56 b.) $3x + 6x^3 + 12x^5 + 24x^7$

$r = 2x^2$

$a_1 = 3x$

$n = 4$

$$S_4 = 3x\left(\frac{1-(2x^2)^4}{1-2x^2}\right)$$

$$= 3x\left(\frac{1-16x^8}{1-2x^2}\right)$$

58.) $a_n = 32\left(\frac{1}{2}\right)^{n-1}$

$a_1 = 32$

$a_2 = 16$

$a_3 = 8$

$a_4 = 4$

$a_5 = 2$

$a_6 = 1$

7.4: Find Sums of Infinite Geometric Series

$$S = \frac{a_1}{1 - r}$$

$$|r| < 1$$

Finite Arithmetic Series: $S_n = \frac{n}{2}(a_1 + a_n)$ convergent

Finite Geometric Series: $S_n = a_1\left(\frac{1-r^n}{1-r}\right)$ convergent

Infinite Arithmetic Series: $1+3+5+7+\cdots$ N/A (diverges)

Infinite Geometric Series: $1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\cdots$

geometric; $r=\frac{1}{2}$

$S_n = \frac{a_1}{1-r}$, $|r|<1$ (converges)

Find the sum of the infinite geometric series, if it exists.

$$\sum_{n=1}^{\infty} 8\left(\frac{1}{5}\right)^{n-1} = 8 + \frac{8}{5} + \frac{8}{25} + \frac{8}{125} + \frac{8}{625} + \cdots$$

$|r| < 1$

$\left|\frac{1}{5}\right| < 1$ ✓

$$S = \frac{8}{1 - \frac{1}{5}} = \frac{8}{\frac{4}{5}}$$

$$= 10$$

$S_2 = 9.6$

$S_3 = 9.92$

$S_4 = 9.98$

$S_5 = 9.996$

Find the sum, if possible.

$4 - 2 + 1 - \frac{1}{2} + \dots$

geometric; $r = -\frac{1}{2}$

$S = \frac{4}{1-\left(-\frac{1}{2}\right)}$

$= \frac{4 \cdot 2}{2 \cdot 1 + \frac{1}{2} \cdot 2} = \frac{8}{2+1}$

$= \frac{8}{3}$

Write the series in summation notation.

$\sum_{k=1}^{\infty}$ rule $\rightarrow a_n = a_1 (r)^{n-1}$

$\sum_{k=1}^{\infty} 4\left(-\frac{1}{2}\right)^{k-1}$

Find the sum if possible.

$9 + 6 + 3 + 0 + -3 + \ldots$

not possible; arithmatic

Write the series in summation notation.

$$\sum_{K=1}^{\infty} -3K + 12$$

Find the sum if possible.

$3 + 0.3 + 0.03 + 0.003 + \ldots$

geometric; $r = \frac{1}{10}$ $\quad S = \frac{3}{1-\frac{1}{10}} = \frac{3}{\frac{9}{10}} = \frac{10}{3}$

$= 3\frac{1}{3}$

Write the series in summation notation

$$\sum_{K=1}^{\infty} 3\left(\frac{1}{10}\right)^{K-1} = \sum_{K=1}^{\infty} 3\left(\frac{1}{10}\right)^{K}\left(\frac{1}{10}\right)^{-1} = \sum_{K=1}^{\infty} 30\left(\frac{1}{10}\right)^{K}$$

Write the sum in summation notation.

$9 + 7 + 5 + \ldots + -5$

arithmetic

$d = -2$

$\frac{-5-9}{-2} = n-1$

$8 = n$

$$\sum_{K=1}^{8} -2K + 11$$

Solve for x.

$$\sum_{n=0}^{\infty} 3\left(\frac{x}{2}\right)^n = 7$$

$$\frac{a_1}{1-r} = S$$

$$\frac{7}{1} = \frac{3 \cdot 2}{2 \cdot 1 - \frac{x}{2} \cdot 2}$$

$$7 = \frac{6}{2-x}$$

$$14 - 7x = 6$$

$$8 = 7x$$

$$x = \frac{8}{7}$$