1b.) $\sum_{i=1}^{\infty} \left(\frac{n}{4}\right)^i = -\frac{3}{7}$

$$\frac{a_1}{1-r} = \frac{-3}{7}$$

$$\frac{\frac{n}{4} \cdot 4}{4 \cdot 1 - \frac{n}{4} \cdot 4} = -\frac{3}{7}$$

$$\frac{n}{4-n} = -\frac{3}{7}$$

$$-3(4-n) = 7n$$

$$-12 + 3n = 7n$$

$$-12 = 4n$$

$$-3 = n$$

13.) $1 + 4x + 16x^2 + 64x^3 + \ldots$

$r = 4x$

$a_1 = 1$

$a_n = 1 \cdot (4x)^{n-1}$

$$\sum_{k=1}^{\infty} (4x)^{n-1}$$

If $|x| < \frac{1}{4}$, $\frac{a_1}{1-r} = \frac{1}{1-4x}$

4.) $\dfrac{\frac{5}{27}}{1-\frac{1}{3}}$

$\dfrac{5}{27-9}$

$\dfrac{5}{18}$

14.) $\sum_{i=1}^{n}(10-3i) = -28$

$-28 = \frac{n}{2}(a_1 + a_n)$

$-28 = \frac{n}{2}(7 + 10 - 3n)$

$-56 = 17n - 3n^2$

$3n^2 - 17n - 56 = 0$

$(3n + 7)(n - 8) = 0$

$n = 8$

10.) $5+10+15+\ldots+90$

Finite arithmetic sum

$n = 18$

$a_1 = 5$

$a_n = 90$

$$S_{18} = \frac{18}{2}(5+90) = 9(95)$$

$$\frac{90-5}{5} = n-1$$

$$18 = n$$

11.) finite geometric

$$\sum_{K=1}^{9} \left(-\frac{1}{3}\right)^{n-1}$$

$$S_n = a_1\left(\frac{1-r^n}{1-r}\right)$$

$$= 1\left(\frac{1-\left(-\frac{1}{3}\right)^9}{1-\left(-\frac{1}{3}\right)}\right)$$

$$\frac{3^9\left(1+\left(\frac{1}{3}\right)^9 3^9\right)}{3^9\left(1+\frac{1}{3}3^9\right)} = \frac{3^9+1}{3^9+3^8}$$

$$\approx .7500381$$

## 7.5: Use Recursive Rules with Sequences

Explicit Rule: gives $a_n$ as a function of the term's position number n in the sequence

Recursive rule: gives the beginning term or terms of a sequence and then a recursive equation that tells how $a_n$ is related to one or more preceding terms

# Recursive Equations for Arithmetic and Geometric Sequences

## Arithmetic Sequence

$a_n = a_{n-1} + d$ (d is the common difference)

## geometric sequence

$a_n = r(a_{n-1})$ (r is the common ratio)

Write the first five terms of the sequence.

1) $a_1 = 2$ $\qquad a_n = a_{n-1} - n^2$

$a_2 = a_{2-1} - 2^2 = 2 - 4 = -2$

$a_3 = a_{3-1} - 3^2 = -2 - 9 = -11$

$a_4 = a_{4-1} - 4^2 = -11 - 16 = -27$

$a_5 = a_{5-1} - 5^2 = -27 - 25 = -52$

Write a recursive rule for the sequence.

2) 3, 12, 48, 192, 768, ...

geometric $a_n = r(a_{n-1})$

$a_1 = 3$, $a_n = (a_{n-1})4$

$a_1 = 3$

$a_2 = 4(3) = 12$

$a_3 = 4(12) = 48$

Write a recursive rule for the sequence.

3) 1, 8, 15, 22, 29, ...

$a_n = a_{n-1} + d$

$a_1 = 1, a_n = a_{n-1} + 7$

Given an explicit rule, write a recursive rule.

4) $a_n = 5 - 3n$

$d = -3$

$a_1 = 2$

$a_n = a_{n-1} - 3$

Given an explicit rule, write a recursive rule.

5) $a_n = 2(3)^{n-1}$

$a_n = (a_{n-1})3$

$a_1 = 2, a_n = (a_{n-1})3$

Given a recursive rule, write an explicit rule.

6) $a_1 = 11 \quad a_n = a_{n-1} + 11$

$a_n = a_1 + (n-1)d$

$= 11 + (n-1)11$

$a_n = 11n$

Given a recursive rule, write an explicit rule.

7) $a_1 = 324$ $a_n = 1/3(a_{n-1})$

$a_n = a_1(r)^{n-1}$

$a_n = 324\left(\frac{1}{3}\right)^{n-1}$