7.3 Analyze Geometric Sequences and Series

For a geometric sequence, the ratio of any term to the previous term is constant. The constant ratio is called the common ratio and is denoted by r.

Tell whether the sequence is geometric

1) 3, 6, 9, 12, 15, 18,... not geometric

2) 1/4, 1/2, 1, 2, 4, 8,... geometric; $r = 2$

3) 3, 12, 48, 192, ... geometric; $r = 4$

Rule for a geometric sequence (nth term)

$a_n = a_1(r)^{n-1}$

$r = \frac{a_2}{a_1}$

Write a rule for the nth term.

4) 6, 18, 54, 162,... $a_1 = 6, r = 3$; $a_n = 6(3)^{n-1}$

5) 3, 6/5, 12/25, 24/125, ... $a_1 = 3, r = \frac{2}{5}$; $a_n = 3\left(\frac{2}{5}\right)^{n-1}$

6) 7, -35, 175, -875, ... $a_1 = 7$ $r = -5$; $a_n = 7(-5)^{n-1}$

$$a_n = 6(3)^{n-1}$$

$$a_n = 6 \cdot 3^n \cdot 3^{-1}$$

$$= 6 \cdot \frac{1}{3} \cdot 3^n$$

$$a_n = 2 \cdot 3^n$$

Write a rule for the nth term of the geometric sequence.

7) $a_5 = 1, r = 1/2$ $16, 8, 4, 2, 1$

$a_n = a_1 \cdot r^{n-1}$

$a_n = 16\left(\frac{1}{2}\right)^{n-1}$

$1 = a_1\left(\frac{1}{2}\right)^{5-1}$

$1 = a_1 \cdot \frac{1}{16}$

$16 = a_1$

8) $a_3 = 10$; $a_6 = 270$

55.) $\sum_{i=1}^{n}(-5+7i)=486$

$S_n=\frac{n}{2}(a_1+a_n)$

$486=\frac{n}{2}(2+(-5+7n))$

$486=\frac{n}{2}(7n-3)$

$972=7n^2-3n$

$7n^2-3n-972=0$

$(7n+81)(n-12)=0$

$n=\frac{-81}{7}$ $n=12$

(63.) $a_1 = 0$

$a_2 = 6$

$a_3 = 12$

$a_1 = 6$

$a_2 = 12$

$a_n = 6n$

45) $\sum_{i=5}^{14} (-54+9i)$

$\frac{n}{2}(a_1 + a_n)$

$\frac{10}{2}(-9+72)$

$a_5 = -54+9(5)$
$= -9$

$a_{14} = -54+9(14)$
$= 72$

61.) $1+3+5+\ldots\ldots+299$

$n = 150$

$a_1 = 1$

$a_n = 299$

$\frac{299-1}{2} = n-1$

$150 = n$

37.) $a_5 = 15 \quad a_9 = 24$

$(5, 15) \quad (9, 24)$

$$m = \frac{24 - 15}{9 - 5}$$

$$= \frac{9}{4}$$

$$a_n = a_1 + (n-1)d$$

$$15 = a_1 + (5-1)\frac{9}{4}$$

$$15 = a_1 + 9$$

$$6 = a_1$$

$$a_n = 6 + (n-1)\frac{9}{4}$$

$$\boxed{a_n = \frac{9}{4}n + \frac{15}{4}}$$

The sum of a Finite Geometric Series

$$S_n = a_1\left(\frac{1-r^n}{1-r}\right)$$

Find the sum of the geometric series.

9) $\sum_{i=1}^{8} 6(-2)^{i-1}$

$a_1 = 6$

$n = 8$

$r = -2$

$$S_8 = 6\left(\frac{1-(-2)^8}{1-(-2)}\right) = 6\left(\frac{1-256}{3}\right)$$

$$= 2(-255)$$

$$= -510$$

$$\sum_{i=1}^{6} 4\left(\frac{3}{2}\right)^{i-1}$$

$$S_6 = 4\left(\frac{1-\left(\frac{3}{2}\right)^6}{1-\frac{3}{2}}\right) = 4\left(\frac{1-\frac{729}{64}}{-\frac{1}{2}}\right)\cdot\frac{64}{64}$$

$$4\left(\frac{64-729}{-32}\right) = \left(\frac{-665}{-32}\right)4 = \frac{665}{8}$$

$$\sum_{i=0}^{7} 12\left(-\frac{1}{2}\right)^i \qquad S_8 = 12\left(\frac{1-\left(-\frac{1}{2}\right)^8}{1-\left(-\frac{1}{2}\right)}\right) = 12\left(\frac{1-\frac{1}{256}}{\frac{3}{2}}\right)$$

$a_1 = 12$

$n = 8$

$r = -\frac{1}{2}$

n: number of terms

$= 12\left(\frac{256-1}{384}\right)$

$= 12\left(\frac{255}{384}\right)$

$= \frac{255}{32}$

$\frac{3}{2} \cdot 256$

$3 \cdot 128$

Arithmetic

$a_4 = 12$

$a_8 = 20$

$\frac{20-12}{8-4} = \frac{8}{4} = 2$

$d = 2$

$a_4 = 12$

$a_3 = 10$

$a_2 = 8$

$a_1 = 6$

$a_n = 6 + (n-1)2$

$a_n = 4 + 2n$

Sum (arithmetic)

$44 + 37 + 30 + \ldots + 2$

$d = -7$

$a_1 = 44$

$a_n = 2$

$2 = 44 + (n-1)(-7)$

$7 = n$

$S_7 = \frac{7}{2}(44 + 2) = 161$

geometric

$a_2 = 24$

$a_5 = 1536$

$24r^3 = 1536$

$r^3 = 64$

$r = 4$

$a_1 = 6$

$a_n = a_1(r)^{n-1}$

$a_n = 6(4)^{n-1}$