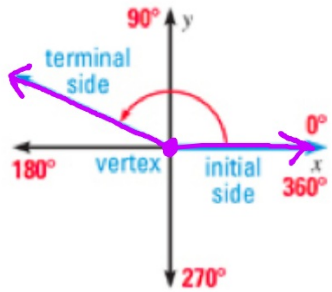
Angles in Standard Position

In a coordinate plane, an angle can be formed by fixing one ray, called the **initial side**, and rotating the other ray, called the **terminal side**, about the vertex.

An angle is in **standard position** if its vertex is at the origin and its initial side lies on the positive x-axis.



Rotations counter-clockwise: positive

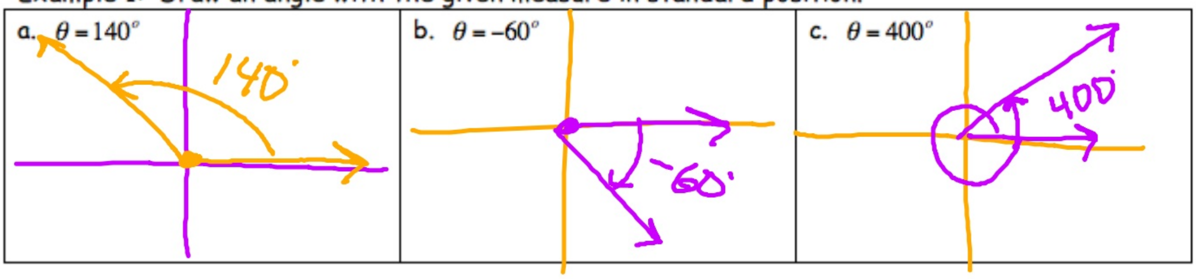
Rotations clockwise: negative

Example 1: Draw an angle with the given measure in standard position.

a. $\theta = 140^\circ$

b. $\theta = -60^\circ$

c. $\theta = 400^\circ$



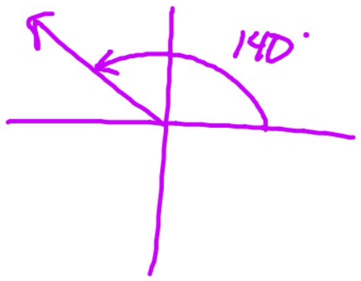
Definition of coterminal angles: angles where the terminal sides coincide.

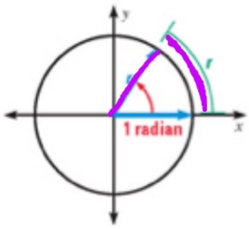
*To find a coterminal angle, add or subtract multiples of 360°.

Example 2: Find one positive and one negative coterminal angle for 140°.

$$140^\circ + 360^\circ = 500^\circ$$

$$140^\circ - 360^\circ = -220^\circ$$



Angles can also be measured in radians. One radian is the measure of an angle in standard position whose terminal side intercepts an arc of length r. Because the circumference of a circle is $2\pi r$, there are 2π radians in a full circle. Therefore: $360^o = 2\pi$ radians and $180^o = \pi$ radians	

$$1 rad = \frac{180}{\pi} = 57.3^\circ$$

Ex 3: Convert from degrees to radians

Ex 4: Convert from radians to degrees

100° $10\cancel{0}^{5} \cdot \frac{\pi}{18\cancel{0}}$ $\quad \frac{5\pi}{9}$	$\frac{7\pi}{6}$ $\frac{7\cancel{\pi}}{\cancel{6}_1} \cdot \frac{\cancel{180}^{30}}{\cancel{\pi}} = 210^{\circ}$

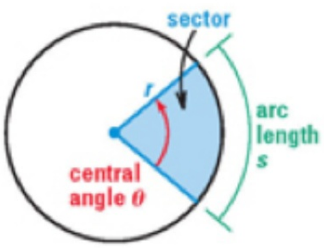
$\frac{10\pi}{18}$

Arc Length and Area of a Sector

The arc length s and area A of a sector with radius r and central angle θ (measured in radians) are as follows.

Arc length: $s = r\theta$

Area: $A = \frac{1}{2}r^2\theta$



Example 5: Find the arc length and area given r = 3 cm and central angle $\theta = 120^\circ$.

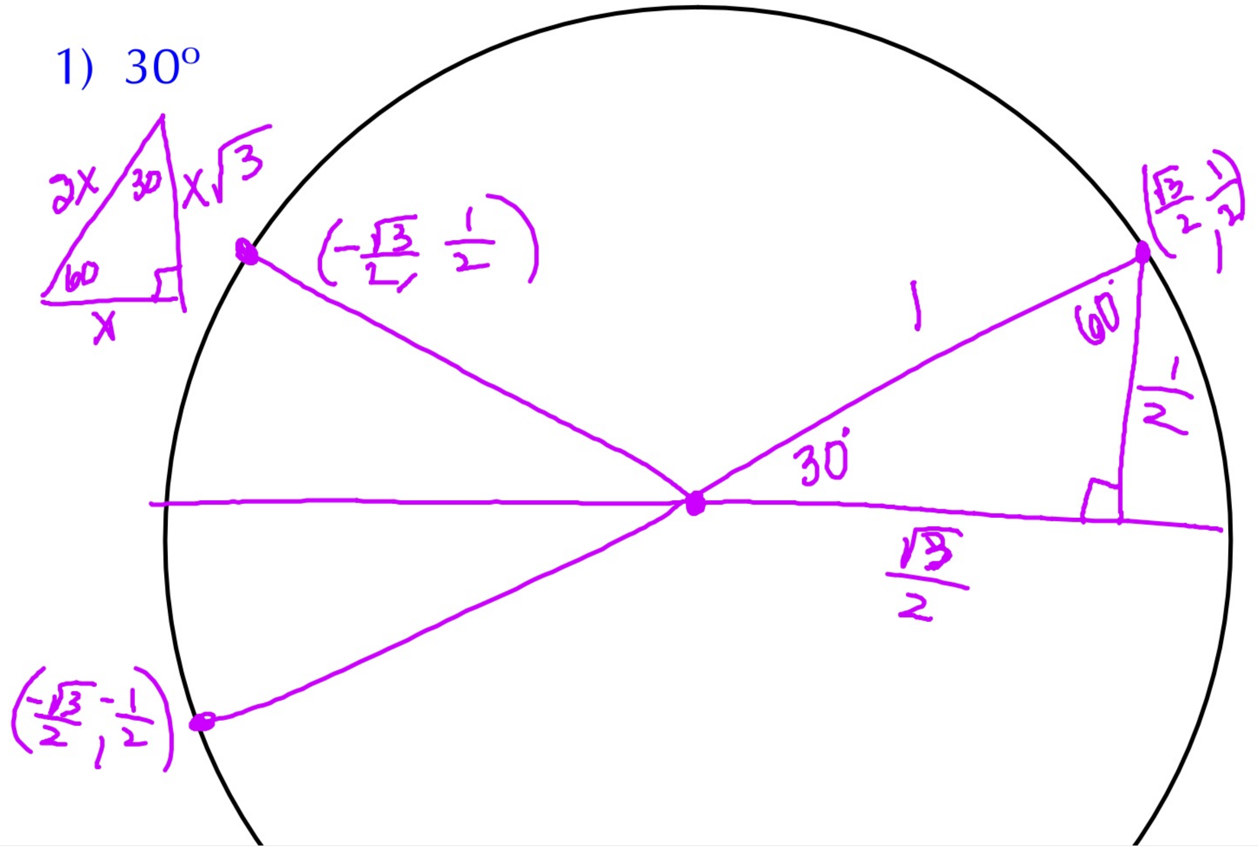
arc length	area
$\theta = \frac{2\pi}{3}$ $S = r\theta = 3\left(\frac{2\pi}{3}\right)$ $= \boxed{2\pi \text{ cm}}$	$A = \frac{1}{2}r^2\theta$ $= \frac{1}{2}(9)\frac{2\pi}{3} = 3\pi \text{ cm}^2$

Example 6: Use a calculator to evaluate the trigonometric expression. Round to 3 decimal places.

$\cos\left(\frac{5\pi}{7}\right)$ $-.623$	$\cot(500^{o})$ -1.192	$\csc\left(-300^{o}\right)$ 1.155	$\sin\left(\frac{11\pi}{5}\right)$ 0.588

$$\frac{1}{\tan 500^{\circ}}$$

1) 30°

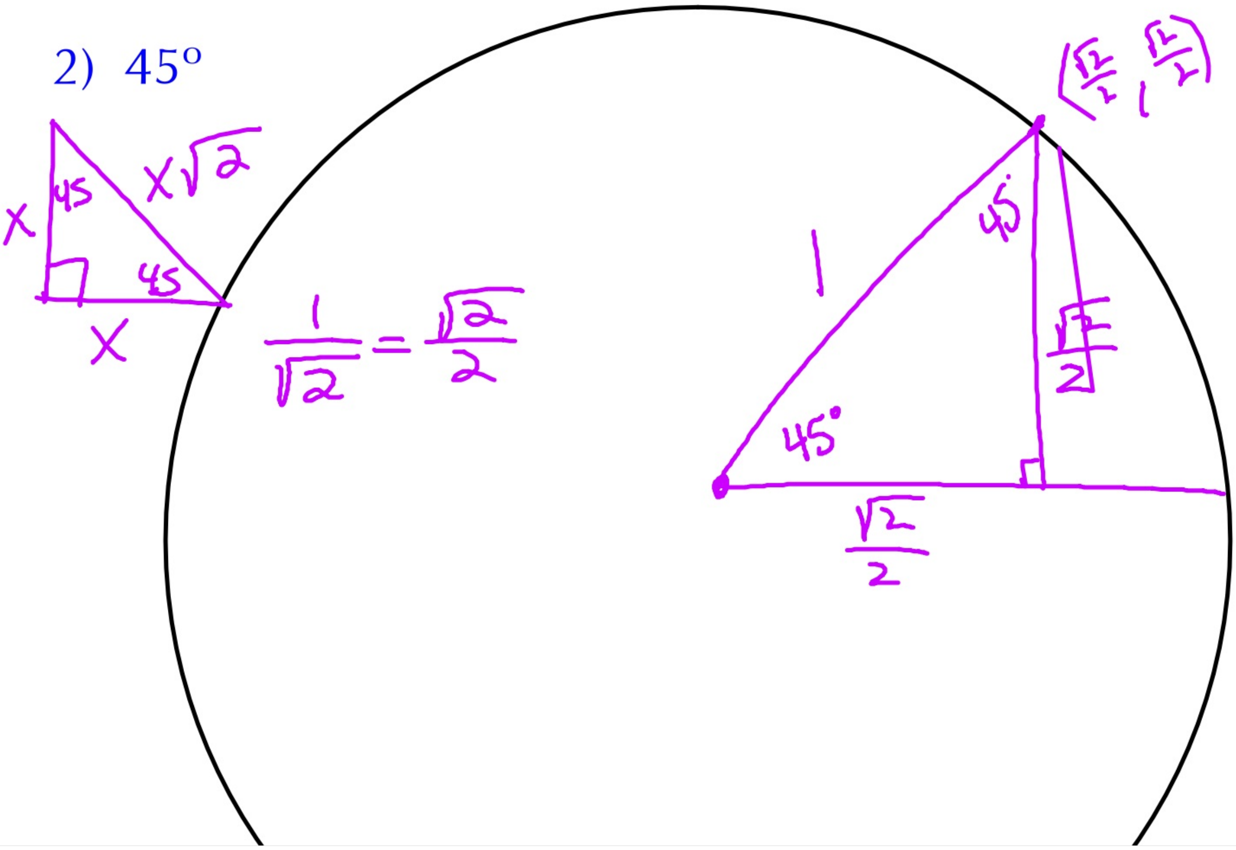

2x
30
$x\sqrt{3}$
60
x
$\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$
$\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$
1
60°
$\frac{1}{2}$
30°
$\frac{\sqrt{3}}{2}$
$\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

For each coordinate of the unit circle, because of SOH CAH TOA, the

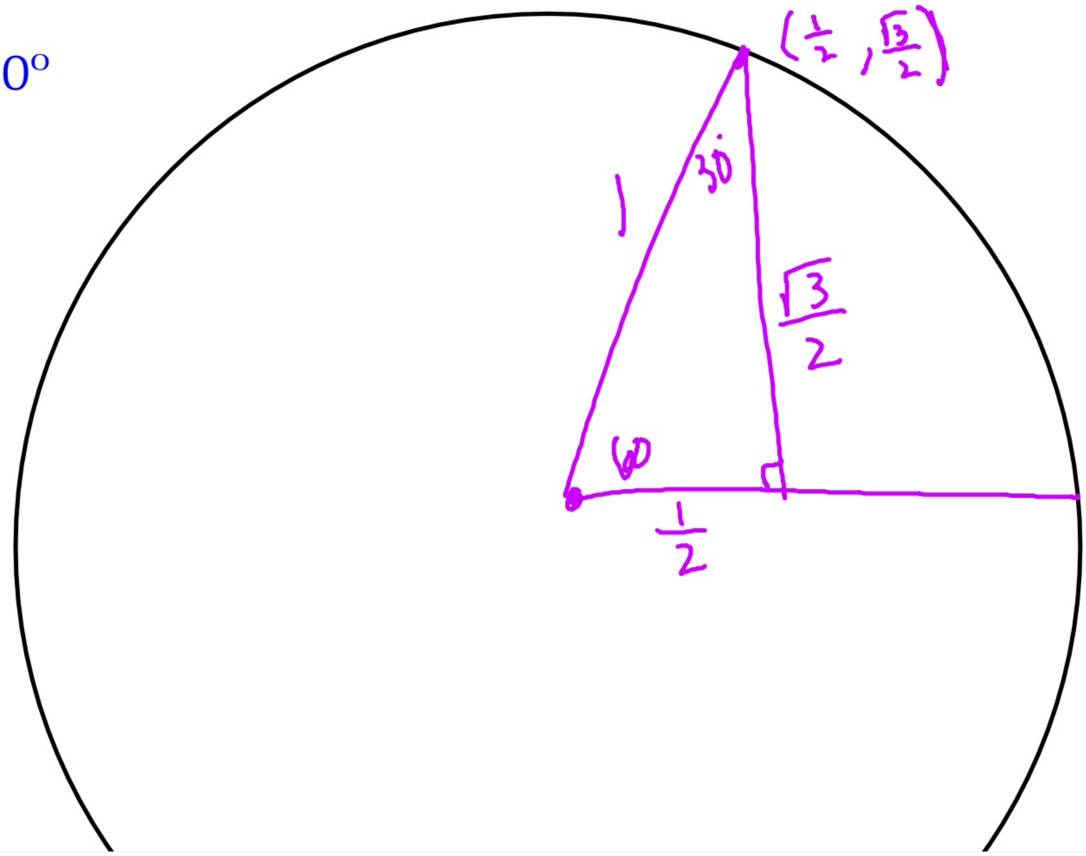
x-coordinate corresponds to cosine

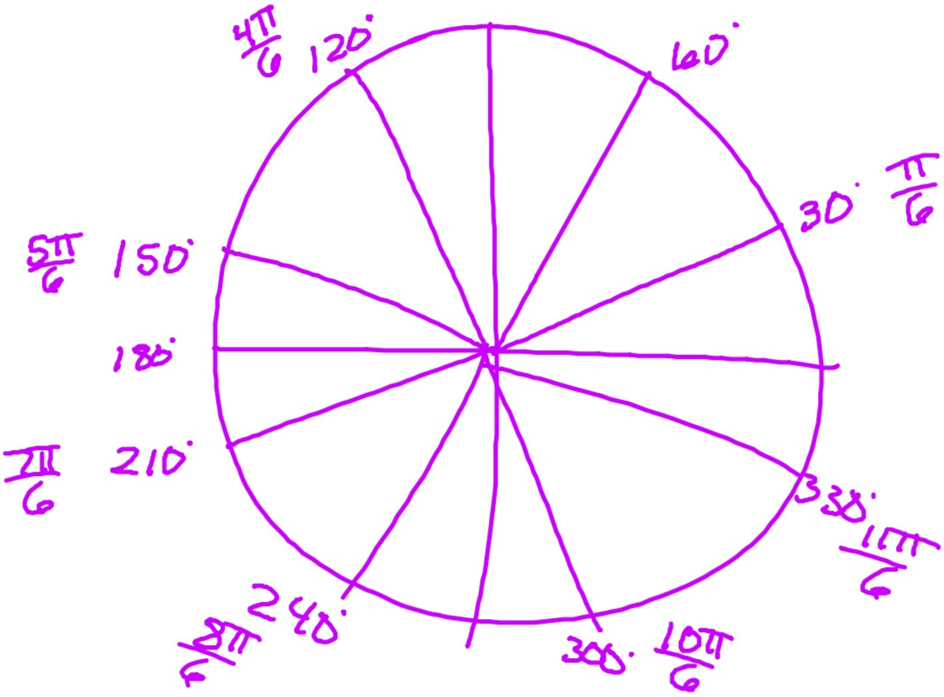
y-coordinate corresponds to sine

2) 45°

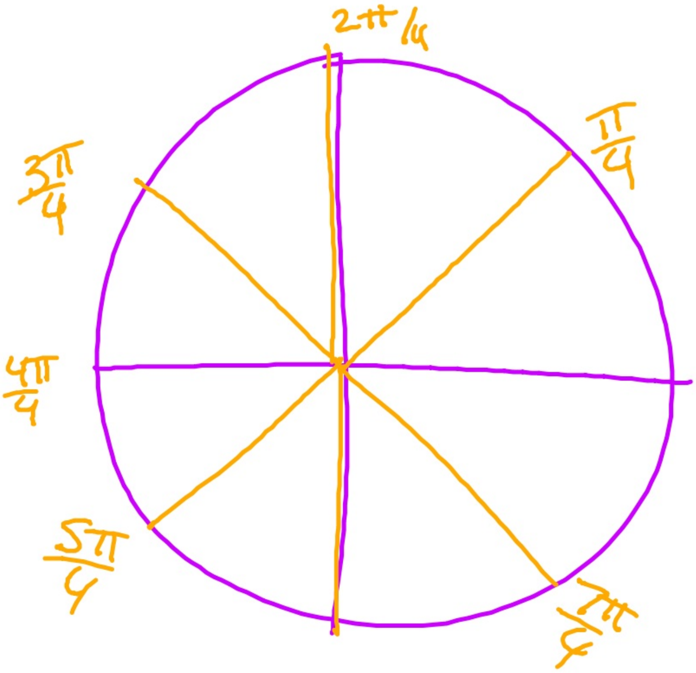


3) 60°

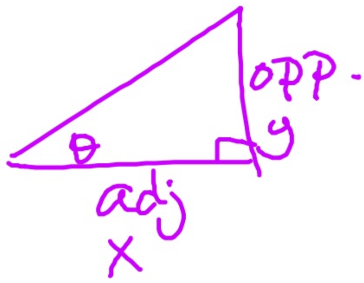




4π/6
120°
60°
30°
π/6
5π/6
150°
180°
7π/6
210°
330°
11π/6
240°
8π/6
300°
10π/6


2π/4
π/4
3π/4
4π/4
5π/4
7π/4

$$\tan\theta = \frac{\text{opp.}}{\text{adj.}} = \frac{y}{x}$$



Quadrantal Angles

A quadrantal angle is an angle in standard position whose terminal side lies on an axis.

ex: List the quadrantal angles on the indicated interval.

a) $[0, 2\pi)$ $0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$

b) $[0^\circ, 360^\circ)$ $0^\circ, 90^\circ, 180^\circ, 270^\circ$