36.) $$\frac{\frac{\sqrt{2}}{2} - \frac{1}{2}}{\frac{\sqrt{2}}{2} + \frac{1}{2}} = \frac{\frac{\sqrt{2}-1}{2}}{\frac{\sqrt{2}+1}{2}}$$

$$\frac{(\sqrt{2}-1)}{\sqrt{2}+1} \frac{\sqrt{2}-1)}{(\sqrt{2}-1)}$$

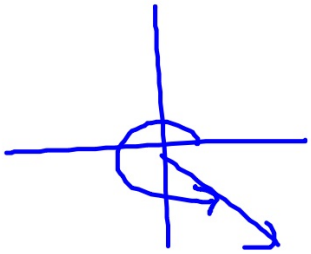
$$\frac{2-2\sqrt{2}+1}{2-1} = 3-2\sqrt{2}$$

22.) $\cos\frac{49\pi}{6} = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$

$\frac{49\pi}{6} - 4(2\pi)$

$\frac{49\pi}{6} - 8\pi$

$\frac{\pi}{6}$



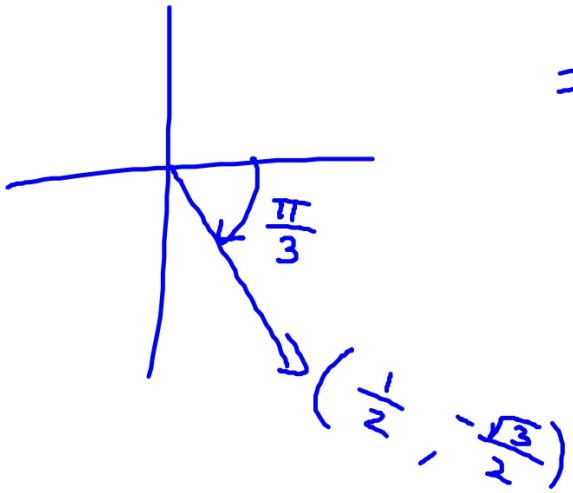
$\frac{15\pi}{4} - 2\pi$

$\frac{15\pi - 8\pi}{4}$

$\frac{7\pi}{4}$

18.) $\cot\left(-\frac{\pi}{3}\right) = \dfrac{\frac{1}{2}}{-\frac{\sqrt{3}}{2}}$

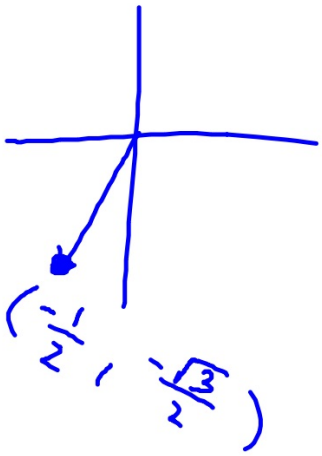
$= -\frac{\sqrt{3}}{3}$



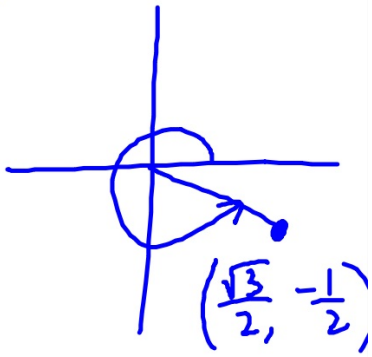
$\tan\theta = \frac{y}{x}$

$\cot\theta = \frac{x}{y}$

32.) $\tan\left(\frac{4\pi}{3}\right) - \tan\left(\frac{11\pi}{6}\right)$



$$\frac{-\frac{\sqrt{3}}{2}}{-\frac{1}{2}} \qquad \frac{-\frac{1}{2}}{\frac{\sqrt{3}}{2}}$$



$$\frac{\sqrt{3}}{1} + \frac{\sqrt{3}}{3}$$

$$\frac{3\sqrt{3}+\sqrt{3}}{3} = \frac{4\sqrt{3}}{3}$$

9.3 Continued

Reference Angle: the acute angle formed by the terminal side of angle θ and the closest x-axis.

reference angle θ'

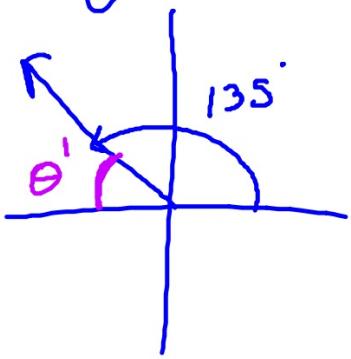
$0 < \theta' < 90^\circ$

$0 < \theta' < \frac{\pi}{2}$

There is never a reference angle for quadrantal angles.

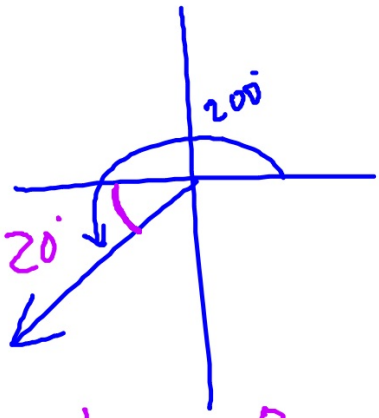
Find the reference angle.

$\theta = 135^\circ$



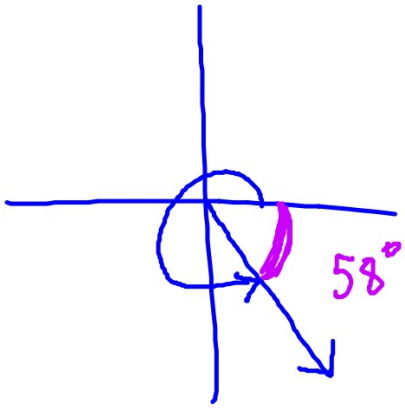
$\theta' = 45^\circ$

$\theta = 200^\circ$



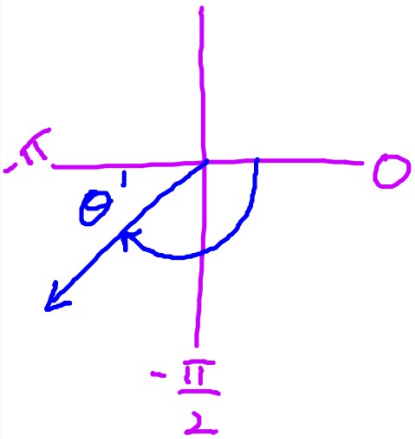
$\theta' = 20^\circ$

$\theta = 302^\circ$



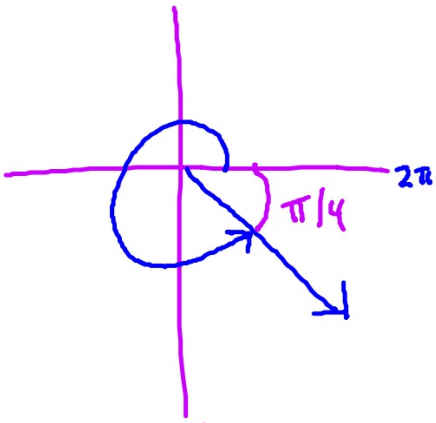
$\theta = 58^\circ$

$\theta = \frac{-2\pi}{3}$



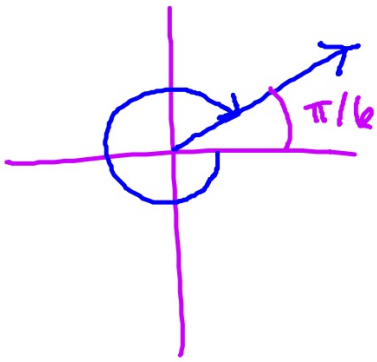
$\theta' = \frac{\pi}{3}$

$\theta = \frac{7\pi}{4}$



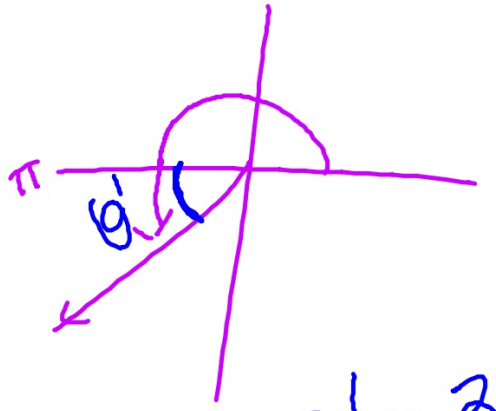
$\theta' = \frac{\pi}{4}$

$\theta = \frac{-11\pi}{6}$



$\theta' = \frac{\pi}{6}$

$$\frac{7\pi}{5}$$

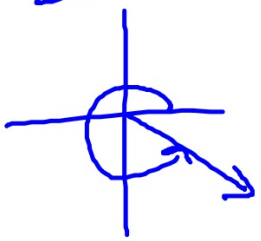


$$\frac{5\pi}{5} = \pi$$

$$\theta' = \frac{2\pi}{5}$$

What is the sign? (Positive or negative)

$\sin 332^\circ$

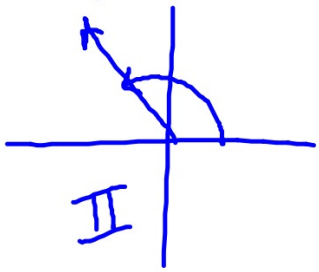


IV

$(-)$

$\sin 332^\circ < 0$

$\cos \frac{2\pi}{3}$

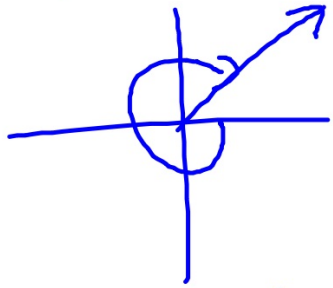


II

$(-)$

$\cos \frac{2\pi}{3} < 0$

$\sin(-332^\circ)$



$\sin(-332^\circ) > 0$

Rewrite using a reference angle, then evaluate.

① $\cos\frac{5\pi}{4} = -\cos\frac{\pi}{4} = \frac{-\sqrt{2}}{2}$

III

$\tan\theta = \frac{y}{x}$

$\cot\theta = \frac{x}{y}$

② $\sin\frac{11\pi}{6} = -\sin\frac{\pi}{6} = \frac{-1}{2}$

IV

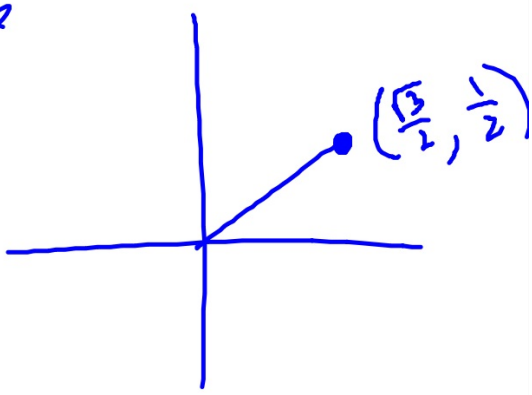
$\frac{\pi}{6}$

③ $\cot\frac{5\pi}{6} = -\cot\frac{\pi}{6}$

$\frac{\sqrt{7}}{7}$

$\sqrt{7}$

$= -\dfrac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}$

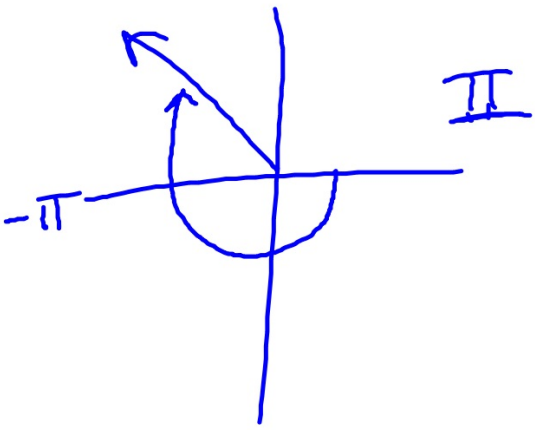


$\frac{\sqrt{2}}{2}$

$\sqrt{2}$

$= -\sqrt{3}$

④ $\tan\left(-\frac{5\pi}{4}\right) = -\tan\frac{\pi}{4}$



$$= -\frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}}$$

$$= -1$$

	0°	30°	45°	60°	90°
$\sin\theta$	0	$1/2$	$\sqrt{2}/2$	$\sqrt{3}/2$	1
$\cos\theta$	1	$\sqrt{3}/2$	$\sqrt{2}/2$	$1/2$	0
$\tan\theta$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	und.

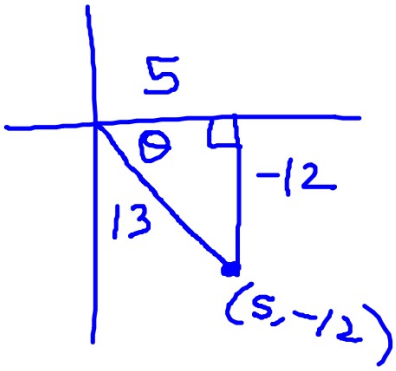
$\sin 240^\circ$

$-\frac{\sqrt{3}}{2}$

$\sqrt{\frac{0}{4}}$ $\sqrt{\frac{1}{4}}$ $\sqrt{\frac{2}{4}}$ $\sqrt{\frac{3}{4}}$ $\sqrt{\frac{4}{4}}$

Use the given point on the terminal side of an angle θ in standard position to evaluate the six trignometric functions of θ.

$(5, -12)$



$\sin\theta = \frac{-12}{13}$ $\csc\theta = \frac{-13}{12}$

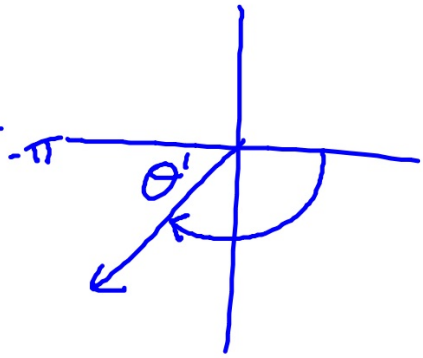
$\cos\theta = \frac{5}{13}$ $\sec\theta = \frac{13}{5}$

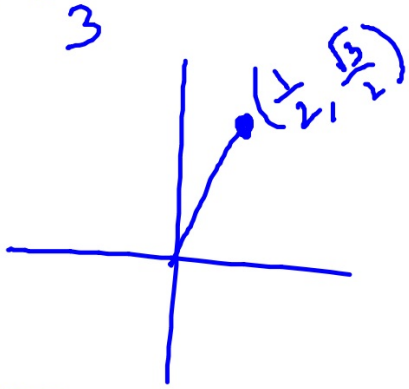
$\tan\theta = \frac{-12}{5}$ $\cot\theta = -\frac{5}{12}$

20.) $\cot\left(-\frac{8\pi}{3}\right) = +\frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{\sqrt{3}}{3}$

$-\frac{8\pi}{3} + 2\pi \quad \theta' = \frac{\pi}{3}$

$-\frac{2\pi}{3}$

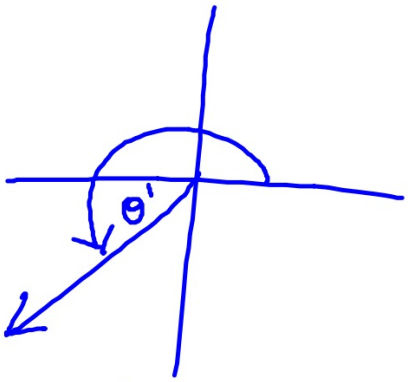




$\tan 30^\circ = \frac{\sqrt{3}}{3}$

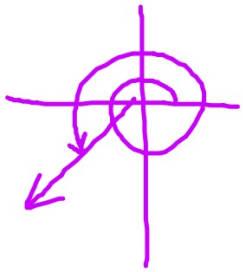
$\cot 30^\circ = \sqrt{3}$

$\cos(202^\circ) = -\cos(22^\circ)$



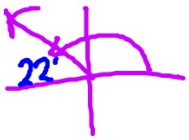
$\theta' = 22^\circ$

$\cos(562^\circ)$ ✓

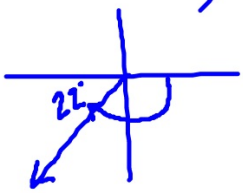


$-\cos(22^\circ)$ ✓

$\cos(158^\circ)$ ✓

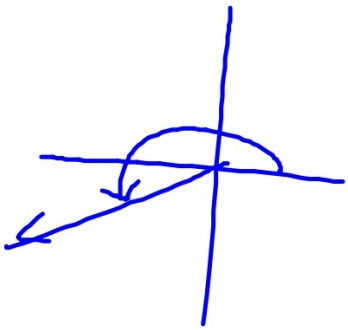


$\cos(-158^\circ)$ ✓



$-\sin(68^\circ)$ ✓

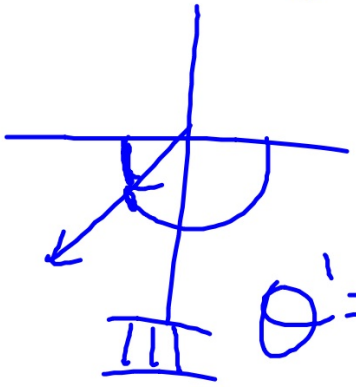
14.) $(\tan 181^\circ)^3 > 0$



(+)

$$\tan\theta = \frac{y}{x} = \frac{\sin\theta}{\cos\theta}$$

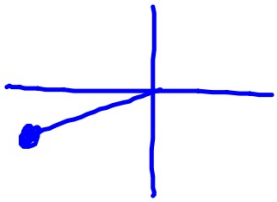
$$\sin(-150^\circ) = -\frac{1}{2}$$

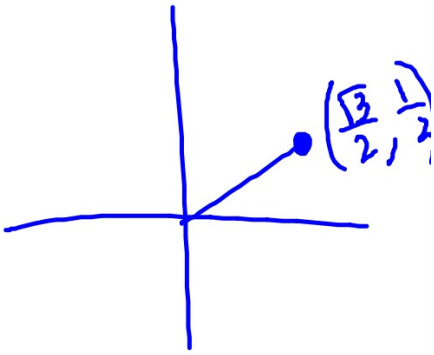


$\theta' = 30^\circ$

III

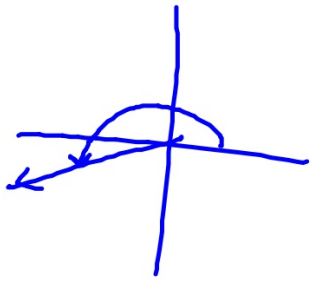
neg



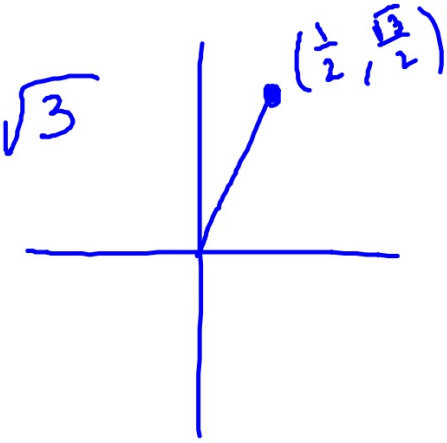


14.) $\tan^3(181^\circ) > 0$

$(\tan 181^\circ)^3$



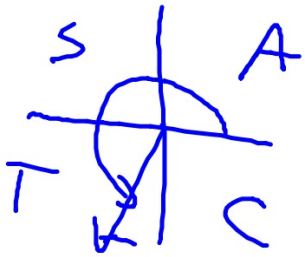
16.) $\tan 240^\circ = \dfrac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}$



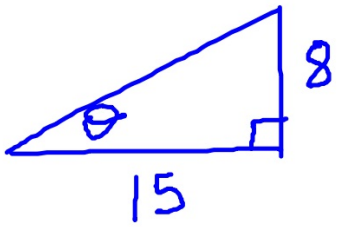
III

$(+)$

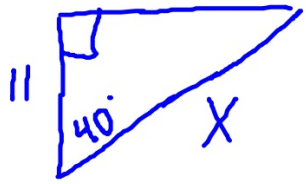
$\theta' = 60^\circ$



1. Write all 6 trig functions



2. Set up to solve but do not solve.

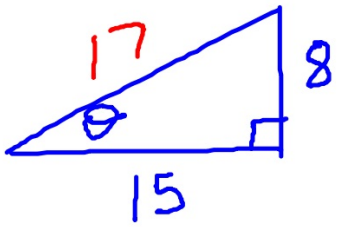


3. find 2 coterminal angles, one (−) and one (+)

a.) 420°

b.) $\frac{17\pi}{6}$

1. Write all 6 trig functions



$\sin\theta = \frac{8}{17}$

$\cos\theta = \frac{15}{17}$

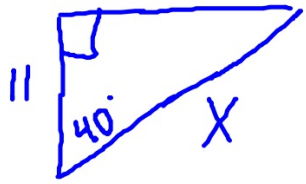
$\tan\theta = \frac{8}{15}$

$\cot\theta = \frac{15}{8}$

$\sec\theta = \frac{17}{15}$

$\csc\theta = \frac{17}{8}$

2. Set up to solve but do not solve.



$\cos 40° = \frac{11}{x}$

or

$x = \frac{11}{\cos 40°}$

3. find 2 coterminal angles, one (−) and one (+)

a.) $420°$

$60°, -300°$

b.) $\frac{17\pi}{6}$

$\frac{5\pi}{6}, \quad -\frac{7\pi}{6}$

④ Convert.

a. 300° b. $\frac{3\pi}{5}$ c. -4π d. -515°

⑤ Find the reference angle.

a. -199°

b. $\frac{21\pi}{8}$

c. $\frac{8\pi}{3}$

④ Convert.

a. 300° $\quad \frac{5\pi}{3}$

b. $\frac{3\pi}{5}$ $\quad 108^\circ$

c. -4π $\quad -720^\circ$

d. -515° $\quad \frac{-515^\circ}{1} \cdot \frac{\pi}{180^\circ}$ $\quad \frac{-103\pi}{36}$

⑤ Find the reference angle.

a. -199° $\quad 19^\circ$

b. $\frac{21\pi}{8}$ $\quad \frac{21\pi}{8} - 2\pi = \frac{5\pi}{8}$

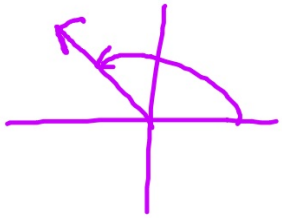
$\boxed{\frac{3\pi}{8}}$

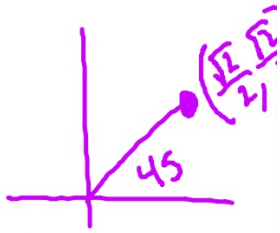
c. $\frac{8\pi}{3}$ $\quad \frac{\pi}{3}$

⑥ $\sec 135^\circ = -\sqrt{2}$

II $\theta' = 45^\circ$

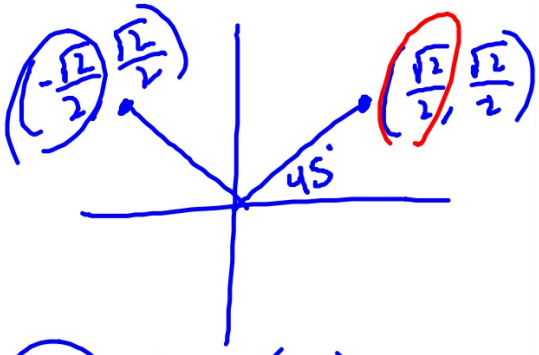
(−)





⑦ $\sin\left(-\frac{2\pi}{3}\right)$ ⑧ $\cot\left(\frac{11\pi}{6}\right)$ ⑨ $\tan\left(\frac{\pi}{2}\right)$

(6) $\sec 135^\circ = -\sqrt{2}$



II neg.

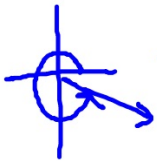
$\theta' = 45^\circ$

(7) $\sin\left(-\frac{2\pi}{3}\right)$

$-\frac{\sqrt{3}}{2}$

(8) $\cot\left(\frac{11\pi}{6}\right)$

$-\sqrt{3}$



$-\cot\frac{\pi}{6} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}$

(9) $\tan\left(\frac{\pi}{2}\right)$

Undefined

10 $\tan(-240^\circ)$

11 $\sin\left(\frac{5\pi}{2}\right)$

12 $\sec\left(\frac{5\pi}{6}\right)$

13 $\cot(225^\circ)$

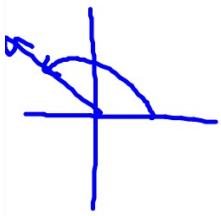
(10) $\tan(-240^\circ)$

$-\sqrt{3}$

(11) $\sin\left(\frac{5\pi}{2}\right)$

1

(12) $\sec\left(\frac{5\pi}{6}\right)$



$-\frac{2\sqrt{3}}{3}$

(13) $\cot(225^\circ)$

1