33. $a_8 = -10$ $a_{20} = -58$ (20, -58) (8, -10)

$a_n - a_{\#} = d(n - n_{\#})$

$-10 - -58 = d(8 - 20)$

$8 - 20$

$-4 = d$

$a_n - (-10) = -4(n - 8)$

$a_n =$ ___

45.) $S_n = \frac{n(a_1 + a_n)}{2}$

$\sum_{i=5}^{14}(-54+9i)$ $\quad S_{\boxed{10}} = \frac{10(-9+72)}{2}$

61.) $1+3+5+7+\ldots\ldots+299$

$a_n = 2n - 1$

$299 = 2n - 1$

$150 = n$

$S_n = \frac{n(a_1 + a_n)}{2}$

$= \frac{150(1+299)}{2}$

$$5 + 10 + 15 + \ldots + 90$$

$a_n = 5n$

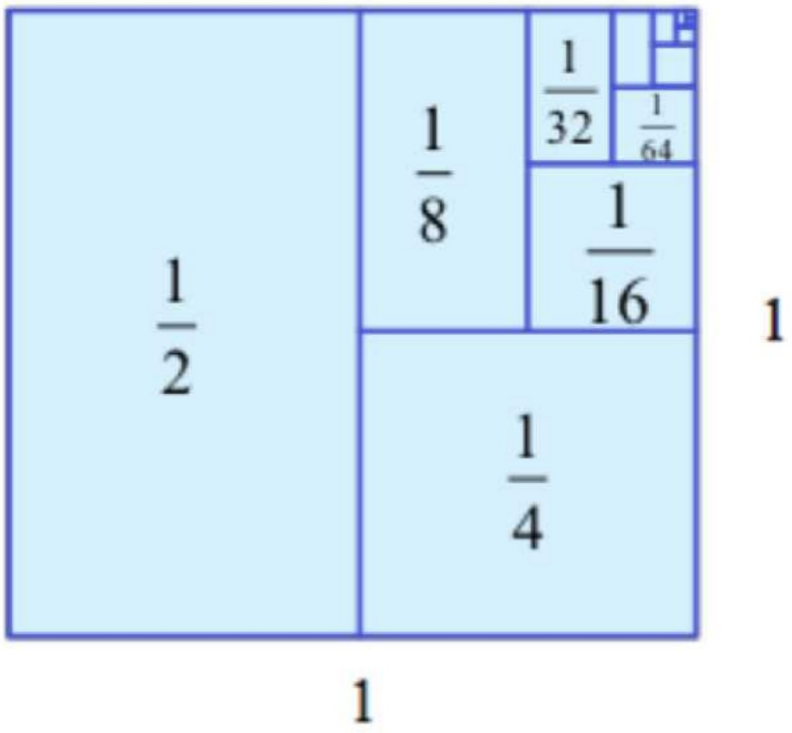
$90 = 5n$

$18 = n$

$$S_{18} = \frac{18(5+90)}{2} \checkmark$$

$$\sum_{n=1}^{18} 5n$$

7.3/7.4 Geometric Sequences and Series



*See printout.

Geometric Sequences

In a **geometric sequence**, the ratio of any term to the previous term is constant. This constant ratio is called the **common ratio** and is denoted by *r*.

ex: Determine if the sequences is geometric. If so, identify the common ratio.

a) 10, 20, 30, 40, 50 . . . NO

b) 3, 6, 12, 24, 48 . . . yes

$r = 2$

$\frac{a_2}{a_1} = r$

c) 16, 12, 9, 27/4 . . .

$\frac{12}{16} = \frac{3}{4}$ $\quad \frac{9}{12} = \frac{3}{4}$ yes

ex: Write the 1st 4 terms of the sequence and sketch the graph.

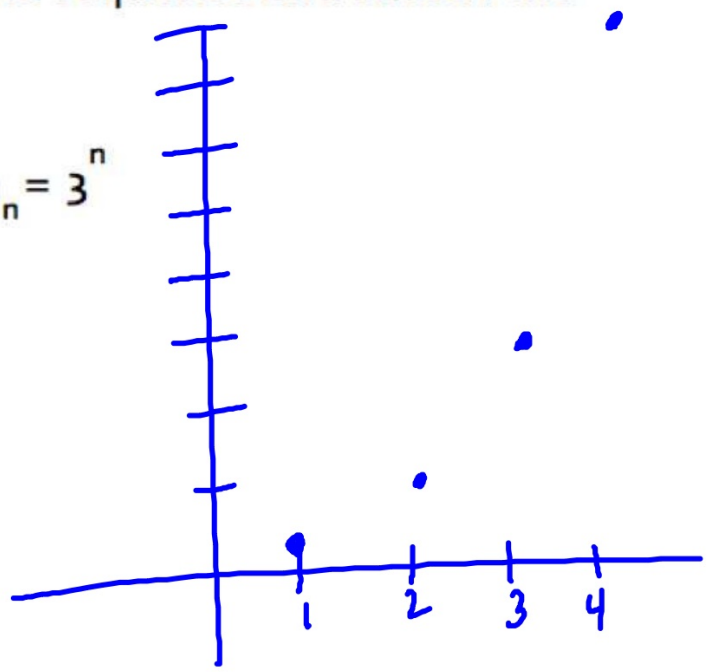
$a_n = 3^n$

$a_1 = 3$

$a_2 = 9$

$a_3 = 27$

$a_4 = 81$



Writing Explicit Rules for Geometric Sequences/Series

*Since geometric sequences have an exponential pattern, the explicit rule is exponential!

Recall Exponential Functions: $y = ab^x$

Explicit Rule: $a_n = a_1 r^{n-1}$

Where:

a_1 1st term

r common ratio

ex: Write an explicit rule for the geometric sequence.

a) 4, 20, 100, 500 . . .

$a_1 = 4$ $\quad a_n = 4 \cdot 5^{n-1}$

$r = 5$

b) $a_2 = 3$, $r = 1/4$

$a_1 = 12$ $\quad a_n = 12\left(\frac{1}{4}\right)^{n-1}$

ex: Write an explicit rule for the geometric sequence.

c) $a_3 = 10, a_6 = 270$

$10r^3 = 270$

$r^3 = 27$

$r = 3$

30 $r = 3$

90 $a_2 = \frac{10}{3}$

270 $a_1 = \frac{10}{9}$

$a_n = \frac{10}{9}(3)^{n-1}$

Writing Recursive Rules for Geometric Sequences/Series

ex: Write a recursive rule for the geometric sequence.

a) 4, 20, 100, 500 . . .

$r = 5$

$a_1 = 4$

$a_n = a_{n-1} \cdot 5$

b) $a_2 = 3$, $r = 1/4$

$a_1 = 12$

$a_n = a_{n-1} \cdot \frac{1}{4}$

The Sum of a FINITE Geometric Sequence/Series

The Sum of a Finite Geometric Series

The sum of the first n terms of a geometric series with common ratio $r \neq 1$ is:

$$S_n = a_1\left(\frac{1 - r^n}{1 - r}\right)$$

S_n	sum of the 1st n terms
n	number of terms in the sum
a_1	1st term in the sequence
r	common ratio

The Sum of an INFINITE Geometric Sequence/Series

The Sum of an Infinite Geometric Series

The sum of an infinite geometric series with first term a_1 and common ratio r is given by

$$S = \frac{a_1}{1 - r}$$

provided $|r| < 1$. If $|r| \geq 1$, the series has no sum.

S	sum of ALL terms
a_1	1st term in the sequence/series
r	common ratio

$a_1 = 32 \quad r = \frac{1}{2}$

$32 + 16 + 8 + 4 + 2 + 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8}$

63.875

$S = \frac{a_1}{1 - r}$

63.984

$S = \frac{32}{1 - \frac{1}{2}} =$

ex: Find the indicated sum, if possible.

a) 1, 2, 4, 8, ... $S_9 = ?$

$$S_9 = 1\left(\frac{1-2^9}{1-2}\right)$$

$$= \frac{-511}{-1} = 511$$

ex: Find the indicated sum, if possible.

b) $\sum_{n=1}^{8} 6\left(-\frac{1}{2}\right)^{n-1}$

$$S_8 = \frac{6\left(1-\left(-\frac{1}{2}\right)^8\right)}{1-\frac{-1}{2}} = \frac{6\left(1-\frac{1}{256}\right)}{\frac{3}{2}}$$

$$4\left(1-\frac{1}{256}\right)$$

$$\frac{4(255)}{256}$$

$$\frac{255}{64}$$

ex: Find the indicated sum, if possible.

c) $\sum_{n=1}^{\infty} 6\left(-\frac{1}{2}\right)^{n-1}$

$\left|-\frac{1}{2}\right| < 1$

$S = \frac{6}{1-\left(-\frac{1}{2}\right)} = 4$

ex: Find the indicated sum, if possible.

d) $4 - 2 + 1 - 0.5 + \ldots$

$r = \frac{-1}{2}$

$S = \frac{4}{1 - \frac{-1}{2}}$

$S = 8/3$

g) $\sum_{n=3}^{8} 6(4)^{n-2}$

finite geo.

$a_1 = 24$

$r = 4$

$n = 6$

$S_6 = 24\left(\frac{1-4^6}{1-4}\right)$

$-8(1-4^6)$

32760

h.) geo.
infinite
$r = 4$

No
sum

ex: Find the indicated sum, if possible.

e) $9 + 6 + 3 + 0 - 3 - \ldots$

Arith. infinite

no sum

ex: Find the indicated sum, if possible.

f) $\sum_{n=0}^{6} n^2 = 0+1+4+9+16+25+36=91$

ex: Express the series using summation notation. Then find the sum or explain why there is no sum.

a) $5 + 15 + 45 + 135 + \ldots$

$$\sum_{n=1}^{\infty} 5 \cdot 3^{n-1}$$

ND SUM

ex: Express the series using summation notation. Then find the sum or explain why there is no sum.

b) 5 + 15 + 45 + 135

$$\sum_{n=1}^{4} 5 \cdot 3^{n-1} = 200$$

ex: Express the series using summation notation. Then find the sum or explain why there is no sum.

c) 100 + 20 + 4 + 4/5 + . . .

infinite geo

$$\sum_{n=1}^{\infty} 100\left(\frac{1}{5}\right)^{n-1} = 125$$

ex: Solve for x.

a) $\sum_{i=1}^{x} 5 - 5i = -50$

$$S_n = \frac{n(a_1 + a_n)}{2}$$

$$-50 = \frac{x(0 + 5 - 5x)}{2}$$

$$-100 = 5x - 5x^2$$

$$5(x^2 - x - 20) = 0$$

$$5(x-5)(x+4) = 0$$

5

ex: Solve for x.

b) $\sum_{n=0}^{\infty} 3\left(\frac{x}{2}\right)^n = 7$

$7 = \frac{3}{1-\frac{x}{2}}$

$7 = \frac{3}{\frac{2-x}{2}}$

geo (infinite)

$r = \frac{x}{2}$

$a_1 = 3$

$7\left(\frac{2-x}{2}\right) = 3$

$7(2-x) = 6$

$14 - 7x = 6$

$x = \frac{8}{7}$

ex: Find the sum of the first 15 three digit whole numbers ending in 5.

$105 + 115 + 125 + \ldots$

Sum
Arith
finite

$a_n = 10n + 95$

$a_{15} = 10(15) + 95$
$= 245$

$a_1 = 105$
$a_{15} = 245$
$n = 15$

$S_{15} = \frac{15(105 + 245)}{2}$

2625

ex: Find the missing terms of the arithmetic sequence.

. . . 37 , 33, 29 , 25 . . .

ex: Find the missing terms of the geometric sequence.

. . . 48, 16 , 16/3 , 48/9 . . .

48 | 27