Sequences (7.1)

A sequence is a function whose domain is a set of consecutive integers. If not specified, the domain starts at 1. The values in the range are called the terms of the sequence.

$a_1 = 2$
$a_2 = 5$

Finite sequence: 2, 5, 8, 11, 14

infinite sequence: 3, 7, 11, 15, ...

a_n: n^{th} term

Series

$\sum$ = sum

When the terms of a sequence are added together, the resulting expression is a series. A series can be finite or infinite.

Finite: 2 + 4 + 6 + 8

Infinite: 2 + 4 + 6 + 8 + ...

$$\sum_{i=1}^{4} 2i = 2(1) + 2(2) + 2(3) + 2(4)$$

(1st, 2nd, 3rd, 4th)

$$\sum_{i=1}^{\infty} 2i = 2 + 4 + 6 + \dots$$

summation notation

Graphing a sequence

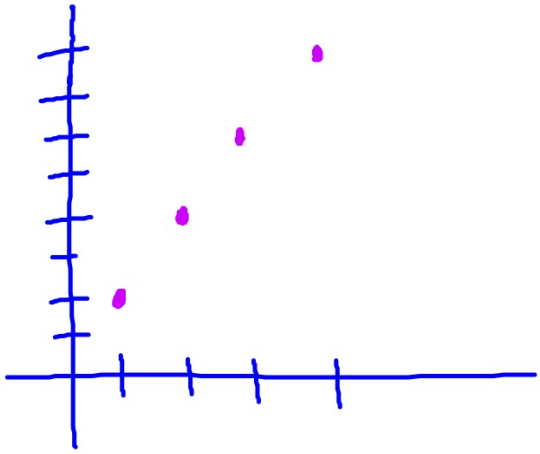
2, 4, 6, 8

$a_1 = 2$ (1, 2)

$a_2 = 4$ (2, 4)

$a_3 = 6$ (3, 6)

$a_4 = 8$ (4, 8)

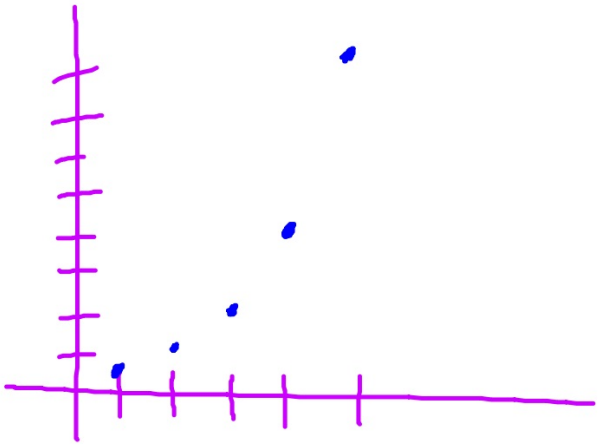


Do not connect the dots. The sequence is a set of points, not a line.

Graphing a sequence

$\frac{1}{2}, 1, 2, 4, 8$

$(1, \frac{1}{2})$
$(2, 1)$
$(3, 2)$
$(4, 4)$
$(5, 8)$



Do not connect the dots. The sequence is a set of points, not a line.

Types of Sequences/Series

Arithmetic

Geometric

Other

Rules can be

explicit $a_n = 16n + 2$

recursive

$a_1 = 2$

$a_n = 3 \cdot a_{n-1}$

Find the first 5 terms of the sequence.

$a_n = 2n - 3$

$-1, 1, 3, 5, 7$

Summation Notation (aka Sigma Notation)

Find the sum

$$\sum_{n=0}^{4} 2^n = 2^0 + 2^1 + 2^2 + 2^3 + 2^4 = 31$$

7.2 Arithmetic Sequences and Series

Arithmetic sequence: A sequence whose consecutive terms have a common difference (slope)

$5, 9, 13, 17, \ldots$ $d = 4$

$16, 14, 12, 10, \ldots$ $d = -2$

Arithmetic sequences are linear $d = a_2 - a_1 = a_3 - a_2$

Writing a rule for the nth term of an arithmetic sequence (explicit formula) (linear equation)

$$y - y_1 = m(x - x_1)$$

$$y = y_1 + m(x - x_1)$$

$$a_n = a_1 + (n-1)d$$

Write a rule for the nth term.

$a_3 = 17 \quad d = 2$

$a_n = a_1 + (n-1)d$

$a_n = 13 + (n-1)2$

$a_1 = 13$
$a_2 = 15$
$a_3 = 17$

$\boxed{a_n = 2n + 11}$

$a_3 = 17 \quad d = 2$

$a_n = a_1 + (n-1)d$

$17 = a_1 + (3-1)2$

$17 = a_1 + 4$

$13 = a_1$

Write a rule for the nth term.

$a_{16} = 2 \qquad a_{20} = 14$

$m = \frac{14-2}{20-16}$

$m = 3$

$d = 3$

$a_n = a_1 + (n-1)d$

$2 = a_1 + (16-1)3$

$2 = a_1 + 45$

$-43 = a_1$

$a_n = -43 + (n-1)3$

$a_n = 3n - 46$

$a_{16} = 2 \quad a_{20} = 14$

$2 = a_1 + (16-1)d \quad 14 = a_1 + (20-1)d$

$2 = a_1 + 15d \quad 14 = a_1 + 19d$

Write a rule for the nth term.

$21, 14, 7, 0, -7, \ldots$

$a_1 = 21$

$d = -7$

$a_n = 21 + (n-1)(-7)$

$a_n = 21 + -7n + 7$

$a_n = -7n + 28$

47.) $\sum_{n=0}^{4} n^3 = 0^3 + 1^3 + 2^3 + 3^3 + 4^3$

The sum of a finite arithmetic series

$$S_n = \frac{n}{2}(a_1 + a_n)$$

S_n : sum of "n" terms

$1+2+3+\ldots 98+99+100$

$1+100=101$

$2+99=101$

$3+98=101$

$50(101)$

5050

Find the sum of the arithmetic series.

$$\sum_{n=1}^{10} 3n-1$$

$n = 10$

$a_1 = 2$

$a_{10} = 29$

$$S_{10} = \frac{10}{2}(2+29)$$

$$= 5(31)$$

$$= 155$$

Find the sum of the finite arithmetic series.

$2 + 6 + 10 + \ldots + 58$

$a_1 = 2$

$a_n = 58$

$n = 15$

$d = 4$

$a_n = a_1 + (n-1)d$

$58 = 2 + (n-1)4$

$15 = n$

$\frac{58-2}{4} = n - 1$

$\frac{56}{4} = n - 1$

$15 = n$

$S_{15} = \frac{15}{2}(2+58)$

$= 15(30)$

$= 450$

The corner section of a football stadium has 15 seats in the first row and 40 rows in all. Each successive row contains two additional seats. How many seats are in this section?

$n = 40$

$a_1 = 15$

$a_{40} = 93$

$108 \times 2 = 216$

$a_n = a_1 + (n-1)d$

$a_{40} = 15 + (40-1)2$

$a_{40} = 93$

$S_{40} = \frac{40}{2}(15 + 93)$

$= 20(108)$

$= 2160$

19 seats
17 seats
15 seats

$\frac{a_{40} + 15}{2} = 40$