Graph simple reciprocal functions

$$y = \frac{1}{x-4} + 5 \qquad y = \frac{-1}{x} - 3$$

Graph not so simple functions

$$y = \frac{x-4}{x^2-x-12} \qquad y = \frac{x^2}{x^2-16} \qquad y = \frac{3x^2}{x+2}$$

Simplifying rational expressions

$$\frac{x^2-9}{x^2-9x+18} \div \frac{3x-6}{12}$$

$$\frac{x}{x-4} - \frac{3}{x+2}$$

$$\frac{\frac{2x-1}{x}}{1+\frac{1}{x^2}}$$

Solve rational equations (check for extraneous)

$$\frac{5}{x} - 2 = \frac{2}{x+3}$$

Average rate of change for a function on an interval

$$g(x) = \frac{x+1}{2x+1} \qquad [2, 5]$$

slope

$$\left(2, \frac{3}{5}\right)$$
$$\left(5, \frac{6}{11}\right)$$

$$\frac{\frac{6}{11} - \frac{3}{5}}{5-2} = \left(\frac{6 \cdot 5}{11 \cdot 5} - \frac{3 \cdot 11}{5 \cdot 11}\right) \div 3$$

$$\frac{30-33}{55} \cdot \frac{1}{3} = \frac{-1}{55}$$

$$(2x)^{-1}$$

$$\frac{1}{2x}$$

$$2x^{-1}$$

$$\frac{2}{x}$$

$$\frac{(2x)^{-1} + 3}{1 + 2x^{-1}}$$

$$\frac{2x \cdot \frac{1}{2x} + 3 \cdot 2x}{2x \cdot 1 + \frac{2}{x} \cdot 2x}$$

$$\frac{1 + 6x}{2x + 4}$$

$$y = \frac{x+1}{x^2-9}$$

x-int: $(-1, 0)$

y-int: $(0, -1/9)$

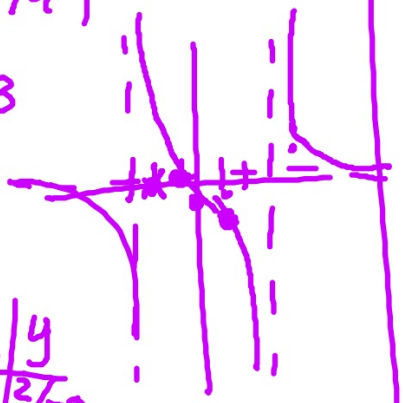
VA: $x = \pm 3$

HA: $y = 0$

SA: none

holes: none

x	y
1	$2/-8$
4	$5/7$



$$y = \frac{x^2-9}{x^2-1}$$

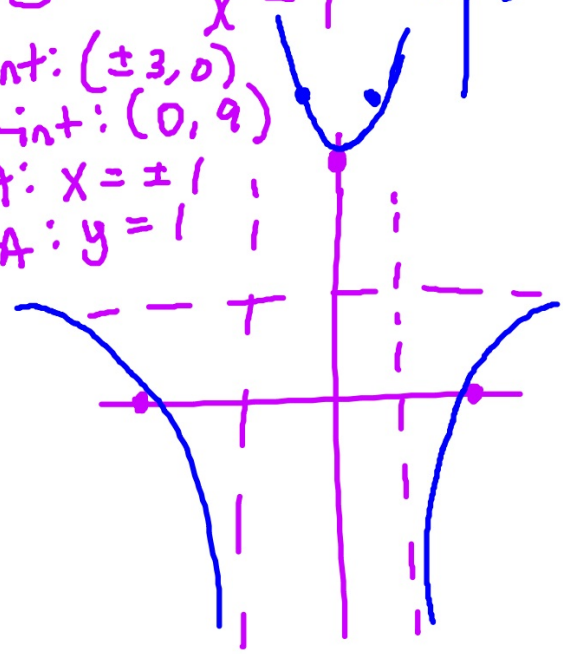
x	y
$\frac{1}{2}$	$\frac{35}{3}$

xint: $(\pm 3, 0)$

y-int: $(0, 9)$

VA: $x = \pm 1$

HA: $y = 1$



15.) $h(x) = \frac{6x^2}{x-2}$

xint: (0,0)

yint: (0,0)

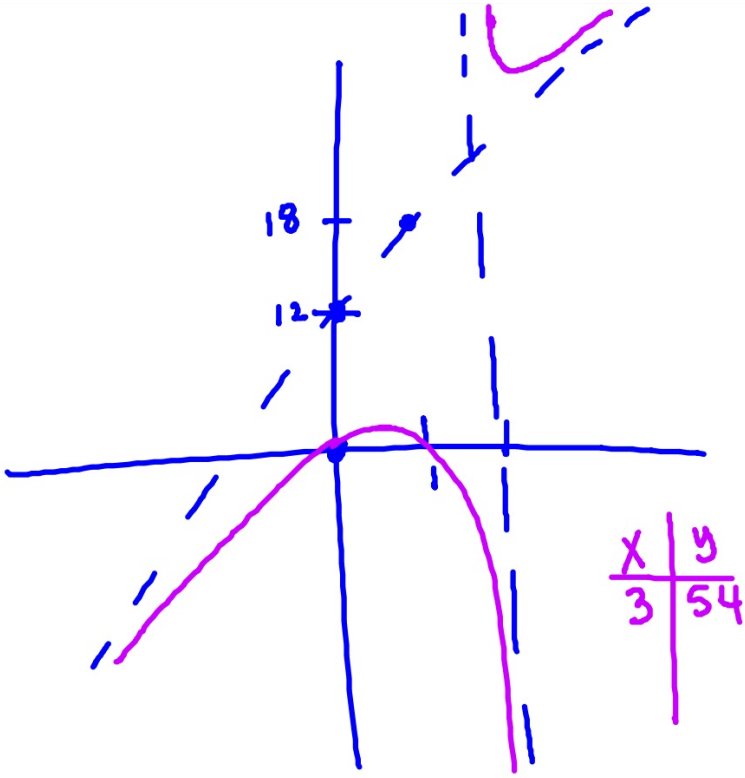
VA: $x = 2$

HA: none

SA:

$$\begin{array}{r} 6x + 12 \\ x-2 \overline{\big) 6x^2 } \\ -6x^2 + 12x \\ \hline 12x \\ -12x + 24 \\ \hline 24 \end{array}$$

$y = 6x + 12$



x	y
3	54

$$y = \frac{x^2 - 4}{x - 2}$$

$$y = \frac{(x+2)\cancel{(x-2)}}{\cancel{(x-2)}}$$

$$y = x + 2$$

hole @ $(2, 4)$

$$y = \frac{x^2 - 4}{x^2 - x - 6}$$

$$y = \frac{\cancel{(x+2)}(x-2)}{(x-3)\cancel{(x+2)}}$$

$$y = \frac{x-2}{x-3}$$

15.

$$\frac{\frac{-x^2\cancel{(4x^2-1)}}{\cancel{4x^2-1}} - \frac{5\cancel{(2x+1)}(2x-1)}{\cancel{2x+1}}}{\frac{3x}{\cancel{4x^2-1}}\cancel{(4x^2-1)}}$$

LCD $(4x^2-1)$

$$\frac{-x^2-5(2x-1)}{3x}$$

$$\frac{-x^2-10x+5}{3x}$$

$$\frac{1}{x^{-2}-4}$$

$$\frac{1\cdot x^2}{x^2\cdot\frac{1}{x^2}-4\cdot x^2}$$

$$\frac{x^2}{1-4x^2}$$

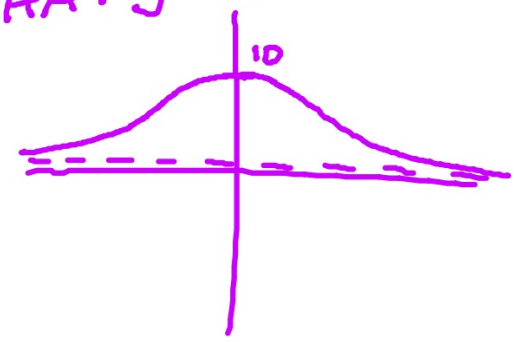
$$\frac{-x^2}{-(4x^2-1)}$$

mountain

$$y = \frac{10}{x^2+1}$$

VA: none y-int: $(0, 10)$

HA: $y = 0$



volcano

$$y = \frac{-2}{(x-4)^2}$$

y-int $\left(0, \frac{-2}{16}\right)$

VA: $x = 4$

HA: $y = 0$

$$(2x)^{-1} \qquad 2x^{-1}$$

$$\frac{1}{2x} \qquad \frac{2}{x}$$

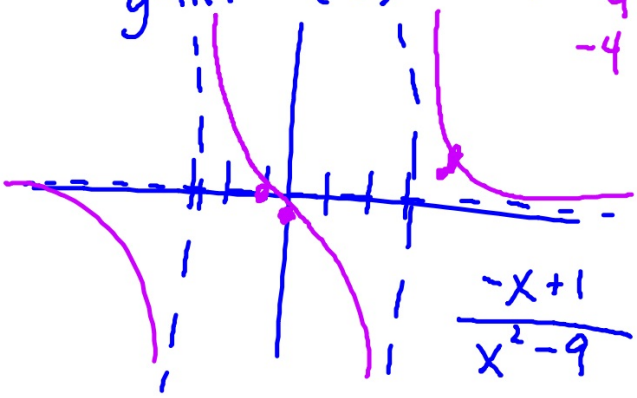
$$\frac{1+(2x)^{-1}}{x+x^{-1}} = \frac{2x\cdot 1 + \frac{1}{2x}\cdot 2x}{2x\cdot x + \frac{1}{x}\cdot 2x} = \frac{2x+1}{2x^2+2}$$

$$y = \frac{x+1}{x^2-9}$$

x-int : $(-1, 0)$

y-int : $(0, -1/9)$

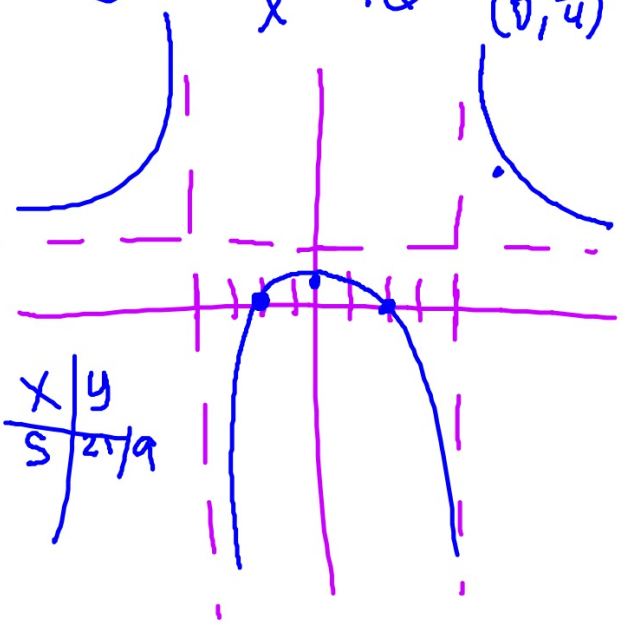
x	y
4	5
-4	-3



$$\frac{-x+1}{x^2-9}$$

$$y = \frac{x^2-4}{x^2-16}$$

$(0, \frac{1}{4})$



x	y
5	21/9

15.)

$$\frac{\frac{-x^2(4x^2-1)}{4x^2-1} - \frac{5(\cancel{2x+1})(2x-1)}{\cancel{2x+1}}}{\frac{3x}{\cancel{4x^2-1}} \cdot \cancel{4x^2-1}}$$

$$\frac{-x^2 - 5(2x-1)}{3x}$$

$$\frac{-x^2-10x+5}{3x}$$

$$\frac{1}{x^{-2}-4}$$

$$\frac{1 \cdot x^2}{x^2 \cdot \frac{1}{x^2} - 4 \cdot x^2}$$

15.) $h(x) = \frac{6x^2}{x-2}$

$0 = \frac{6x^2}{x-2}$; $0 = 6x^2$

$0 = x$

x int $(0,0)$

y int $(0,0)$

VA $x = 2$

HA none

SA $y = 6x + 12$

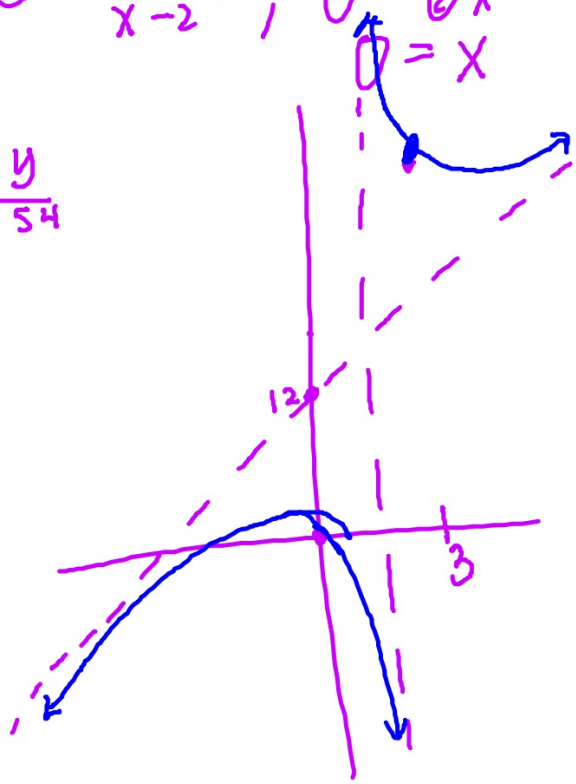
D : $\{x \mid x \neq 2\}$

symmetry

$$\frac{6(-x)^2}{(-x)-2} = \frac{6x^2}{-x-2}$$

$$\begin{array}{r} 6x + 12 \\ x-2 \overline{\big) 6x^2} \\ -6x^2 + 12x \\ \hline 12x \\ -12x + 24 \\ \hline \end{array}$$

x	y
3	54



30.) $\frac{2(x+7)}{x+4}^{2(x+4)} - 2^{2(x+4)} = \frac{2x+20}{2(x+4)}^{2(x+4)}$ LCD: $2(x+4)$

$4(x+7) - 4(x+4) = 2x+20$

$4x+28-4x-16 = 2x+20$

$-8 = 2x$

$-4 = x$

No solution

$\frac{2(x+10)}{2(x+4)}$

$$y = \frac{-3x^2}{x^2-25}$$

x-int: (0,0)

y-int: (0,0)

VA: $x = \pm 5$

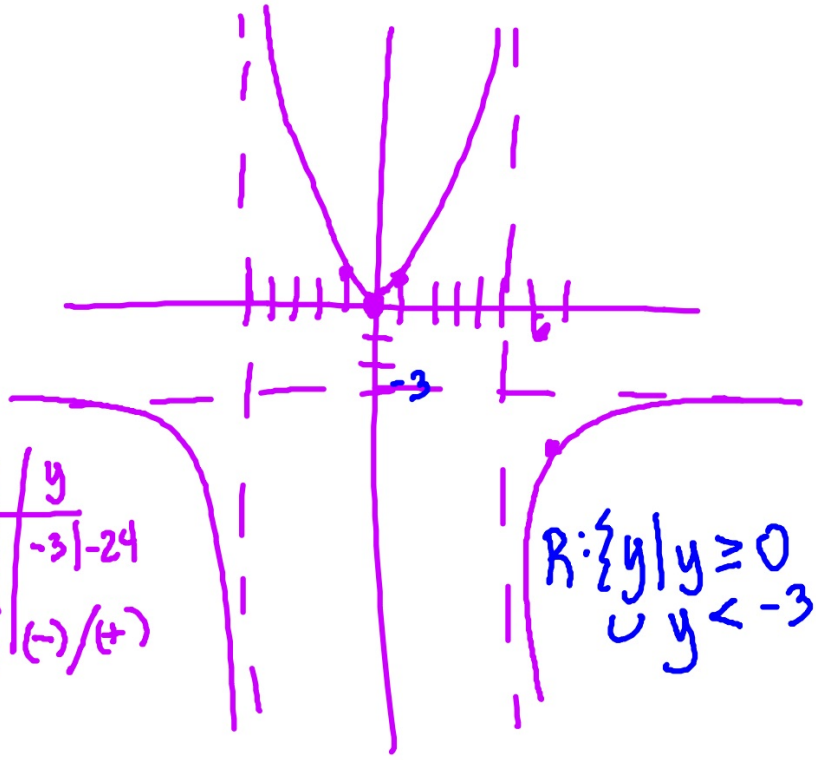
HA: $y = -3$

SA: none

D: $\{x \mid x \neq \pm 5\}$

even

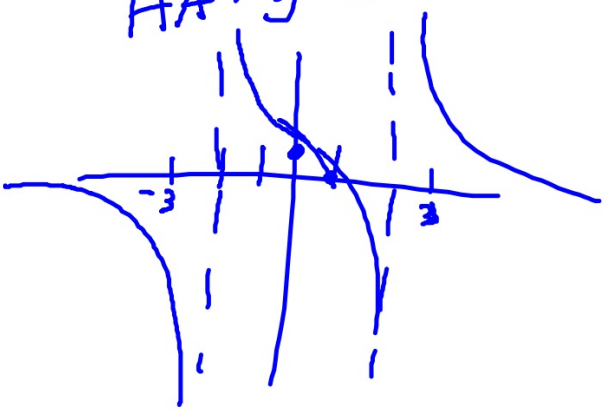
x	y
1	-3 | -24
-1	
6	(-)/(+)



R: $\{y \mid y \geq 0 \cup y < -3\}$

$$y = \frac{x-1}{x^2-4}$$

HA: $y=0$



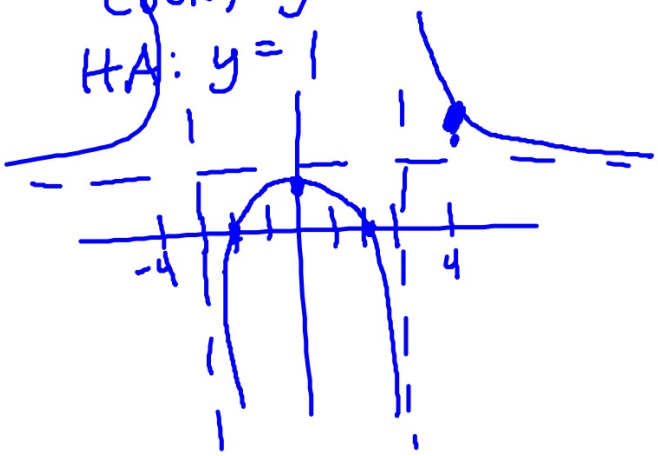
$$y = \frac{x^2-4}{x^2-9}$$

yint $\left(0, \frac{4}{9}\right)$

xint $(\pm 2, 0)$

even; y-axis

HA: $y=1$

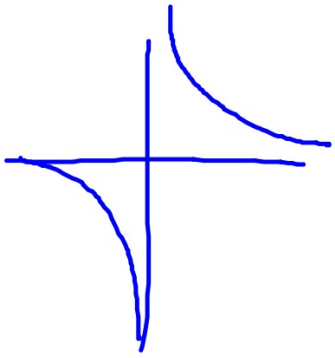


$$y = \frac{4}{x}$$

$$y(-x) = \frac{4}{-x}$$

$(1, 4)$

$(-1, -4)$



$$y = \frac{x^3 - 2x}{x^5 + x^7} \quad \text{even}$$

$$y = \frac{(-x)^3 - 2(-x)}{(-x)^5 + (-x)^7}$$

$$y = \frac{-x^3 + 2x}{-x^5 - x^7} = \frac{\cancel{-1}(x^3 - 2x)}{\cancel{-1}(x^5 + x^7)}$$

$$\frac{\frac{x^2}{1-4x^2} - \frac{5}{2x+1}}{\frac{3x}{4x^2-1}}$$

$$\frac{\frac{-x^2(4x^2-1)}{4x^2-1} - \frac{5}{2x+1}(2x+1)(2x-1)}{\frac{3x}{4x^2-1} \cdot 4x^2-1}$$

$$\frac{-x^2 - 5(2x-1)}{3x}$$

$$\frac{-x^2 - 10x + 5}{3x}$$

11

$$\frac{x+5}{(x+3)(x-2)}\cdot\frac{(x+2)}{(x+2)} - \frac{x-7}{(x-2)(x+2)}\cdot\frac{(x+3)}{(x+3)}$$

$$\frac{x^2+7x+10-(x^2-4x-21)}{(x+3)(x-2)(x+2)}$$

$$\frac{11x+31}{(x+3)(x-2)(x+2)}$$