5.6: Solving Rational Equations

1 $18(x+3)\frac{4}{x+3} + \frac{5}{6}^{18(x+3)} = \frac{2\ 3}{18}18(x+3)$

$72 + 15(x+3) = 23(x+3)$

$72 + 15x + 45 = 23x + 69$

$48 = 8x$

✓ $\boxed{6 = x}$

$x \neq -3$

Steps

1) Find the LCM of all denominators

LCM: $18(x+3)$

2) Multiply the LCM to each term

3) Solve for x

4) Check for extraneous solutions (why?) denominators can't be zero

(2) $$\frac{(x+5)(x+3)\,2x}{x+5} - \frac{x^2-x-10\,(x+5)(x+3)}{\cancel{x^2+8x+15}\;(x+5)(x+3)} = \frac{3\,(x+5)(x+3)}{x+3}$$

LCM
$(x+5)(x+3)$

$$2x(x+3) - (x^2-x-10) = 3(x+5)$$

$$2x^2+6x-x^2+x+10 = 3x+15$$

$$x^2+4x-5=0$$

$$(x+5)(x-1)=0$$

$$x=1, \cancel{-5}$$ extraneous

3 $x(x-5)\left[- \frac{8}{x-5} \right. = \frac{3}{x} x(x-5)$ LCM : $x(x-5)$

$x(x-5) - 8x = 3(x-5)$

$x^2 - 5x - 8x = 3x - 15$

$x^2 - 16x + 15 = 0$

$(x-15)(x-1) = 0$

$\boxed{x = 1, 15}$

4 $\frac{2}{x+1} - \frac{1}{x-1} = \frac{-2}{x^2-1}$ $(x-1)(x+1)$

LCD $(x-1)(x+1)$

$2(x-1) - 1(x+1) = -2$

$2x - 2 - x - 1 = -2$

$\cancel{x = 1}$

no solution

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$$\frac{6x^{\cancel{(x+4)}(x-1)}}{\cancel{x+4}} + 4^{(x+4)(x-1)} = \frac{2x+2^{(x+4)\cancel{(x-1)}}}{x-1}$$

LCD: $(x+4)(x-1)$

$$6x(x-1) + 4(x^2+3x-4) = (2x+2)(x+4)$$

$$6x^2 - 6x + 4x^2 + 12x - 16 = 2x^2 + 10x + 8$$

$$8x^2 - 4x - 24 = 0$$

$$4(2x^2 - x - 6) = 0$$

$$4(2x+3)(x-2) = 0$$

$$x = 2, -\frac{3}{2}$$