7.) $\frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}, \ldots$

$d = \frac{1}{2}$

45.) $S_n = \frac{n}{2}(a_1 + a_n)$ $\qquad \sum_{i=5}^{14}(-54+9i)$

$S_n = \frac{10}{2}(-9 + 72)$ $\qquad \frac{630}{2}$

$-54+9(5)$

-9

$-54+9(14)$

$-54+126$ $\quad 72$

55.) $\sum_{i=1}^{n}(-5+7i) = 486$

$$S_n = \frac{n}{2}(a_1 + a_n)$$

$$a_n = -5 + 7n$$

$$486 = \frac{n}{2}(2 + a_n)$$

$$486 = \frac{n}{2}(2 + {}^{-}5 + 7n)$$

$$486 \cdot 2 = n(-3 + 7n)$$

$$972 = -3n + 7n^2$$

$$7n^2 - 3n - 972 = 0$$

$$(7n + 81)(n - 12) = 0$$

$$n = 12$$

7.3 Analyze Geometric Sequences and Series

For a geometric sequence, the ratio of any term to the previous term is constant. The constant ratio is called the common ratio and is denoted by r.

Tell whether the sequence is geometric

1) 3, 6, 9, 12, 15, 18,... not geometric

2) 1/4, 1/2, 1, 2, 4, 8,... geo. $r = 2$

3) 3, 12, 48, 192, ... geo. $r = 4$

Rule for a geometric sequence (nth term)

$$a_n = a_1(r)^{n-1}$$

Write a rule for the nth term.

4) 6, 18, 54, 162,... $r = 3, a_1 = 6$ $\quad a_n = 6(3)^{n-1}$

5) 3, 6/5, 12/25, 24/125, ... $\quad a_n = 3\left(\frac{2}{5}\right)^{n-1}$

$\frac{6}{5} \div 3 = \frac{6}{5} \cdot \frac{1}{3} = \frac{2}{5}$

6) 7, -35, 175, -875, ...

$r = -5$ $\quad a_n = 7(-5)^{n-1}$

Write a rule for the nth term of the geometric sequence.

7) $a_5 = 1$, $r = 1/2$

$$a_5 = a_1\left(\frac{1}{2}\right)^{5-1}$$

$$1 = a_1\left(\frac{1}{2}\right)^4$$

$$1 = a_1\left(\frac{1}{16}\right)$$

$$16 = a_1$$

$$a_n = 16\left(\frac{1}{2}\right)^{n-1}$$

8) $a_3 = -48$, $a_5 = -768$

$(3,-48)$ $(5,-768)$

$$-48 = a_1(r)^{3-1} \qquad -768 = a_1(r)^{5-1}$$

$$\frac{-48}{r^2} = a_1 \qquad -768 = \frac{-48}{r^2}(r)^4$$

$$\sqrt{\frac{+768}{+48}} = \sqrt{r^2}$$

$$4 = r$$

$$-3 = a_1$$

$$\boxed{a_n = -3(4)^{n-1}}$$

$a_3 = 10 \qquad a_6 = 270$

$10 = a_1(r)^2$

$\frac{10}{r^2} = a_1$

$\frac{10}{9} = a_1$

$270 = a_1(r)^5$

$270 = \frac{10}{r^2} \cdot r^5$

$27 = r^3$

$3 = r$

$a_n = \frac{10}{9}(3)^{n-1}$

The sum of a Finite Geometric Series

$$S_n = a_1\left(\frac{1-r^n}{1-r}\right)$$

Find the sum of the geometric series.

10) $\displaystyle\sum_{i=1}^{8} 6(-2)^{i-1}$

$$S_8 = 6\left(\frac{1-(-2)^8}{1--2}\right) = 6\left(\frac{1-256}{3}\right) = 2(-255)$$

$$= -510$$

7.4: Find Sums of Infinite Geometric Series

$$S = \frac{a_1}{1-r}$$

$$|r| < 1$$

Find the sum of the infinite geometric series, if it exists.

11.) $$\sum_{n=1}^{\infty} 8\left(\frac{1}{5}\right)^{n-1} = 8 + \frac{8}{5} + \frac{8}{25} + \frac{8}{125} + \frac{8}{625}$$

Since $r = \frac{1}{5}$, the infinite sum exists.

$$S = \frac{8 \cdot 5}{5 \cdot 1 - \frac{1}{5} \cdot 5} = \frac{40}{5-1}$$

$$= 10$$

9.9968

Find the sum of the infinite geometric series, if it exists.

Applications

A rubber ball is dropped from a height if 60 feet. Each bounce takes it to 2/3 of its previous height. What is the total vertical distance the ball travels?

$$S = \frac{a_1}{1-r} = \frac{60 \cdot 3}{3 \cdot 1 - \frac{2}{3} \cdot 3} = \frac{180}{3-2}$$

$$= 180 \text{ ft.}$$

A pendulum that is released to swing freely travels 25 inches on the first swing. On each successive swing, the pendulum travels 85% as far as the previous swing. What is the total distance of the pendulum swings?

$$S = \frac{25}{1 - \frac{17}{20}}$$

$$= \frac{500}{20-17}$$

$$= \frac{500}{3} \text{ in}$$

$$.85 = \frac{85}{100} = \frac{17}{20}$$

$$S = \frac{25}{1-.85}$$

$$= \frac{25}{.15} \times 100 = \frac{2500}{15}$$

$$= \frac{500}{3} \text{ in}$$