3.4 Inverse Functions

f and g are inverse functions if

$$(f \circ g)(x) = \quad x$$

AND

$$(g \circ f)(x) = \quad x$$

Verifying Inverse Functions

1. Algebraically

Show: $(f \circ g)(x) = x$ AND $(g \circ f)(x) = x$

ex: Show $f(x) = 4x + 9$ and $g(x) = \dfrac{x-9}{4}$ are inverses, algebraically.

$$f(g(x)) = f\left(\frac{x-9}{4}\right)$$
$$= \cancel{4}\left(\frac{x-9}{\cancel{4}}\right) + 9$$
$$= x - 9 + 9$$
$$= x$$

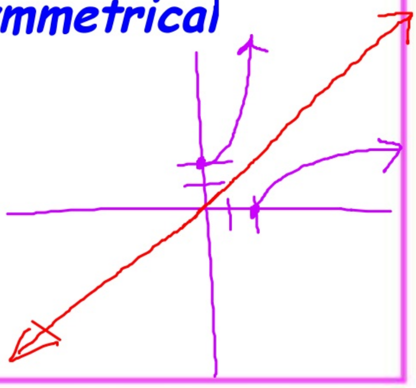
Since f(g(x)) = x and g(f(x)) = x, the functions are inverses

$$g(f(x)) = g(4x+9)$$
$$= \frac{4x + \cancel{9} - \cancel{9}}{4}$$
$$= x$$

Verifying Inverse Functions

2. Graphically

Show: *the functions are symmetrical with the line y = x*



Verifying Inverse Functions

3. Numerically (NOT A PROOF)

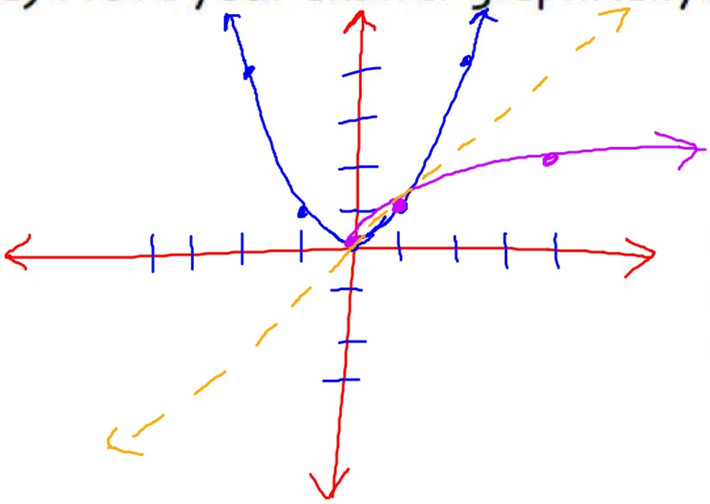
Show:

the x-coordinate and the y-coordinate switch places if the functions are inverses ex: If f and g are inverse functions and f contains the point (2, -3) then g must contain the point (-3, 2).

ex: Are $f(x) = x^2$ and $g(x) = \sqrt{x}$ inverses?

x	$g(x)$
0	0
1	1
4	2

a) Prove your answer graphically.



No. f(x) and g(x) are not symmetrical with the line y= x

the functio
are not sym

ex: Are $f(x) = x^2$ and $g(x) = \sqrt{x}$ inverses?

a) Prove your answer algebraically.

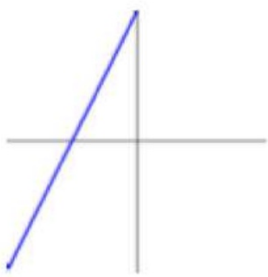
$f(g(x)) = (\sqrt{x})^2$
$= x$

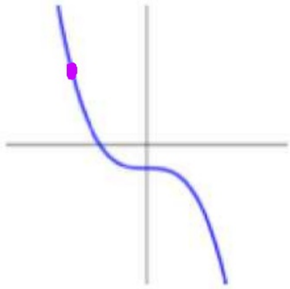
$g(f(x)) = \sqrt{x^2}$
$= |x|$

No. f(g(x)) = x
but g(f(x)) does
not equal x

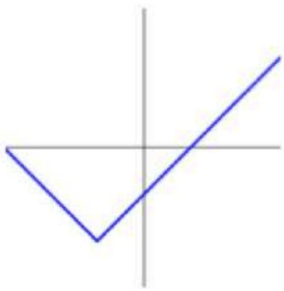
The Existence of an Inverse

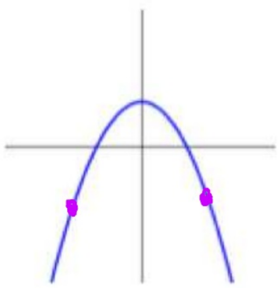
Examples of functions that DO have inverses:





Examples of functions that DO NOT have inverse functions.





The Existence of an Inverse

A function has an inverse function if it passes BOTH the vertical line test (VLT) and horizontal line test(HLT).

One-To-One *(always increasing or always decreasing)*

A function is one-to-one if it passes BOTH the vertical line test (VLT) and horizontal line test(HLT).

Inverse Notation

$x^{-1} = \frac{1}{x}$

$$\boxed{f^{-1}(x)}$$

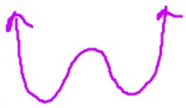
NOTE: $f^{-1}(x) \neq [f(x)]^{-1}$ or $\frac{1}{f(x)}$

3.4 Notes

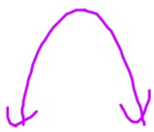
Determine whether each function has an inverse function.

1. $f(x) = x + 1$ Yes

2. $f(x) = x^4 - x^2 + 7$ No



3. $f(x) = -3x^2 + 4x + 5$ No

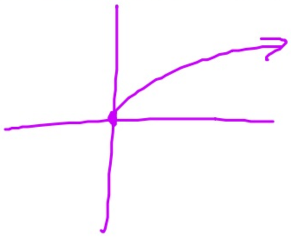


*See printout.

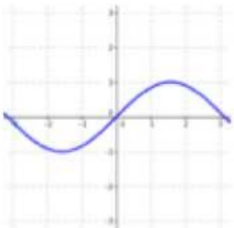
3.4 Notes

Determine whether each function has an inverse function.

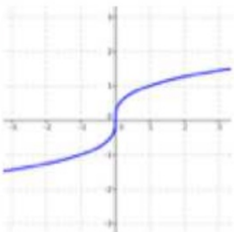
4. $f(x) = \sqrt{x}$



Yes

5. 

No

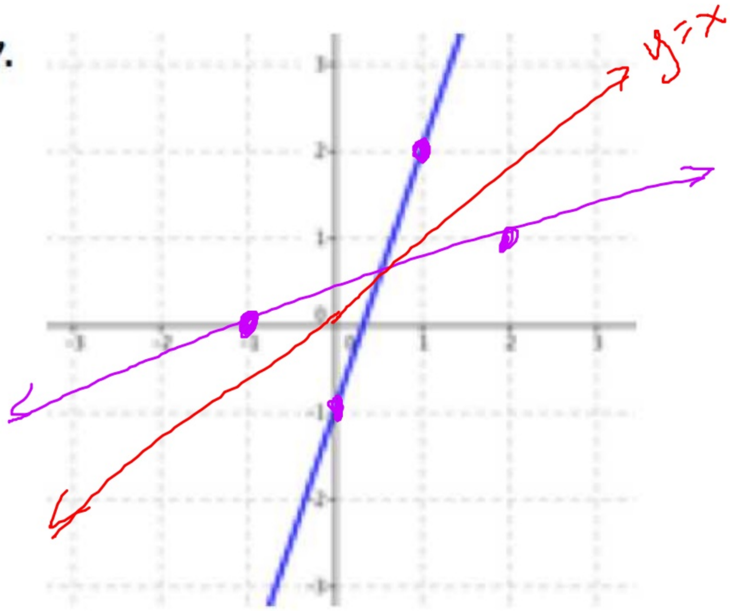
6. 

Yes

3.4 Notes

Sketch the inverse function, if it exists.

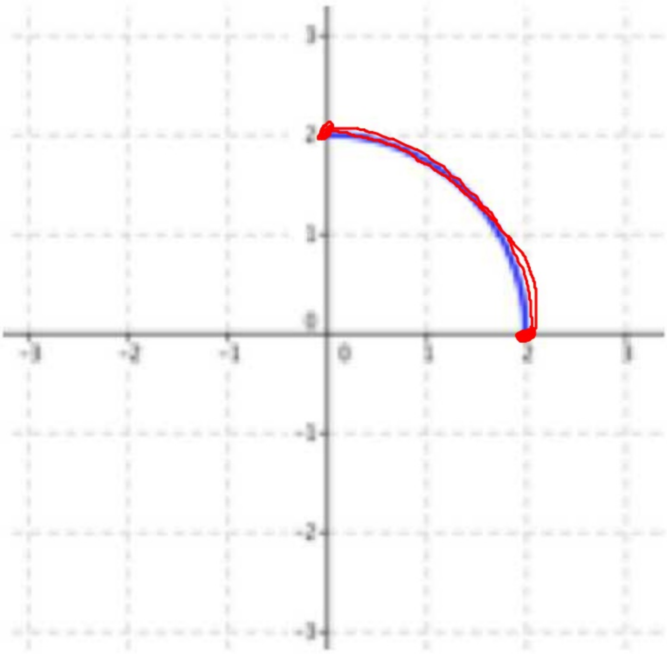
7.



3.4 Notes

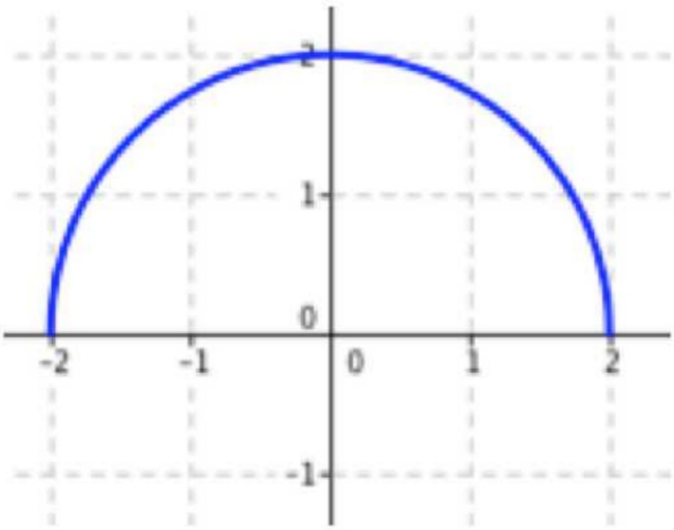
Sketch the inverse function, if it exists.

8.



3.4 Notes

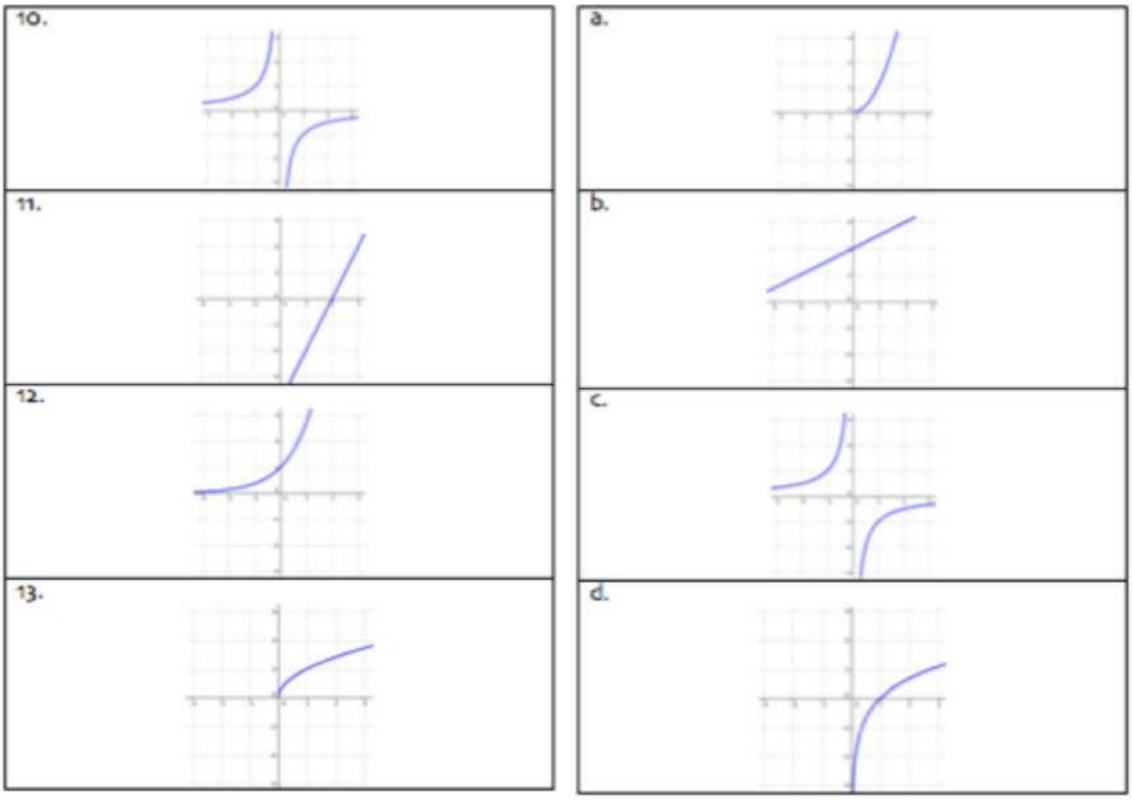
Sketch the inverse function, if it exists.

9. 

Does not exist

3.4 Notes

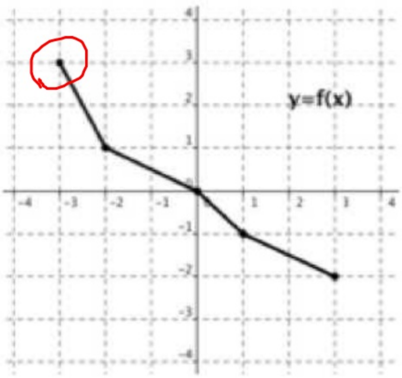
Match each graph with the graph of its inverse.



3.4 Notes

Find the indicated values, if possible.

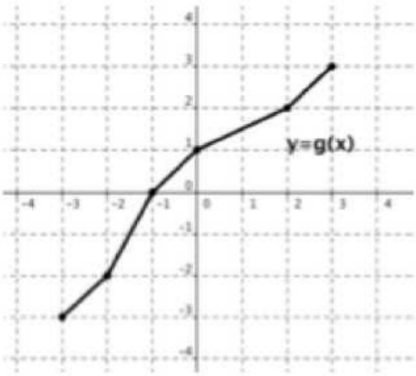
$f(3) = -2$



y-coord.

14. $f^{-1}(3) = -3$

15. $g^{-1}(0)$

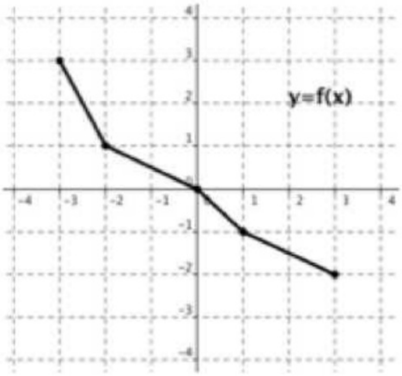


16. $g^{-1}(1)$

17. $f^{-1}(0)$

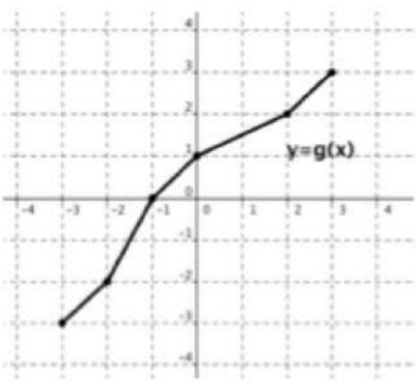
3.4 Notes

Find the indicated values, if possible.



18. $(f \circ g)(-1)$

19. $(g \circ f)(-2)$

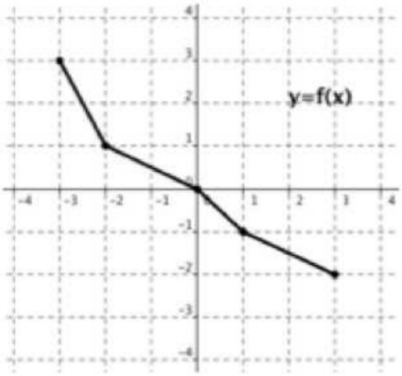


20. $(f \circ g^{-1})(1)$

21. $(g \circ f^{-1})(-1)$

3.4 Notes

Find the indicated values, if possible.



22. $\left(f \circ f^{-1}\right)(2)$

23. $\left(g^{-1} \circ f^{-1}\right)(2)$