Analyzing Polynomial Functions

Polynomial Degrees and Number of Turning Points

Polynomial Type	Degree	Maximum Number of Turning Points
Constant	0	0
Linear	1	0
Quadratic	2	1
Cubic	3	2
n^{th} Degree Polynomial	n	$n-1$

*See printout.

ex 1) Determine the degree and state the maximum number of turning points.

a) $f(x) = 2x^3 + 5x^2 - 9$

degree: 3 max turn: 2

b) $f(x) = 9 - 6x^2$

degree: 2 max turn: 1

ex 1) Determine the degree and state the maximum number of turning points.

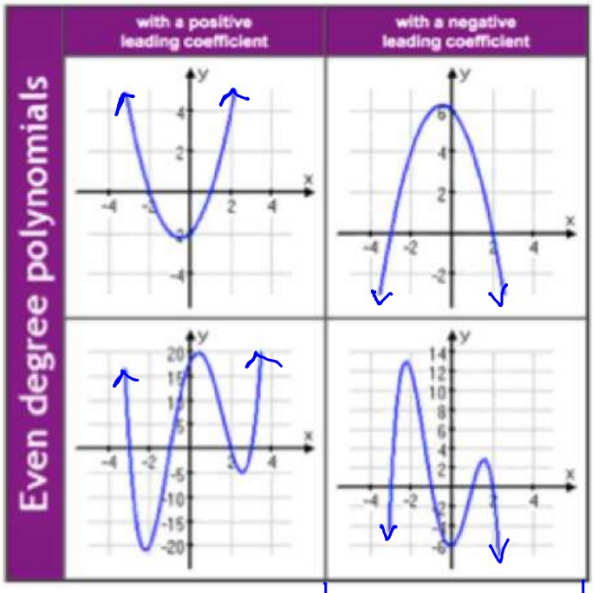
c) $f(x) = (x-2)^5(x+3)^6$

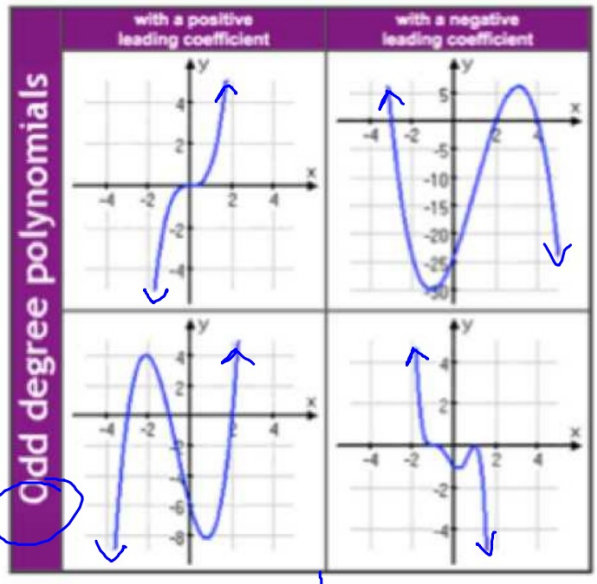
degree: 11 ; maxturn 10

d) $f(x) = 5(1-x^2)^7$

degree: 14 ; maxturn 13

End Behavior





$x \to -\infty$ $y \to \infty$
$x \to \infty$ $y \to \infty$

$x \to -\infty$ $y \to -\infty$
$x \to +\infty$ $y \to -\infty$

$x \to -\infty$ $y \to -\infty$
$x \to \infty$ $y \to \infty$

$x \to -\infty$ $y \to \infty$
$x \to \infty$ $y \to -\infty$

Stating End Behavior

$x \to -\infty, \quad y \to$ $+\infty$ or $-\infty$

$x \to \infty, \quad y \to$ $+\infty$ $-\infty$

ex 2) Determine the end behavior of each polynomial.

a) $f(x) = 2x^3 + 5x^2 - 9$

degree: odd

l.c.: +

$x \to -\infty$ $y \to -\infty$

$x \to \infty$ $y \to \infty$

b) $f(x) = 9 - 6x^2$

degree: even

l.c.: (−)

$x \to -\infty$ $y \to -\infty$

$x \to \infty$ $y \to -\infty$

ex 2) Determine the end behavior of each polynomial.

c) $f(x) = (x-2)^5(x+3)^6$

degree: odd

l.c. : +

$x \to -\infty$ $y \to -\infty$

$x \to \infty$ $y \to \infty$

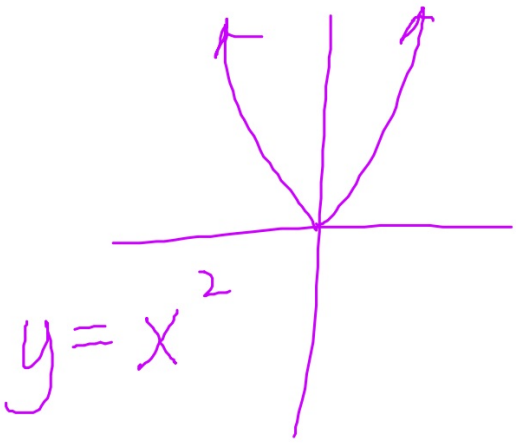
d) $f(x) = 5(1-x^2)^7$

degree: 14

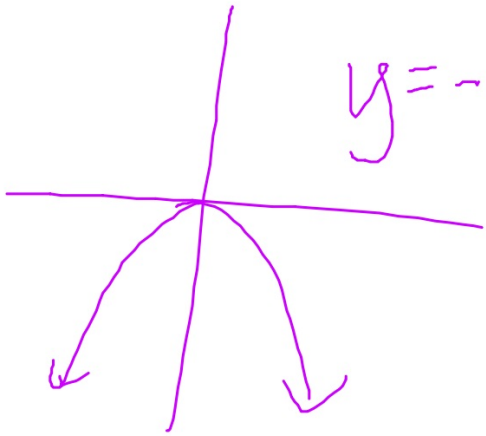
l.c. : (−)

$x \to -\infty$ $y \to -\infty$

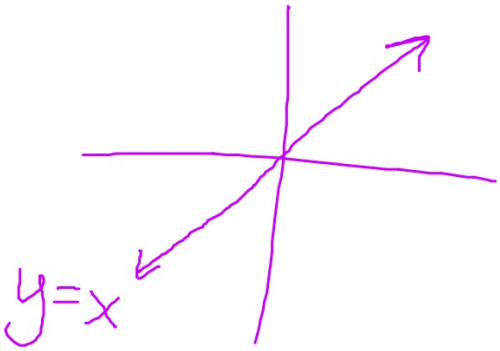
$x \to +\infty$ $y \to -\infty$



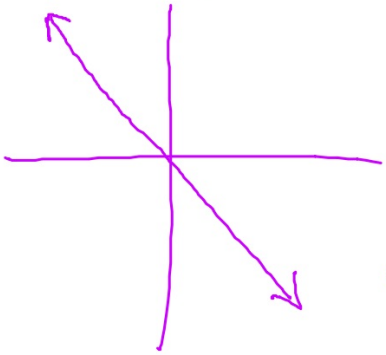
$y = x^2$



$y = -x^2$

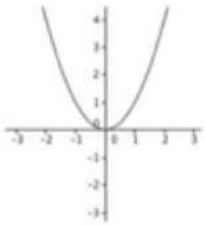
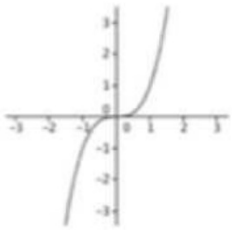


$y = x$



$y = -x$

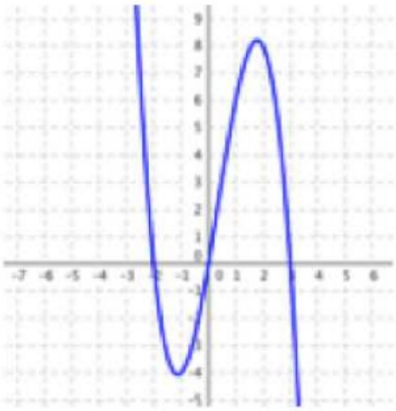
Bouncing and Crossing Zeros

In the graph below the graph "bounces" off the x-axis at x=0. 	In the graph below the graph "crosses" the x-axis at x=0. 

ex 3) Using the graph of the polynomial function,

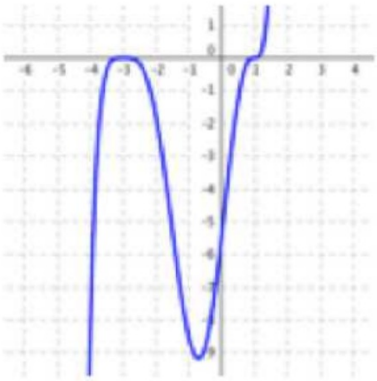
1. State the degree of the polynomial.
2. Find the zeros. State the multiplicity if greater than 1.
3. State the end behavior.
4. Determine whether the graph "crosses" the x-axis or "bounces" off the x-axis at each zero.

a) $f(x) = -x(x+2)(x-3)$

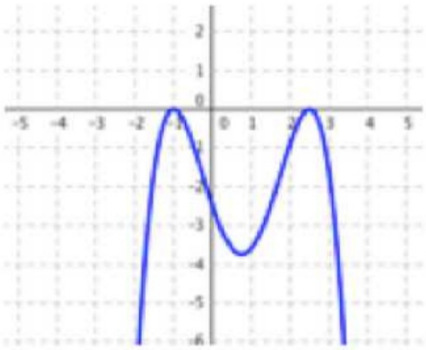


1. 3
2. $-2, 0, 3$
3. $x \to -\infty \quad y \to \infty$
   $x \to \infty \quad y \to -\infty$
4. all are crossing

b) $f(x) = \frac{1}{15}(x+3)^4 (x-1)^3$



c) $f(x) = -\frac{1}{10}(2x-5)^2(x+1)^2$

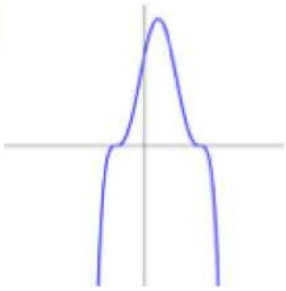


- A graph "crosses" the x-axis at a zero if the multiplicity of that zero is ____odd____.

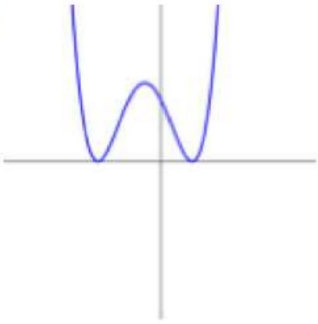
- A graph "bounces" off the x-axis at a zero if the multiplicity of that zero is ____even____.

ex 4) Which of the following graphs could represent the polynomial $f(x) = a(x-b)^2(x-c)^3$?

a)



c)



b)

d)