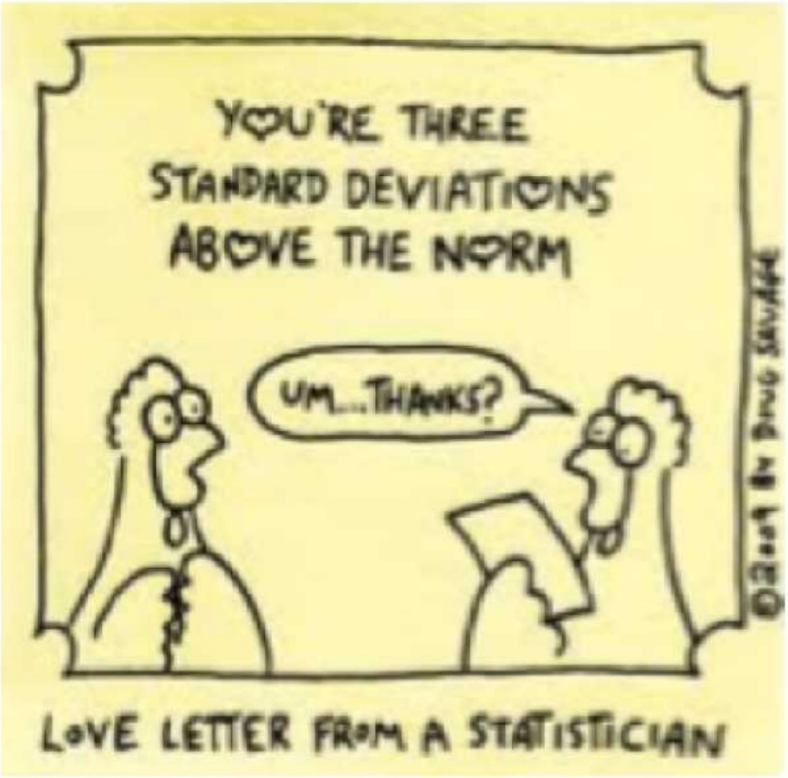
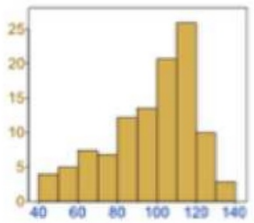
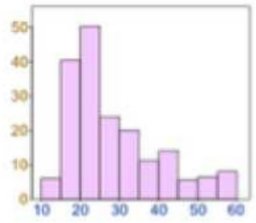
Empirical Rule & Z Scores



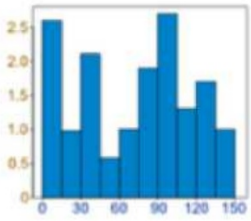
Normal Distribution



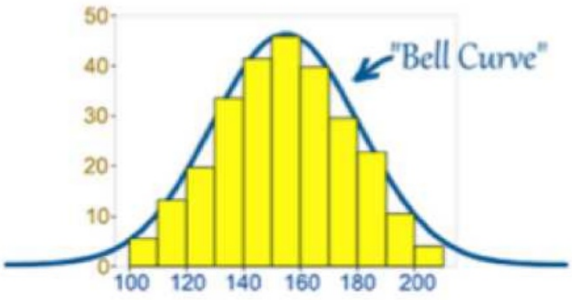
Data is "skewed ~~right~~" left



Data is "skewed ~~left~~" right

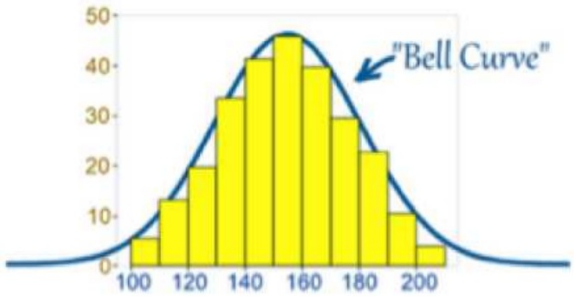


Data is "jumbled"



"Normally Distributed Data"

Normal Distribution

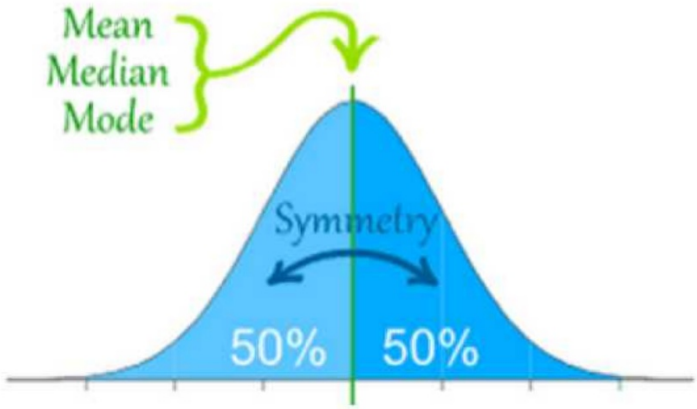


Data is "normally distributed" if it tends to be around a central value with no bias left or right.

Many types of data closely follow a Normal Distribution:

- heights of people
- size of products produced by machines
- errors in measurements
- blood pressure
- marks on a test

The Normal Distribution has:



- mean = median = mode
- symmetry about the center
- 50% of values less than the mean and 50% greater than the mean

Standard Deviation & Empirical Rule

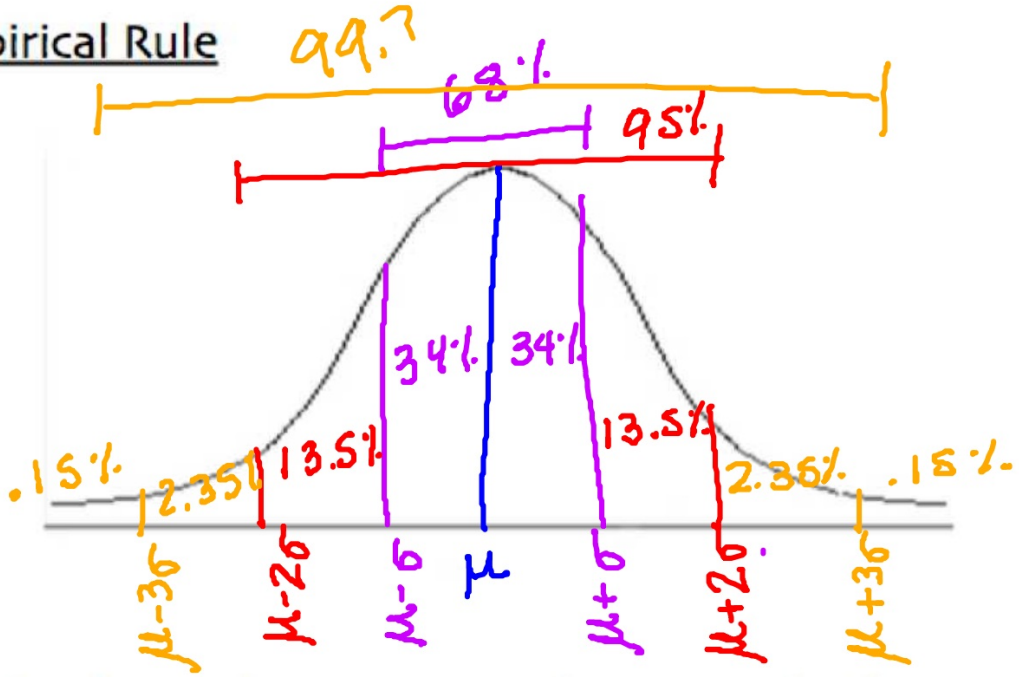
Standard Deviation (σ) is a measure of how spread out data is from the mean.

sigma

Data that is normally distributed will always follow the Empirical Rule. That is...

- About 68% of all values fall within 1 standard deviation of the mean (likely)
- About 95% of all values fall within 2 standard deviations of the mean (very likely)
- About 99.7% of all values fall within 3 standard deviations of the mean (almost certainly)

The Empirical Rule



Mu (μ) - used to represent the mean of a data set

Sigma (σ) - used to represent standard deviation

Ex 1: The mean speed of vehicles along a stretch of highway is 56 mph with a standard deviation of 4 mph. About what percentage of the vehicles will be between a speed of…

a) 44 mph and 68 mph

3σ 99.7%

Ex 1: The mean speed of vehicles along a stretch of highway is 56 mph with a standard deviation of 4 mph. About what percentage of the vehicles will be between a speed of...

b) 48 mph and 64 mph

2σ

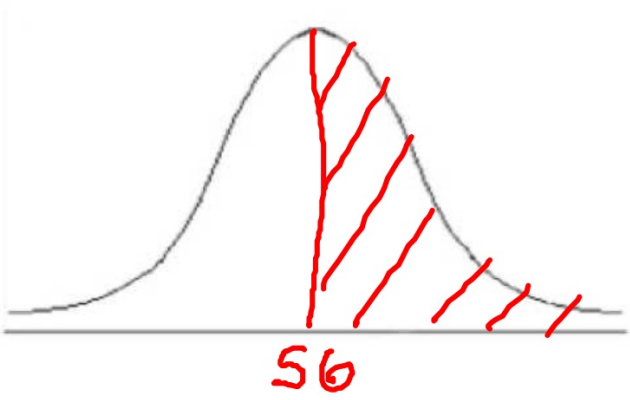
95%

Ex 1: The mean speed of vehicles along a stretch of highway is 56 mph with a standard deviation of 4 mph. About what percentage of the vehicles will be between a speed of…

c) 44 mph and 60 mph

83.85%

Ex 1: The mean speed of vehicles along a stretch of highway is 56 mph with a standard deviation of 4 mph. About what percentage of the vehicles will be between a speed of…



d) more than 56 mph

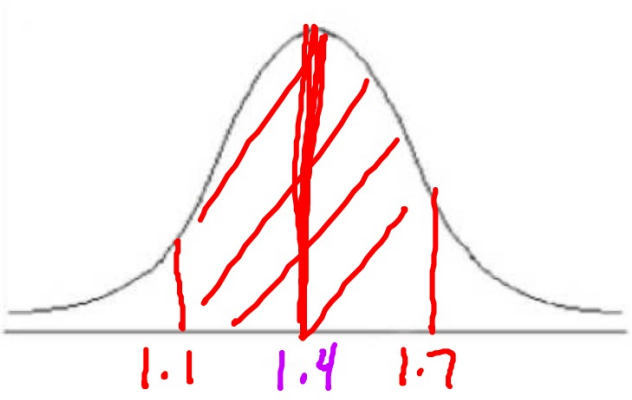
50%

Ex 1: The mean speed of vehicles along a stretch of highway is 56 mph with a standard deviation of 4 mph. About what percentage of the vehicles will be between a speed of...

e) less than 68 mph

99.85 %.

Ex 2: 95% of students at school are between 1.1m and 1.7m tall. Assuming this data is normally distributed calculate the mean and standard deviation.



$2\sigma = .3$

$\sigma = .15$

Standard Scores

A z-score is the number of standard deviations away from the mean.

Z-scores are also known as a standard scores and are represented by the lowercase Greek letter sigma, σ .

To calculate a z-score use the formula:

$$z = \frac{x - \mu}{\sigma}$$

What to know about z-scores…

- Z-scores DO NOT have units
- Z-scores are used to compare values that are measured on different scales, with different units, or different populations
- Z-scores should be rounded to two decimal places

Ex 3: The best 800 m time, run by Gertrud Bacher of Italy, was 129 seconds which was faster than the mean (137 seconds). The standard deviation for the qualifying times was 5.0 seconds. The winning long jump by the Russian Yelena Prokhorova was 60 cm longer than the mean. The standard deviation was 30 cm. Who's feat was more impressive?

Bacher

$$z = \frac{129 - 137}{5}$$

$$z = -1.6$$

Prokhorova

$$z = \frac{60}{30}$$

$$\boxed{z = 2}$$

more impressive

Ex 4: The mean IQ score is 100 with a standard deviation of 15. Galileo's IQ was z=5.67 as a z-score. Find his IQ.

$$Z = \frac{X - \mu}{\sigma}$$

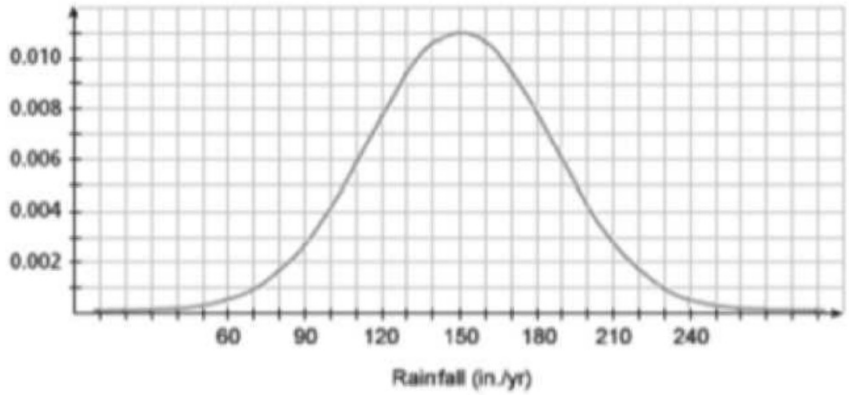
$$5.67 = \frac{X - 100}{15}$$

$$185.05 = X$$

$$185 = X$$

Ex 5:

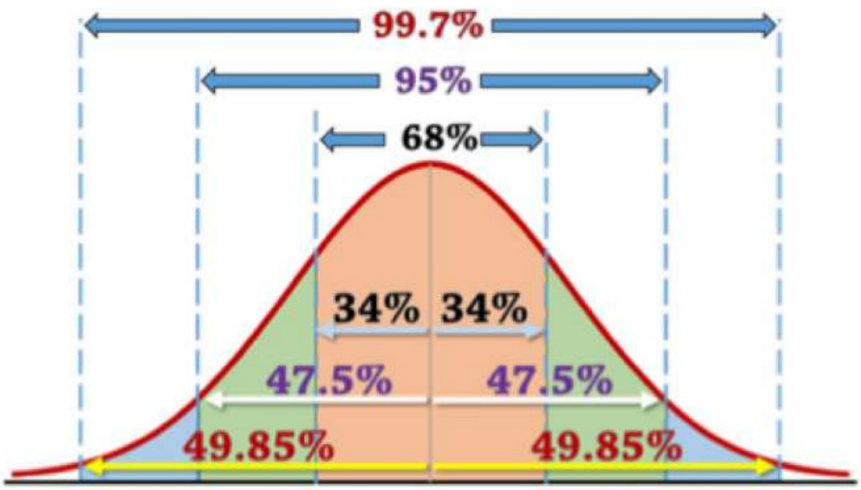
The area under the normal curve below is equal to 1 unit. Each grid square has an area of 0.01 unit.



Suppose the annual rainfall in an Alaskan town is normally distributed with a mean of 150 inches per year and a standard deviation of 36 inches per year. Use the graph to approximate the probability that the rainfall in a given year is less than 140 inches. Express your answer as a percent.

Ⓐ 51%
Ⓒ 50%
Ⓑ 61%
Ⓓ 39%

Area Under The Standard Normal Curve



Area Under the Curve = 1

Using the Z - Table

Standard Normal Probabilities

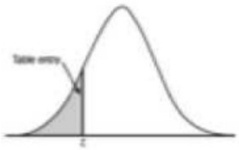


Table entry for z is the area under the standard normal curve to the left of z.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

Standard Normal Probabilities

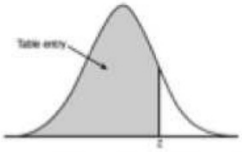


Table entry for z is the area under the standard normal curve to the left of z.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

Using the Z - Table

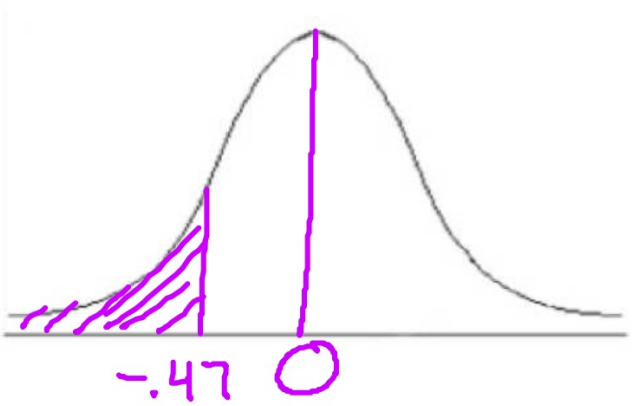
The data presented on the z-table represents the AREA under the standard normal curve to the LEFT of a particular z-score.

The value of the area also represents the probability:

$P(z < \text{z-score})$

Ex 6: Sketch the area described and find the ~~z-score.~~ probability

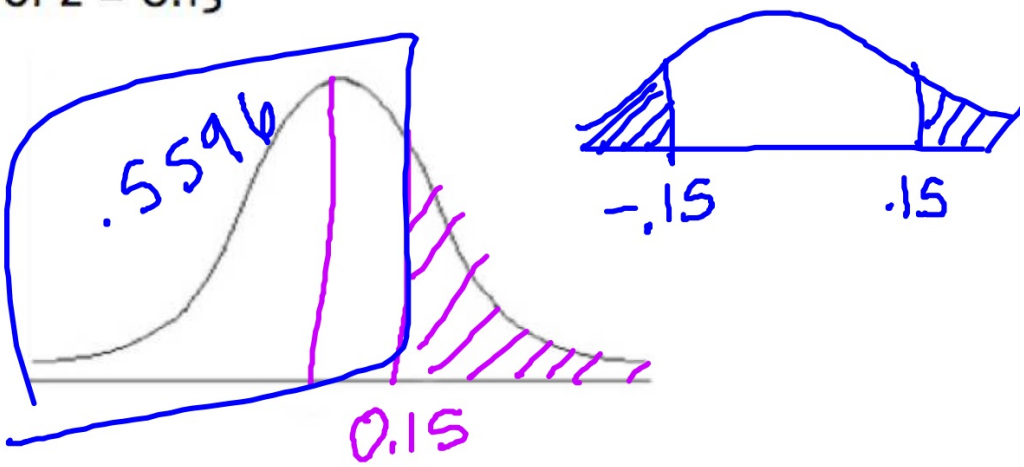
a) To the left of z = -0.47



$P(z < -.47) = .3192$

Ex 6: Sketch the area described and find the z-score.

b) To the right of z = 0.15

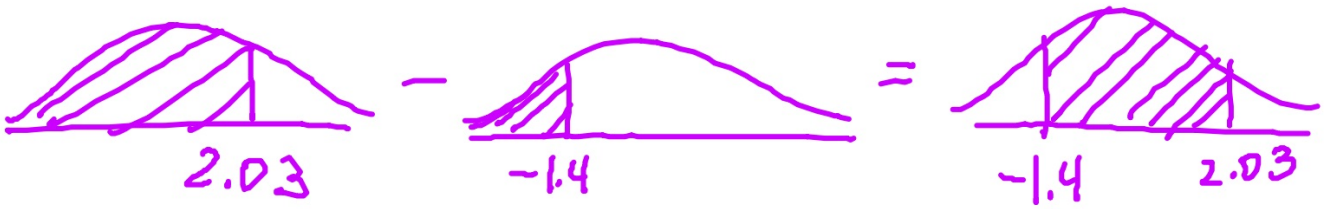


$1 - .5596 = .4404$

$P(Z > .15) = .4404$

Ex 6: Sketch the area described and find the z-score.

c) Between z = -1.40 and z = 2.03



.9788 − .0808 = .898

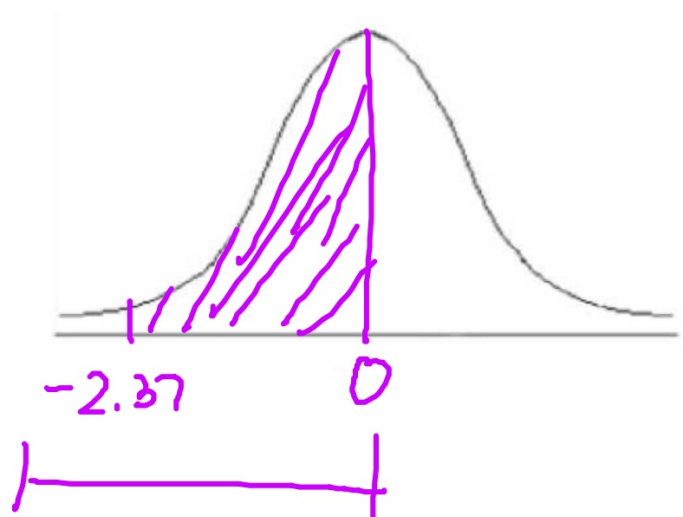
Ex 7: Find the indicated probabilities

a) $P(z < 3.20)$

.9993

Ex 7: Find the indicated probabilities

b) $P(-2.37 < z < 0)$



$.5 - .0089 = .4911$

Percentiles

A percentile is a measure used in statistics indicating the value below which a given percentage of observations in a group of observations fall. Percentiles are often used in reporting scores of standardized tests like the SAT or ACT.

Example: You are the fourth tallest person in a group of 20

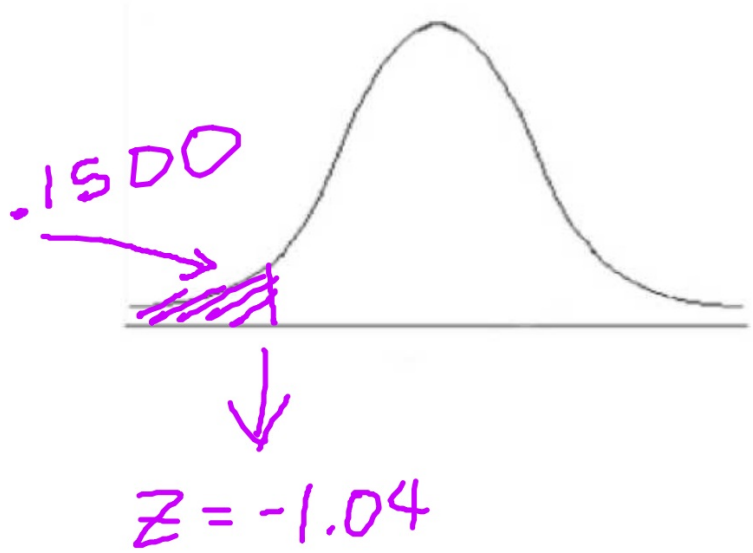
80% of people are shorter than you:

That means you are at the **80th percentile**.

If your height is 1.85m then "1.85m" is the 80th percentile height in that group.

Ex 8: Sketch the area described and find the z-score.

a) 15^{th} percentile



Ex 8: Sketch the area described and find the z-score.

b) 97.5^{th} percentile

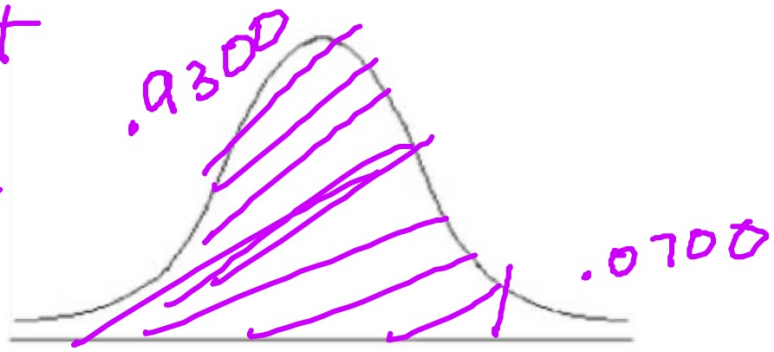
$z = +1.96$

Ex 8: Sketch the area described and find the z-score.

c) upper 7^{th} percentile

equivalent

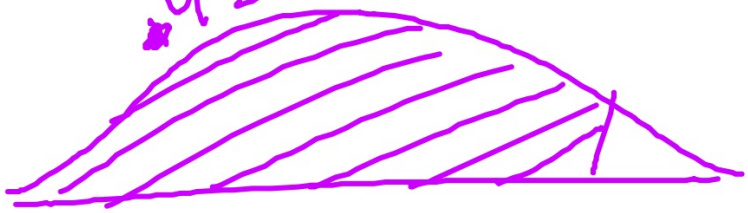
93^{rd} percentile



$Z = 1.48$

95th percentile

.9500



$\frac{1.64 + 1.65}{2}$ (Average)

$z = 1.645$