8.1 Apply The Distance & Midpoint Formulas, 8.3 Circles Notes

Distance Formula

KEY CONCEPT *For Your Notebook*

The Distance Formula

The distance d between (x_1, y_1) and (x_2, y_2) is $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

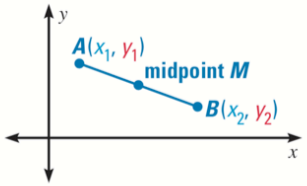
Midpoint Formula

KEY CONCEPT *For Your Notebook*

The Midpoint Formula

A line segment's *midpoint* is equidistant from the segment's endpoints. The **midpoint formula**, shown below, gives the midpoint of the line segment joining $A(x_1, y_1)$ and $B(x_2, y_2)$.

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$



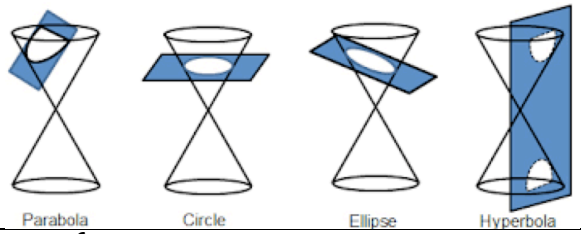
In words, each coordinate of M is the mean of the corresponding coordinates of A and B.

ex: (0, 6), (5, -4)

a) Find the distance between the two points.

b) Find the midpoint of the line segment joining the two points.

conic section - a figure formed by the intersection of a plane and a double-napped cone.



circle - locus of points equidistant from a center

Standard Form:

_____

Where:

(h, k):_____

r:_____

ex: Sketch. Then state the center and radius.

a) $(x-1)^2 + (y-3)^2 = 4$

b) $x^2 + (y+5)^2 = 9$

ex: Complete the square.

a) $x^2-8x+13$	b) $x^2+10x-1$	c) $2x^2-12x-7$	d) $-3x^2+12x+5$

ex: Rewrite from general to standard form. Then sketch and state the center and radius.

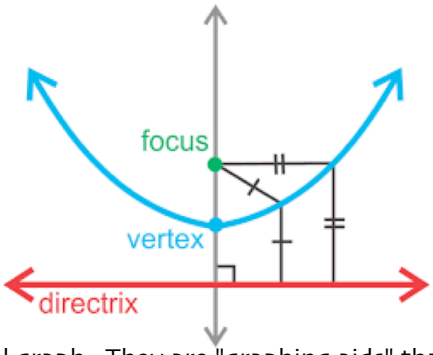
a) $x^2+y^2+2x-6y+5=0$	b) $x^2+y^2+6x-4y+12=0$

ex: Write an equation in standard form of the circle with the given characteristics.

a) center: (6, 4) Area: 9π	b) Endpoints of a diameter: (-7, -1), (-9, 5)	c) Center: (4, 3) Lies tangent to the line y=6

8.2 Parabolas Notes

parabola - locus of points equidistant from a focus and directrix



*The focus and directrix are not the actual graph. They are "graphing aids" that define the points on the parabola.

Standard Form

Where

vertex:_____

Opens: UP/DOWN

_____

p>o:_____

p<o:_____

Opens: RIGHT/LEFT

_____

$|p|$:_____

$|4p|$:_____

ex: Sketch & state the vertex, focus and directrix.

a) $(x-5)^2 = 8(y+3)$	b) $(y+1)^2 = -4(x+2)$	c) $(y-3)^2 = 12(x-2)$

ex: Rewrite from general to standard form then sketch.

a) $y^2 + 2x + 6y + 1 = 0$	b) $4x^2 + 8x - 5y - 6 = 0$	c) $y^2 + 12x - 6y - 27 = 0$

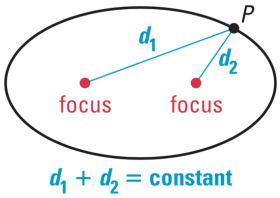
ex: Write an equation in standard form of the parabola with the given characteristics.

a) vertex: (2, 3) focus: (2, -1)	b) vertex: (1, 2) directrix: x=-1	c) focus: (4, 0) directrix: x=8

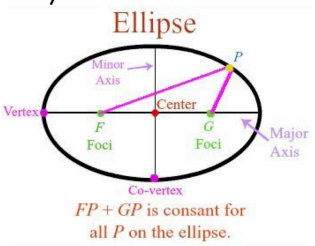
d)	e)
vertex: (3, -7) latus rectum length: 42 opens left	directrix: y=-2 right endpoint of the latus rectum: (1,0)

8.4 Ellipses, 8.5 Hyperbolas Notes

ellipse - locus of points P in a plane such that the sum of distances between P and two fixed points, called the foci, is constant



Ellipse Vocabulary



Standard Form

Horizontal Ellipse	Vertical Ellipse
$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$	$\frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1$

Where:

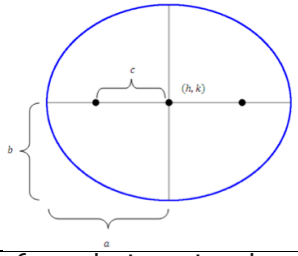
center:_____

horizontal distance:_____

vertical distance:_____

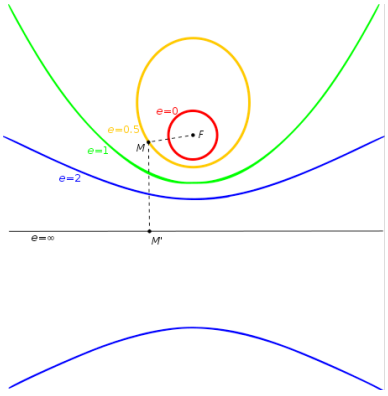
Foci - the foci of the ellipse lie on the major axis at a distance c units from the center

$c^2 = a^2 - b^2, \quad a > b$



Eccentricity - a measure of how much the conic section deviates from being circular

$e = \left|\frac{c}{a}\right|$



*The eccentricity of an ellipse is $0 < e < 1$.

ex: Sketch. State the center, foci, major axis length, minor axis length, vertices and eccentricity.

a) $\frac{(x+4)^2}{25}+\frac{(y+1)^2}{4}=1$

Center	
Foci	
Major Axis Length	
Minor Axis Length	
Vertices	
Eccentricity	

b) $(x-1)^2+\frac{(y+4)^2}{9}=1$

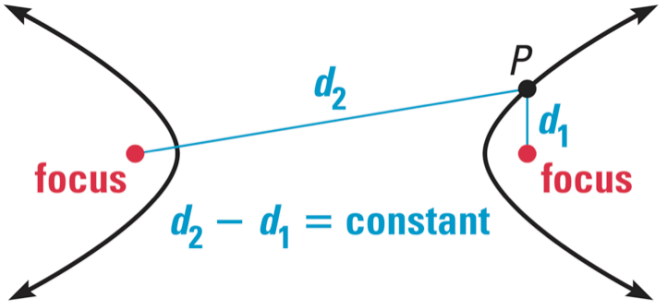
Center	
Foci	
Major Axis Length	
Minor Axis Length	
Vertices	
Eccentricity	

ex: Rewrite in standard form.

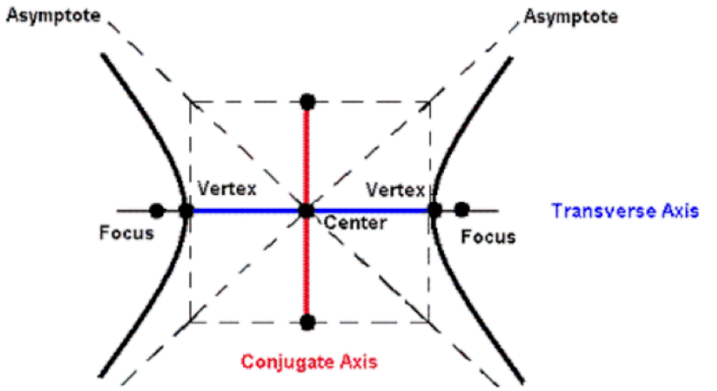
a) $4x^2+y^2-32x-4y+52=0$

b) $4(x-10)^2+21(y+2)^2=12$

hyperbola - locus of points P in a plane such that the difference of distances between P and two fixed points, called the foci, is constant



Hyperbola Vocabulary



Standard Form

Opens Left & Right	Opens Up & Down
$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$	$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$

Where:

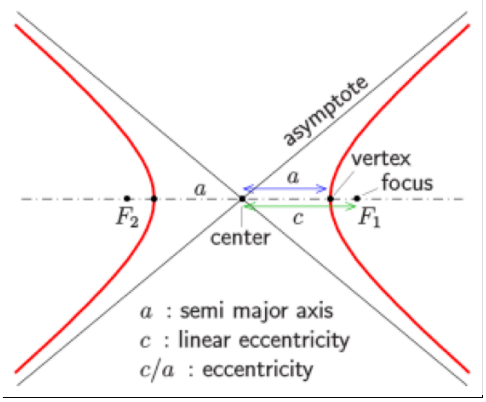
center:_____

horizontal distance:_____

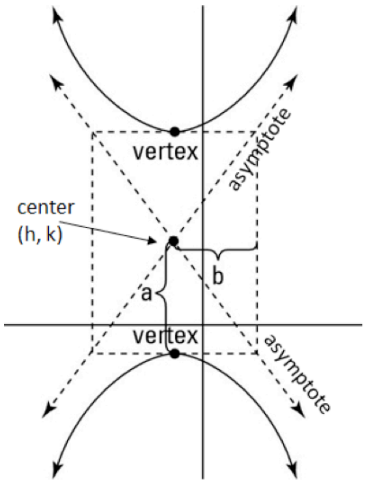
vertical distance:_____

Foci - the foci of the hyperbola lie on the transverse axis at a distance c units from the center

$c^2 = a^2 + b^2$



Asymptotes



Equations:

ex: Sketch. State the center, foci, vertices and asymptotes.

a) $\frac{(x-2)^2}{4} - \frac{y^2}{4} = 1$

Center	
Foci	
Vertices	
Asymptotes	

b) $(y+1)^2 - \frac{(x+3)^2}{4} = 1$

Center	
Foci	
Vertices	
Asymptotes	

ex: Rewrite in standard form.

$$4x^2 - 9y^2 - 16x + 18y - 65 = 0$$

8.4 Ellipses, 8.5 Hyperbolas - Day 2

Writing Equations

Ellipse	Hyperbola
NEED:	NEED:

REVIEW

ex: Sketch. State the center, foci and asymptotes (if applicable).

a) $\frac{(y+3)^2}{4} - \frac{(x-1)^2}{9} = 1$	b) $\frac{(x+2)^2}{16} + y^2 = 1$

ex: Write the standard form equation of the conic section with the given characteristics.

a) ellipse vertices: (0, 2) & (8, 2) minor axis length: 6	b) ellipse foci: (3, 0) & (-3, 0) major axis length: 12

c) ellipse verticies: (0, 5) & (0, -5) passes through the point (4, 2)	d) hyperbola vertices: (8, 14) & (8, -10) conjugate axis length: 12
e) hyperbola foci: (0, 8) & (0, -8) asymptotes: y=4x, y=-4x	f) circle center: (2, -5) tangent to the x-axis
g) parabola focus: (2, 3) directrix: y=7	

8.6 Classifying Conic Sections Notes

REVIEW - Standard Form Equations

Circle

Parabola

Ellipse

Hyperbola

Examples of Conic Sections in Standard Form

1. Circles

$2x^2+2y^2+16x-20y+32=0$	$19x^2+19y^2+14y-6x+88=0$

2. Parabolas

$4x^2-6y-8x-2=0$	$y^2-2x-4y+10=0$

3. Ellipses

$3x^2+2y^2+16x-12y-6=0$	$-4x^2-6y^2+3x+2y+12=0$

4. Hyperbolas

$5y^2-3x^2-2x-6y+8=0$	$3x^2-y^2+9=0$

General Form:_____

If B=0, then....

1. Circle when_____

2. Parabola when_____

3. Ellipse when_____

4. Hyperbola when_____

ex: Classify.

a) $y^2+4x^2-5y+4x-8=0$	b) $3x^2-3y^2-6x+4y-4=0$

ex: Classify and sketch.

a) $y^2+x+10y+26=0$	b) $x^2+y^2-6x+4y+12=0$
c) $4x^2-y^2-24x+2y+39=0$	d) $4x^2+9y^2+40x+36y+100=0$

8.7 Nonlinear Systems & Optimization Notes

Nonlinear Systems

Line and Circle System:

0 Solutions	1 Solution	2 Solutions

Line and Parabola System:

0 Solutions	1 Solution	2 Solutions

Circle and Parabola System:

0 Solutions	1 Solution	2 Solutions

3 Solutions	4 Solutions

Systems of Equations

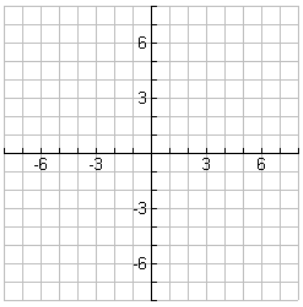
Ex: Is the point a solution to the system?

a) (1, 2)	b) (2, 7)
$4y^2 + 34x + y - 52 = 0$ $2x + y - 4 = 0$	$x^2 + y^2 \le 9$ $x^2 > -4(y-1)$

Ex: Solve graphically.

$(x-3)^2 + y^2 = 1$

$y^2 = -4(x-2)$



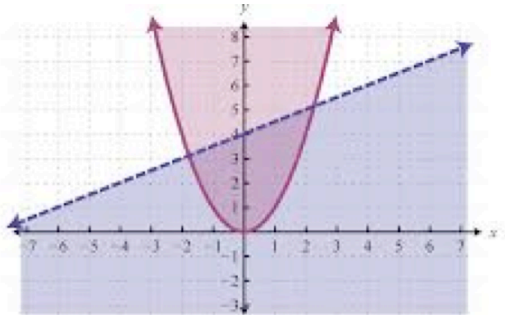
Ex: Solve algebraically.

a) $x^2 + y^2 = 13$
$y = x - 1$

b) $-2y^2 + x + 2 = 0$
$x^2 + y^2 - 1 = 0$

c) $x^2 - y^2 - 16x + 39 = 0$
$x^2 - y^2 - 9 = 0$

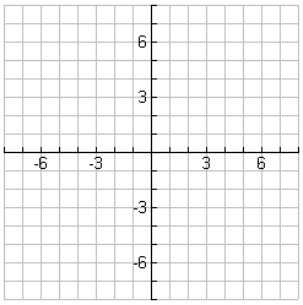
Systems of Inequalities



The solution is where the shading overlaps.

Ex: Solve graphically.

a) $x^2 + y^2 \leq 9$
$x^2 > -4(y-1)$



b) $(y+3)^2 < 8(x+2)$
$x > 2$