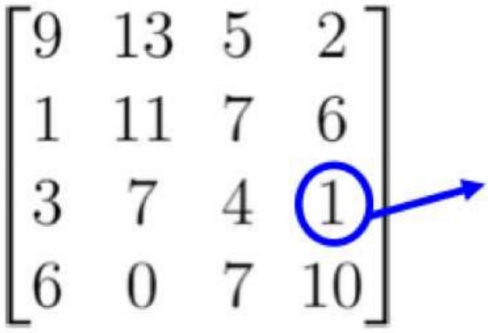
Matrices, Determinants, Cramer's Rule & 3x3 Word Problems

*See printout.

What is a Matrix?

A matrix is a rectangular array of variables or constants organized in rows and columns, enclosed by **brackets**. Each value in the matrix is referred to as an element.



The element 1 is in Row 3 and Column 4 and is depicted by the name a_{34}

ex: Name the circled element.

a) $\begin{bmatrix} 9 & 13 & 5 & 2 \\ 1 & 11 & \textcircled{7} & 6 \\ 3 & 7 & 4 & 1 \\ 6 & 0 & 7 & 10 \end{bmatrix}$

a_{23}

b) $\begin{bmatrix} 9 & 13 & 5 & 2 \\ 1 & 11 & 7 & 6 \\ \textcircled{3} & 7 & 4 & 1 \\ 6 & 0 & 7 & 10 \end{bmatrix}$

a_{31}

Matrix Dimensions

A matrix is often described by its dimensions. A matrix with m rows and n columns is known as an "m x n matrix."

ex: State the dimensions of each matrix.

a) $\begin{bmatrix} 3 & -1 & 5 \\ 0 & 4 & 2 \end{bmatrix}$

2x3

b) $\begin{bmatrix} 7 & 8 \\ 0.5 & -10 \end{bmatrix}$

2x2

Special Matrices

Row Matrix - one row

ex: $\begin{bmatrix} 1 & 2 \end{bmatrix}$

Column Matrix - one column

ex: $\begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}$

Special Matrices

Square Matrix - same number of columns and rows

ex: $\begin{bmatrix} 7 & 8 \\ 0.5 & -10 \end{bmatrix}$

Zero Matrix - every element is zero

ex: $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Determinants

A determinant is a matrix function that only accepts a **square matrix**.

Notation for the Determinant of Matrix A:

$$\det(A) \qquad |A| \qquad D_A$$

*The determinant of a 2x2 square matrix is called a **second-order determinant**.

*The determinant of a 3x3 square matrix is called a **third-order determinant**.

ex: Evalutate.

a) $\begin{vmatrix} 5 & -4 \\ 8 & 9 \end{vmatrix}$

$= 5 \cdot 9 - 8 \cdot -4$

$= 45 + 32$

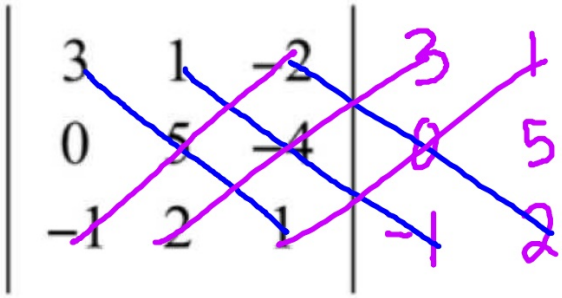
$= 77$

ex: Evalutate.

b) $\begin{vmatrix} 0 & 6 \\ 4 & -11 \end{vmatrix} = 0 \cdot -11 - 4 \cdot 6 = -24$

ex: Evalutate.

c)

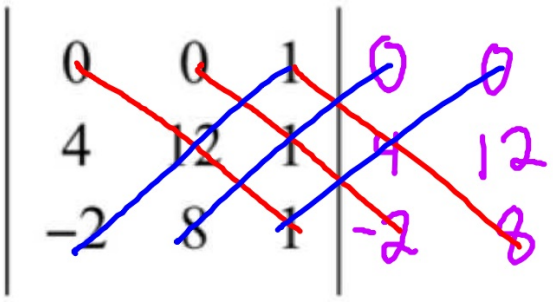


$(15 + 4 + 0) - (10 - 24 + 0)$

$19 + 14 = 33$

ex: Evalutate.

d)



$(0+0+32)-(-24+0+0)$

$32+24$

56

Cramer's Rule

Cramer's Rule is an algebraic method for solving systems of equations using determinants of matrices.

$$ax+by=c$$
$$dx+ey=f$$

$$D=\begin{vmatrix} a & b \\ d & e \end{vmatrix} \quad D_x=\begin{vmatrix} c & b \\ f & e \end{vmatrix} \quad D_y=\begin{vmatrix} a & c \\ d & f \end{vmatrix}$$

$$x=\frac{D_x}{D} \quad y=\frac{D_y}{D}$$

ex: Solve the system using Cramer's Rule, if possible.

a) $x - 3y = 4$

$5x + 7y = 8$

$D = \begin{vmatrix} 1 & -3 \\ 5 & 7 \end{vmatrix} = 22$

$D_x = \begin{vmatrix} 4 & -3 \\ 8 & 7 \end{vmatrix} = 52$

$D_y = \begin{vmatrix} 1 & 4 \\ 5 & 8 \end{vmatrix} = -12$

$x = \frac{D_x}{D} = \frac{52}{22} = \frac{26}{11}$

$y = \frac{D_y}{D} = \frac{-12}{22} = \frac{-6}{11}$

$\left(\frac{26}{11}, \frac{-6}{11}\right)$

ex: Solve the system using Cramer's Rule, if possible.

b) $7x - 7y = 8$

$-3x + 3y = 2$

$x = \frac{D_x}{0} = \text{und.}$

$D = \begin{vmatrix} 7 & -7 \\ -3 & 3 \end{vmatrix} = 0$

$\emptyset$

ex: Solve the system using Cramer's Rule, if possible.

c) $4x - 3y = 11$

$6x + 5y = 7$

$D = \begin{vmatrix} 4 & -3 \\ 6 & 5 \end{vmatrix} = 38$

$D_x = \begin{vmatrix} 11 & -3 \\ 7 & 5 \end{vmatrix} = 55 + 21 = 76$

$D_y = \begin{vmatrix} 4 & 11 \\ 6 & 7 \end{vmatrix} = 28 - 66 = -38$

$x = \frac{76}{38} = 2$

$y = \frac{-38}{38} = -1$

$(2, -1)$

ex: Solve the system using Cramer's Rule, if possible.

d) $x + 2y + 3z = -5$

$3x + y - 3z = 4$

$-3x + 4y + 7z = -7$

$(-1, 1, -2)$

$D = \begin{vmatrix} 1 & 2 & 3 \\ 3 & 1 & -3 \\ -3 & 4 & 7 \end{vmatrix} \begin{matrix} 1 & 2 \\ 3 & 1 \\ -3 & 4 \end{matrix} = 40$

$D_z = -80$

$D_x = \begin{vmatrix} -5 & 2 & 3 \\ 4 & 1 & -3 \\ -7 & 4 & 7 \end{vmatrix} \begin{matrix} -5 & 2 \\ 4 & 1 \\ -7 & 4 \end{matrix} = (-35 + 42 + 48) - (-21 + 60 + 56)$

$= -40$

$D_y = \begin{vmatrix} 1 & -5 & 3 \\ 3 & 4 & -3 \\ -3 & -7 & 7 \end{vmatrix} \begin{matrix} 1 & -5 \\ 3 & 4 \\ -3 & -7 \end{matrix} = (28 - 45 - 63) - (-36 + 21 - 105)$

40

ex: Solve the system using Cramer's Rule, if possible.

e) $x + y + z = 1$

$2x - 5y + z = 7$

$2y - 4z = -10$

$y = \frac{D_y}{D} = -1$

$D = \begin{vmatrix} 1 & 1 & 1 \\ 2 & -5 & 1 \\ 0 & 2 & -4 \end{vmatrix} \begin{matrix} 1 & 1 \\ 2 & -5 \\ 0 & 2 \end{matrix} = (20 + 4) - (2 - 8)$

$24 - (-6) = 30$

$D_y = -30$

ex: Sandy has nickels, dimes and quarters that amount to $3.75 in change. She has three more quarters than dimes but twice as many nickels as quarters. How many dimes, nickels and quarters does Sandy have?