

$$2.) y = \frac{(x+2)(x-2)}{2x(x-2)(x+2)} = \frac{x+2}{2x(x+2)}$$

2a) domain
 $\{x \mid x \neq 0, 2, -2\}$

Hole
 $(2, \frac{1}{4})$

Hole
 $(-2, -\frac{1}{4})$

$$4.) \quad \begin{array}{r} \textcircled{y=x} \\ \underline{x^3-1} \overline{) x^4 + 0x^3 + 2x^2 + 0x + 0} \\ \underline{-x^4} + x \\ \hline x \end{array}$$

$$4.) \quad \frac{\textcircled{x^2}(\textcircled{x^2+2})}{(x-1)(x^2+x+1)}$$

$$\boxed{x=0}$$

$$(0,0)$$

$$6. f(x) = \frac{x^3 - 4x^2 - 4x + 16}{x^3 - 5x^2 + 4x}$$
$$= \frac{(x-4)(x-2)(x+2)}{x(x-4)(x-1)}$$

(4, 1)

$$\frac{(2)(6)}{4(3)}$$

16.)

$$y = \frac{(2x-1)(x-7)}{x(x^2-10x+21)}$$

$$y = \frac{2x^2 - 15x + 7}{x^3 - 10x^2 + 21x}$$

$$5.) y = \frac{(x-8)(x+8)}{x-8} = x+8$$

hole: (8, 16)

$$2.) \frac{(3-2x)(3+2x)}{(2x-3)(3x+2)}$$

$$\frac{-\cancel{(2x-3)}(3+2x)}{\cancel{(2x-3)}(3x+2)}$$

$$-\frac{3+2x}{3x+2}$$

$$\frac{-3-2x}{3x+2}$$

$$\begin{array}{l}
 5.) \quad \frac{\cancel{7}^1 x^2 y^3}{\cancel{7}_1} \cdot \frac{\cancel{15}^5 y^4}{\cancel{4}_2 x} \cdot \frac{\cancel{6}^{21} x y}{\cancel{35}_5} \\
 \\
 \frac{x^3 y^8}{2x} = \frac{x^2 y^8}{2}
 \end{array}$$

$$7.) \quad \begin{array}{l} x^2 \cdot x + \frac{1}{x^2} \cdot x^2 \\ \hline x^2 \cdot 1 + \frac{1}{x} \cdot x^2 \end{array} \quad \frac{x^3 + 1}{x^2 + x}$$

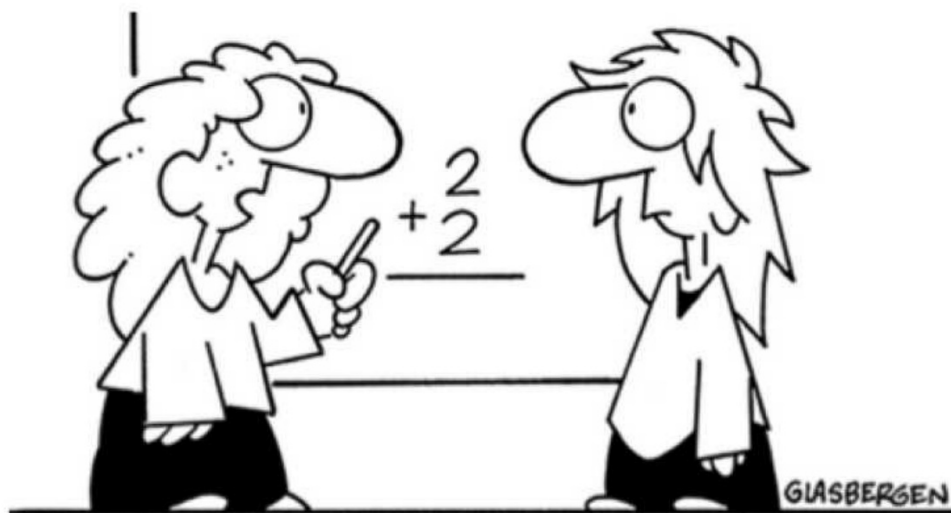
$$\frac{(x+1)(x^2-x+1)}{x(x+1)}$$

$$\frac{\cancel{x+1}}{x}$$

$$5.) \frac{\cancel{7}x^2y^4}{\cancel{3}9y} \cdot \frac{\cancel{3}5y^5}{\cancel{4}xy^2} \cdot \frac{\cancel{6}xy}{\cancel{35}81} \frac{1}{2} + \frac{1}{8}$$

$$\frac{x^3y^{10}}{2xy^3} = \frac{x^2y^7}{2}$$

5.3 Graphs of Rational Functions - cont.



"First they build up your confidence with simple addition and subtraction, then they slam you with algebra and calculus. It's quite a clever scheme."

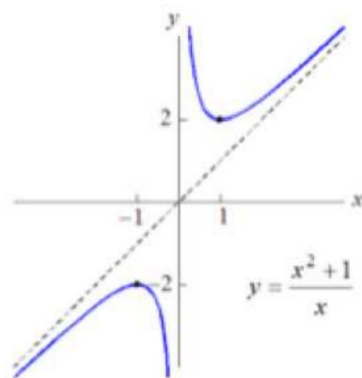
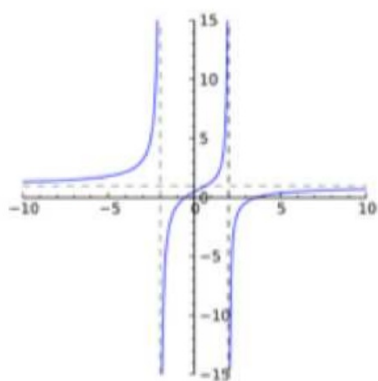
*See printout.

HW:

Sketching Requirements:

- x-intercepts
- y-intercept
- asymptotes
- holes
- plot at least one point per "piece"

Number of VA and the Number of "Pieces"



Number of Vertical Asymptotes	Number of "Pieces"
0	1
1	2
2	3
n	$n + 1$

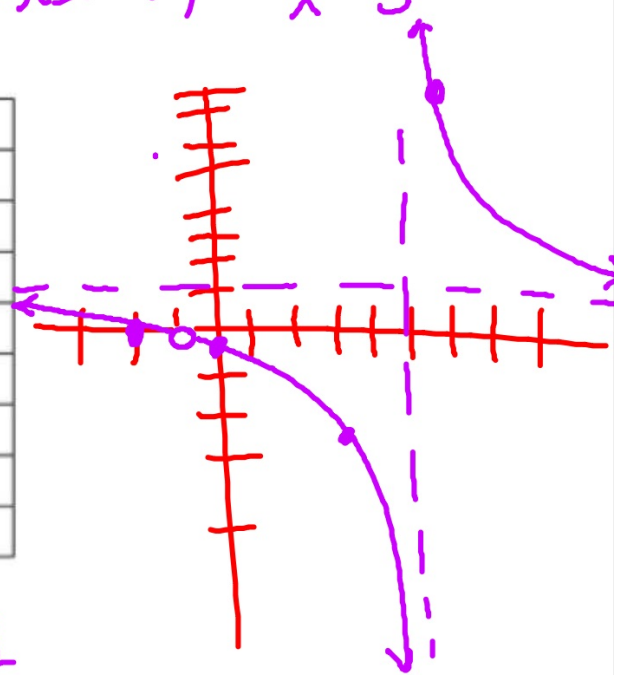
ex: Sketch and state the domain, range and end behavior.

$$a) f(x) = \frac{x^2 + 3x + 2}{x^2 - 4x - 5} = \frac{(x+2)(x+1)}{(x-5)(x+1)} = \frac{x+2}{x-5}$$

x-int	$x = -2$
y-int	$y = -2/5$
VA	$x = 5$
HA	$y = 1$
SA	none
Holes	$(-1, -1/6)$
Domain	$\{x \mid x \neq 5, -1\}$
Range	$\{y \mid y \neq 1, -1/6\}$
End Behavior	

$$\begin{aligned} x \rightarrow -\infty & \quad y \rightarrow 1 \\ x \rightarrow \infty & \quad y \rightarrow 1 \end{aligned}$$

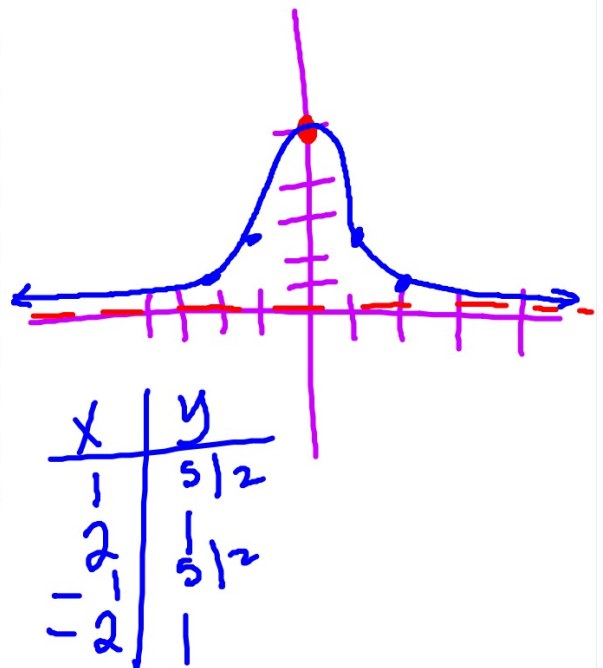
x	y
3	-5/2
6	8



ex: Sketch and state the domain, range and end behavior.

b) $f(x) = \frac{5}{x^2 + 1}$

x-int	none
y-int	(0, 5)
VA	none
HA	$y = 0$
SA	none
Holes	none
Domain	$\{x x \in \mathbb{R}\}$
Range	$\{y 0 < y \leq 5\}$
End Behavior	$x \rightarrow -\infty \quad y \rightarrow 0$ $x \rightarrow \infty \quad y \rightarrow 0$



ex: Sketch and state the domain, range and end behavior..

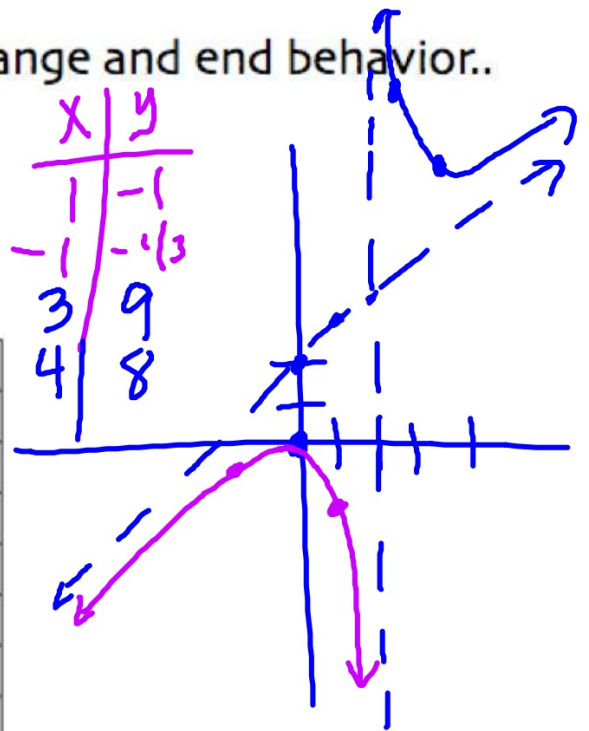
c) $y = \frac{x^2}{x-2}$

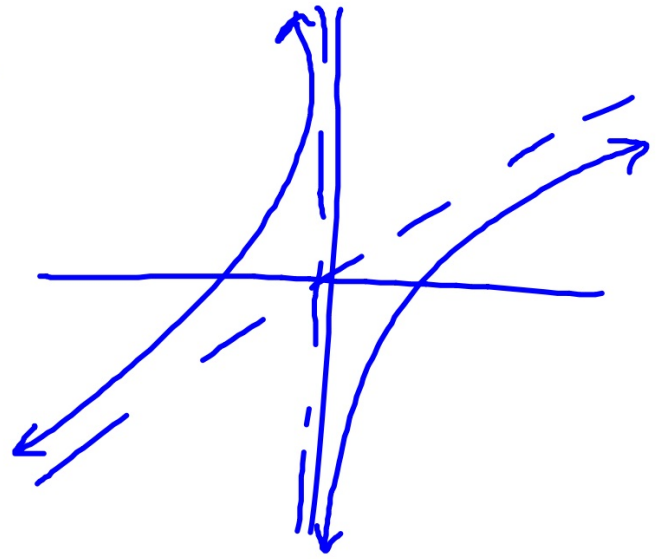
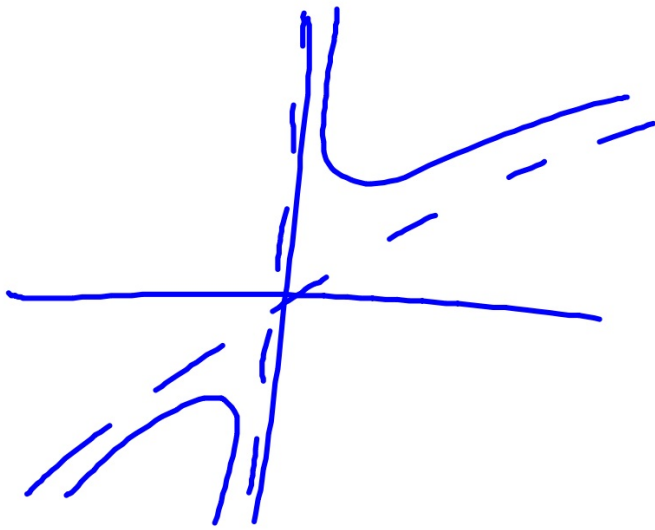
$$\begin{array}{r|rr} 2 & 1 & 0 & 0 \\ & & 2 & 4 \\ \hline & 1 & 2 & \cancel{4} \end{array}$$

$y = x + 2$

x	y
1	-1
-1	-1/3
3	9
4	8

x-int	(0,0)
y-int	(0,0)
VA	x=2
HA	none
SA	y=x+2
Holes	none
Domain	{x x ≠ 2}
Range	SKIP
End Behavior	x → -∞ y → -∞ x → ∞ y → ∞

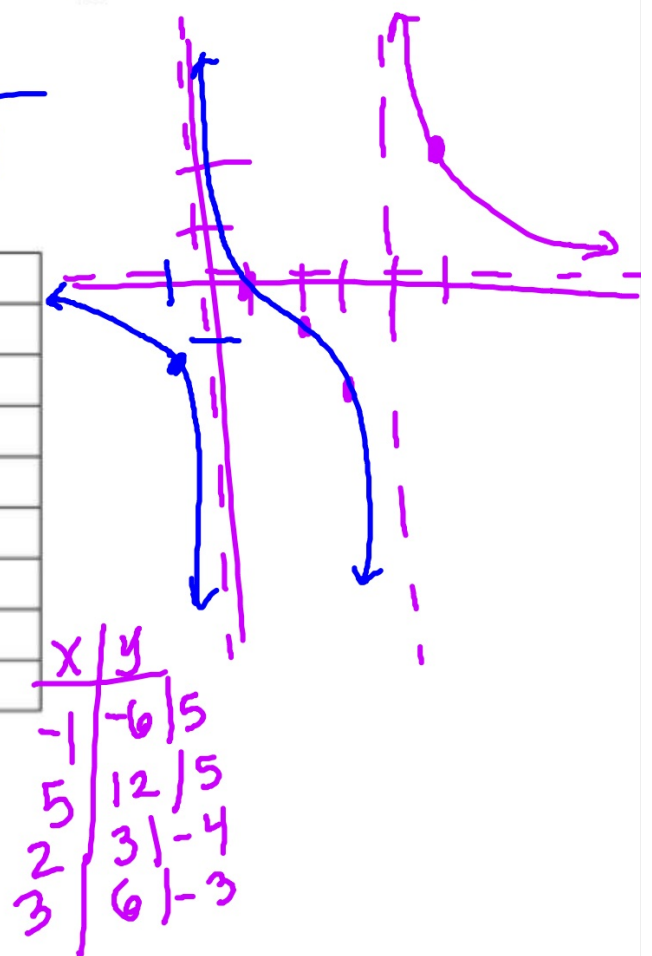




ex: Sketch and state the domain, range and end behavior..

$$d) y = \frac{3x-3}{x^2-4x} = \frac{3(x-1)}{x(x-4)}$$

x-int	(1, 0)
y-int	none
VA	$x=0, x=4$
HA	$y=0$
SA	none
Holes	none
Domain	$\{x \mid x \neq 0, 4\}$
Range	$\{y \mid y \in \mathbb{R}\}$
End Behavior	$x \rightarrow -\infty \quad y \rightarrow 0$ $x \rightarrow \infty \quad y \rightarrow 0$

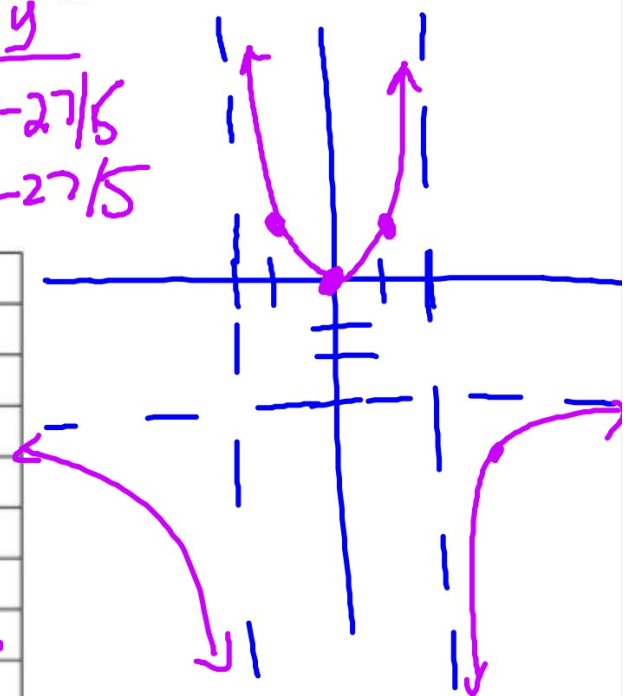


ex: Sketch and state the domain, range and end behavior..

e) $f(x) = \frac{-3x^2}{x^2 - 4}$

$$\begin{array}{c|c} x & y \\ \hline 3 & -27/5 \\ -3 & -27/5 \end{array}$$

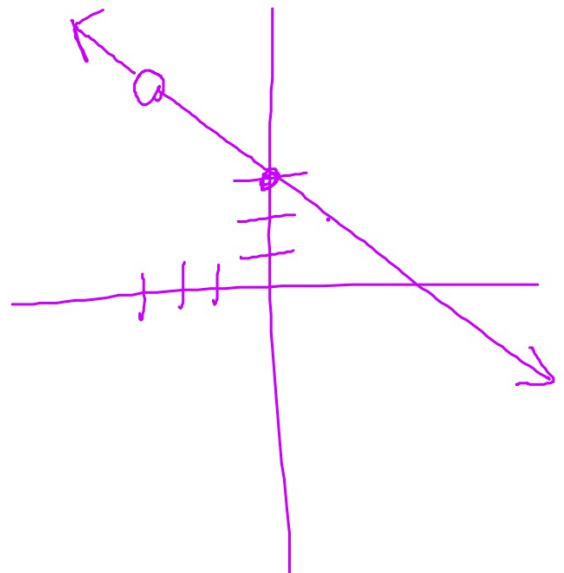
x-int	$(0, 0)$
y-int	$(0, 0)$
VA	$x = \pm 2$
HA	$y = -3$
SA	none
Holes	none
Domain	$\{x \mid x \neq 2, -2\}$
Range	$\{y \mid y \geq 0, y < -3\}$
End Behavior	$x \rightarrow -\infty \quad y \rightarrow -3$ $x \rightarrow \infty \quad y \rightarrow -3$



ex: Sketch and state the domain, range and end behavior..

$$f) f(x) = \frac{9 - x^2}{x + 3} = \frac{(3-x)(3+x)}{\cancel{x+3}} = 3 - x$$

x-int	(3, 0)
y-int	(0, 3)
VA	none
HA	none
SA	none
Holes	(-3, 6)
Domain	$\{x \mid x \neq -3\}$
Range	$\{y \mid y \neq 6\}$
End Behavior	$x \rightarrow -\infty \quad y \rightarrow \infty$ $x \rightarrow \infty \quad y \rightarrow -\infty$



ex: State the end behavior.

$$\text{a) } f(x) = \frac{x^2 + 3x + 2}{x^2 - 4x - 5}$$

ex: State the end behavior.

b) $y = \frac{x^2}{x-2}$

ex: State the end behavior.

c)

