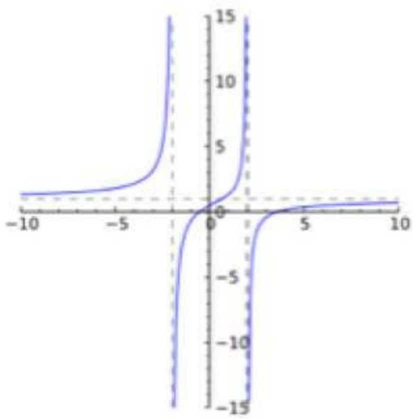
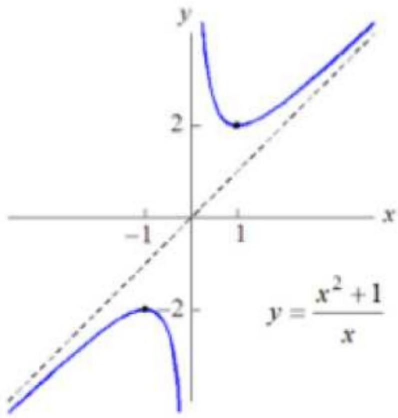
5.3 Graphs of Rational Functions





When Sketching Rational Functions You Must Find:

- x-intercept(s) (real zeros)
- y-intercept
- asymptotes (HA, VA, SA)
- holes

*See printout.

x-intercepts

ex: Find the x-intercepts, if any.

Simplify $f(x)=\dfrac{x^2-x-12}{x^2-2x-8}$

set num. = 0

$f(x)=\dfrac{\cancel{(x-4)}(x+3)}{\cancel{(x-4)}(x+2)}$

$x+3=0$

$x=-3$

$f(x)=\dfrac{x+3}{x+2}$

y-intercept

ex: Find the y-intercept, if any.

$$f(x)=\frac{x^2-x-12}{x^2-2x-8}$$

plug in
$x=0$

$$f(0)=\frac{-12}{-8}$$

$$=\frac{3}{2}$$

Finding Horizontal Asymptotes

To find the horizontal asymptote, compare the degree of the numerator and denominator. Three cases arise:

Case	Degree		Degree	Asymptote
1	Numerator	$<$	Denominator	$y = 0$
2	Numerator	$>$	Denominator	none
3	Numerator	$=$	Denominator	varies

Rational functions can have at most ONE HA

$y = \frac{x^3}{x+1}$ $x \to \infty$ $y \to \infty$

$y = \frac{1}{x^3}$ $x \to \infty$ $y \to 0$

Remembering Horizontal Asymptotes

$\boxed{\text{BOBO} \quad \text{BOTN} \quad \text{EATSDC}}$

BOBO	BOTN	EATSDC
bigger	bigger	exponents
on	on	are
bottom	top	the
zero	none	same
case 1	case 2	divide coefficients
$y=0$	NoHA	$y=\frac{2}{3}$

$$y=\frac{2x+5}{3x+1}$$

Horizontal Asymptotes

ex: Find the HA, if any.

a) $y = \dfrac{16x+1}{4x^2-2}$ bobo : HA: $y=0$

b) $y = \dfrac{16x^2+1}{4x^2-2}$ eatsdc : HA: $y=4$

c) $y = \dfrac{16x^3+1}{4x^2-2}$ botn: no HA

Finding Vertical Asymptotes

To find vertical asymptotes:

1. Simplify.
2. Set the simplified denominator = 0.

Rational functions can have more than one VA

Vertical Asymptotes

ex: Find the VA, if any.

a) $y = \dfrac{x^2 - 4x - 5}{x^2 - 1}$

Simplify :

$$y = \frac{x-5}{x-1}$$

Set den. $= 0$; $x = 1$

ex: Find the VA, if any.

b) $f(x) = \dfrac{3}{(x+8)^2}$

$(x+8)^2 = 0$

VA: $x = -8$

c) $f(x) = \dfrac{x+1}{x^2+1}$

$x^2 + 1 = 0$

$x^2 = -1$

no VA

d) $y = \dfrac{x-7}{x^3-8} = \dfrac{x-7}{(x-2)(x^2+2x+4)}$

$(x-2)\cancel{(x^2+2x+4)} = 0$

$x = 2$

ex: Find the VA, if any.

e) $y = \dfrac{x-2}{x^3-8}$

$y = \dfrac{1}{x^2+2x+4}$

no VA

imaginary

Finding Slant (Oblique) Asymptotes

*A rational function has a slant asymptote when the degree of the numerator is EXACTLY one greater than the degree of the denominator AND the denominator is NOT a factor of the numerator.

To find slant asymptotes:

1. Divide. There MUST be a remainder.
2. Ignore the remainder.

Rational functions can at most one SA

ex: Is it possible for a rational function to have both a slant and horizontal asymptote? Explain.

No, because when the degree of the numerator is more than the denominator, there is no HA.

ex: Find the SA, if any.

a) $y = \dfrac{x^2 - 2x + 3}{x + 3}$

$$
\begin{array}{r|rrr}
-3 & 1 & -2 & 3 \\
 & \downarrow & -3 & 15 \\
\hline
 & 1 & -5 & \cancel{18}
\end{array}
$$

$y = 1x - 5$

ex: Find the SA, if any.

b) $f(x) = \dfrac{x^2 - 3x + 7}{x - 2}$

$$
\begin{array}{r|rrr}
2 & 1 & -3 & 7 \\
  &   & 2 & -2 \\
\hline
  & 1 & -1 & \cancel{8}
\end{array}
$$

$y = x - 1$

ex: Find the SA, if any.

c) $f(x)=\dfrac{x+5}{x^2+4x+8}$

no SA

ex: Find the SA, if any.

d) $y = \dfrac{x^2 - 9}{x + 3}$

$y = \dfrac{x - 3}{1}$

no SA

Finding Holes

To find holes:

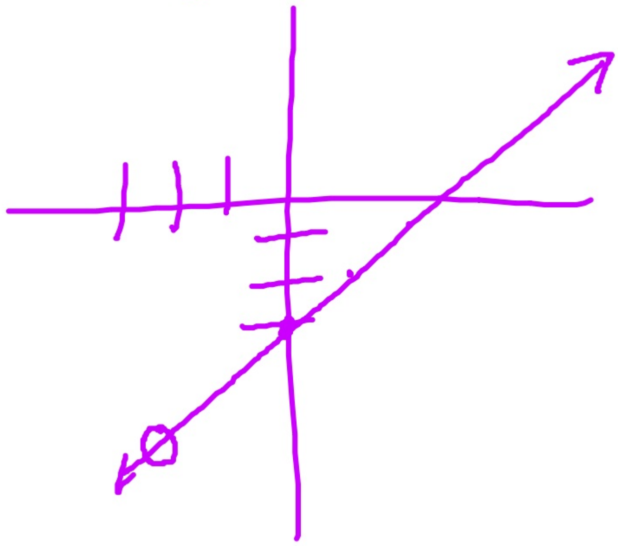
1. Factor completely.
2. If the numerator and denominator share a common factor a hole exists.
3. The hole exists at the zero of the common factor.
4. To find the y-value, plug in x into the SIMPLIFIED version.

Rational functions can have more than one hole ex: Find all holes, if any.

a) $y = \frac{x^2 - 9}{x + 3}$

$y = \frac{(x-3)\cancel{(x+3)}}{\cancel{(x+3)}} = y = x - 3$

$(-3, -6)$



ex: Find all holes, if any.

b) $y = \dfrac{x^2 + 4x - 5}{x^3 - 1}$

$$y = \frac{(x+5)\cancel{(x-1)}}{\cancel{(x-1)}(x^2+x+1)}$$

hole:

$(1, \ 2)$

ex: Find the x-intercepts, y-intercept, asymptotes and holes, if any.

$$f(x) = \frac{5x^2 - 9x - 18}{x^2 - 3x}$$