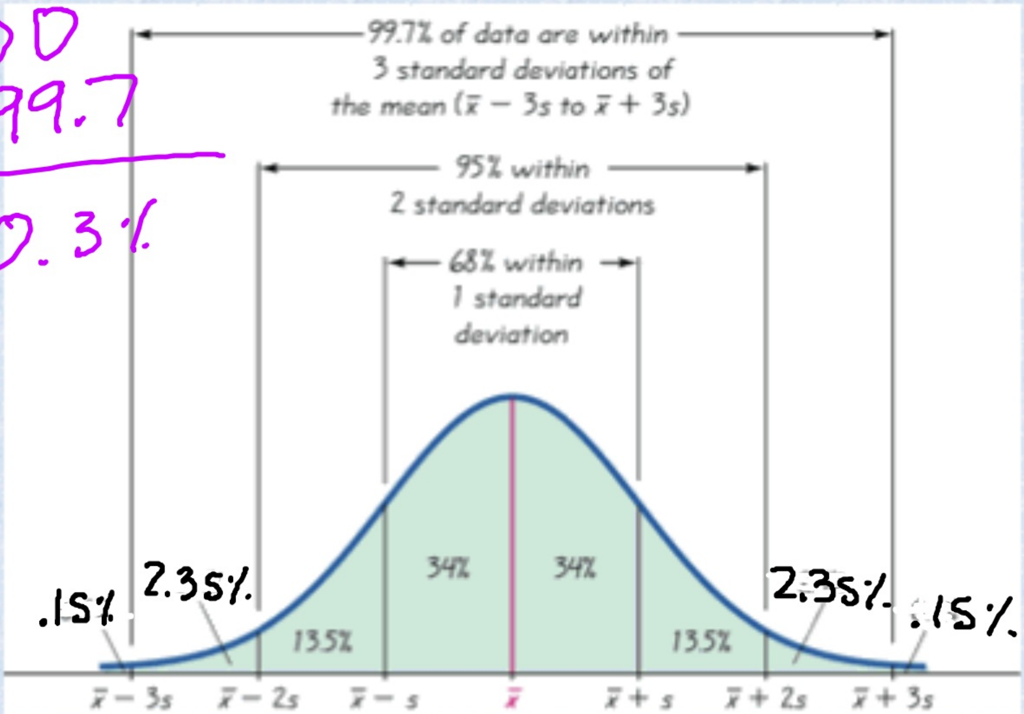
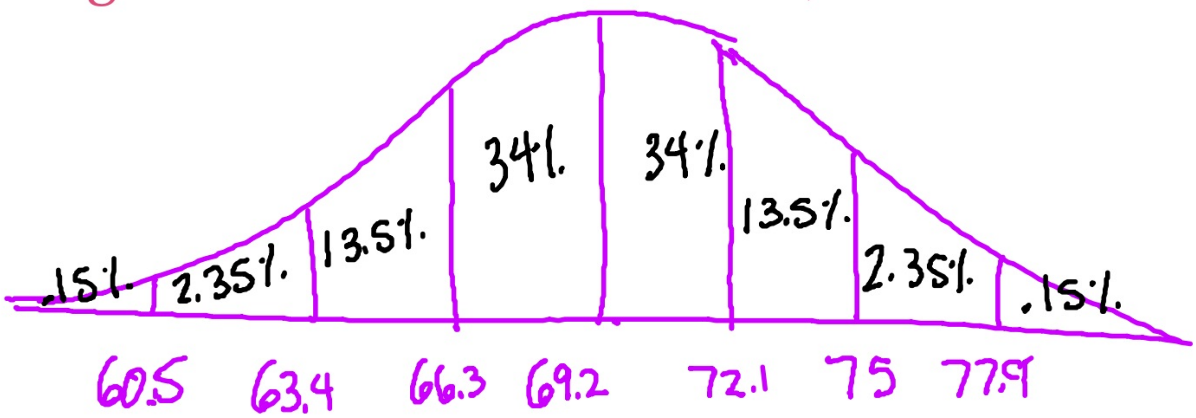


FIGURE 2-13

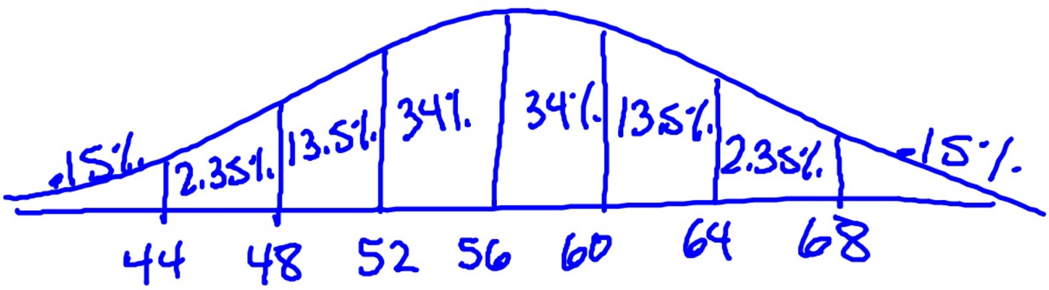
Heights of men: mean = 69.2 in, st dev. = 2.9 in



$\boxed{60 \text{ in}}$

$$z = \frac{60 - 69.2}{2.9} = -3.17$$

The mean speed of vehicles along a stretch of highway is 56 mph with a standard deviation of 4 mph. About what percent of the vehicles will be between a speed of



a) 44 mph and 68 mph

99.7%

b) 48 mph and 64 mph

95%

c) 44 mph and 60 mph

83.85%

Measures of Relative Standing

z-score: (standard score) is the number of standard deviations that a value x is above/below the mean

$$Z = \frac{X - \mu}{\sigma}$$

population mean

population st. dev.

μ : mu

σ : sigma

Z-scores do not have any units. They measure the distance of each data value from the mean in standard deviations.

Using z-scores makes it possible to compare values that are measured on different scales, with different units, or different populations.

Z-scores should be rounded to two decimal places.

The best 800-m time, run by Getrud Bacher of Italy, was 129 seconds which was faster than the mean (137 seconds). The standard deviation for the qualifying times was 5.0 seconds.

The winning long jump by the Russian Yelena Prokhorova was 60 cm longer than the mean. The standard deviation was 30 cm.

WHO'S FEAT WAS MORE IMPRESSIVE? To compare, we need to look how many standard deviations better than the mean each performance was--ie the Z-score.

Bacher's Z-score

$$z = \frac{129s - 137s}{5s} = -1.6$$

Prokhorova's Z-score

$$z = \frac{60cm}{30cm} = 2$$

The mean IQ score is 100 with a standard deviation of 15. Galileo's IQ was z = 5.67 as a z-score. Find his IQ.

$$Z = \frac{X - \mu}{\sigma}$$

$$5.67 = \frac{X - 100}{15}$$

$$185 = X$$

Vince Vaugn's height (as a z-score) is approx. $z = 2.86$. If men's heights are normally distributed with a mean of 69 in and a st. dev. of 2.8 in, what is Vince Vaugn's height?

$$2.86 = \frac{x - 69}{2.8}$$

$$77\text{in} = x \longrightarrow 6'5''$$

$$\boxed{72\text{in} = 6\text{ft}}$$

$$72.4\text{in} \neq 72'4''$$

$$72\tfrac{2}{5}$$

$$6'5'' \qquad \tfrac{2}{5} \cdot 12$$

FlorenceFreshman took college placement exams in French and math.

The French exam had a mean of 72 and a std dev of 8, while the math exam had a mean of 68 and a std of 12.

She scored an 82 on the French exam and an 86 on the math. On which exam did she do better compared to the other freshman?

French

$$z = \frac{82 - 72}{8}$$

$$z = 1.25$$

Math

$$z = \frac{86 - 68}{12}$$

$$z = 1.50$$

Math z-score is higher