

7.5: Use Recursive Rules with Sequences

Explicit Rule: gives a_n as a function of the term's position number n in the sequence

$$a_n = 2 - 5n$$

Recursive rule: gives the beginning term or terms of a sequence and then a recursive equation that tells how a_n is related to one or more preceding terms

$$a_1 = 4$$

$$a_n = a_{n-1} + 5$$

Recursive Equations for Arithmetic and Geometric Sequences

Arithmetic Sequence

$a_n = a_{n-1} + d$ (d is the common difference)

$$a_1 = 4$$

$$a_n = a_{n-1} + 3$$

Geometric Sequence

$a_n = r(a_{n-1})$ (r is the common ratio)

$$\begin{aligned} a_2 &= a_{2-1} + 3 \\ &= 4 + 3 = 7 \end{aligned}$$

$$\begin{aligned} a_3 &= a_2 + 3 \\ &= 7 + 3 \\ &= 10 \end{aligned}$$

Write the first five terms of the sequence.

1) $a_1 = 2$ $a_n = a_{n-1} - n^2$

$$a_2 = a_{2-1} - 2^2 = a_1 - 4 = 2 - 4 = -2$$

$$a_3 = -11$$

$$a_4 = -27$$

$$a_5 = -52$$

Write a recursive rule for the sequence.

2) 3, 12, 48, 192, 768, ...

$$r = 4$$

$$\begin{cases} a_1 = 3 \\ a_n = 4 \cdot a_{n-1} \end{cases}$$

$$a_n = r(a_{n-1})$$

$$a_2 = 4 \cdot a_1 = 4 \cdot 3 = 12$$

Write a recursive rule for the sequence.

3) 1, 8, 15, 22, 29, ...

$$d = 7$$

$$\begin{array}{l} a_1 = 1 \\ a_n = a_{n-1} + 7 \end{array}$$

Given an explicit rule, write a recursive rule.

4) $a_n = 5 - 3n$

Arithmetic
 $d = -3$

$$\begin{aligned} a_1 &= 2 \\ a_n &= a_{n-1} - 3 \end{aligned}$$

Given an explicit rule, write a recursive rule.

5) $a_n = 2(3)^{n-1}$

$$\boxed{\begin{array}{l} a_1 = 2 \\ a_n = 3 \cdot a_{n-1} \end{array}}$$

$$\begin{array}{l} \swarrow \\ a_2 = 3 \cdot a_1 = 3 \cdot 2 = 6 \\ a_3 = 3 \cdot a_2 = 3 \cdot 6 = 18 \end{array}$$

$$\begin{array}{l} \searrow \\ a_2 = 2 \cdot 3^1 = 6 \\ a_3 = 2 \cdot 3^2 = 18 \end{array}$$

Given a recursive rule, write an explicit rule.

6) $a_1 = 15$ $a_n = a_{n-1} + 11$

Arithmetic $d=11$

$$a_n = 4 + 11n$$

$$\begin{aligned} a_2 &= 4 + 11(2) \\ &= 26 \end{aligned}$$

$$\begin{aligned} \checkmark a_1 &= 15 \\ a_2 &= a_1 + 11 \\ &= 15 + 11 \\ &= 26 \end{aligned}$$

Given a recursive rule, write an explicit rule.

6) $a_1 = 324$ $a_n = \frac{1}{3}(a_{n-1})$

$$a_n = a_1 \cdot r^{n-1}$$

$$a_n = 324 \left(\frac{1}{3}\right)^{n-1}$$