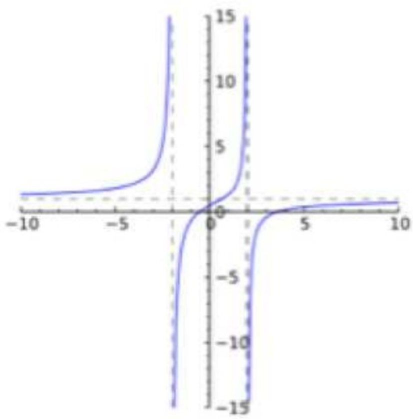
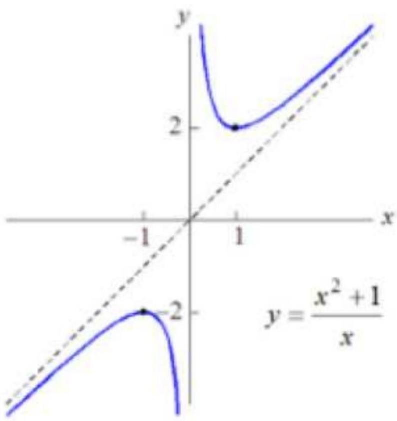
5.3 Graphs of Rational Functions





When Sketching Rational Functions You Must Find:

- x-intercept(s)
- y-intercept
- asymptotes (HA, VA, SA)
- holes

*See printout.

x-intercepts (real zeros)

ex: Find the x-intercepts, if any.

$$f(x) = \frac{x^2 - x - 12}{x^2 - 2x - 8}$$

x int

$0 = \frac{\cancel{(x-4)}(x+3)}{\cancel{(x-4)}(x+2)}$

$0 = \frac{x+3}{x+2}$

$x = -3$ or $(-3, 0)$

y-int

$\frac{-12}{-8} = \frac{3}{2}$

$y = \frac{3}{2}$ or

$(0, \frac{3}{2})$

Finding Horizontal Asymptotes

To find the horizontal asymptote, compare the degree of the numerator and denominator. Three cases arise:

Case	Degree		Degree	Asymptote
1	Numerator	$<$	Denominator	$y=0$
2	Numerator	$>$	Denominator	none
3	Numerator	$=$	Denominator	$y=\frac{a}{b}$

Rational functions can have at most ONE HA

Remembering Horizontal Asymptotes

BOBO BOTN EATSDC

Bigger	*Bigger*	*Exponents*
On	*On*	*Are*
Bottom	*Top*	*The*
o (zero)	*None*	*Same*
		Divide
		Coefficints

$\frac{x}{x^2+1}$

HA: $y=0$

$\frac{x^2+1}{x}$

no HA

$\frac{2x+5}{5x-1}$

HA: $y=\frac{2}{5}$

Horizontal Asymptotes

ex: Find the HA, if any.

a) $y = \dfrac{16x+1}{4x^2-2}$ $y=0$

bobo

b) $y = \dfrac{16x^2+1}{4x^2-2}$ $y=4$

eats dc

c) $y = \dfrac{16x^3+1}{4x^2-2}$ no HA

both

Finding Vertical Asymptotes

To find vertical asymptotes:

1. Simplify.
2. Set the simplified denominator = 0.

Rational functions can have more than one VA

Vertical Asymptotes

ex: Find the VA, if any.

a) $y = \dfrac{x^2 - 4x - 5}{x^2 - 1}$

$$y = \frac{(x-5)\cancel{(x+1)}}{(x-1)\cancel{(x+1)}}$$

$$x - 1 = 0$$

$$\boxed{x = 1}$$

ex: Find the VA, if any.

b) $f(x) = \dfrac{3}{(x+8)^2}$

$x+8=0$

$\boxed{x=-8}$

c) $f(x) = \dfrac{x+1}{x^2+1}$

$x^2+1=0$

$x^2=-1$

NO VA

d) $y = \dfrac{x-7}{x^3-8}$

$\dfrac{x-7}{(x-2)(x^2+2x+4)}$

$\boxed{x=2}$

ex: Find the VA, if any.

e) $y = \dfrac{x-2}{x^3-8}$

Finding Slant (Oblique) Asymptotes

*A rational function has a slant asymptote when the degree of the numerator is EXACTLY one greater than the degree of the denominator AND the denominator is NOT a factor of the numerator.

$$\frac{x^2}{x-5}$$

To find slant asymptotes:

1. Divide. There MUST be a remainder.
2. Ignore the remainder.

Rational functions can at most one SA

ex: Is it possible for a rational function to have both a slant and horizontal asymptote? Explain. No...

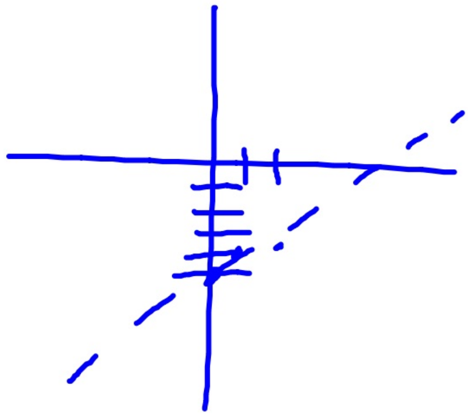
BOTN: When the degree of the numerator is larger than the denominator, the function has no HA. A HA only occurs when you have a BOBO or EATS DC.

ex: Find the SA, if any.

a) $y = \dfrac{x^2 - 2x + 3}{x + 3}$

$$\begin{array}{r|rrr} -3 & 1 & -2 & 3 \\ & & -3 & 15 \\ \hline & 1 & -5 & \cancel{18} \end{array}$$

$$\boxed{y = x - 5}$$



ex: Find the SA, if any.

b) $f(x)=\dfrac{x^2-3x+7}{x-2}$

ex: Find the SA, if any.

c) $f(x)=\dfrac{x+5}{x^2+4x+8}$

ex: Find the SA, if any.

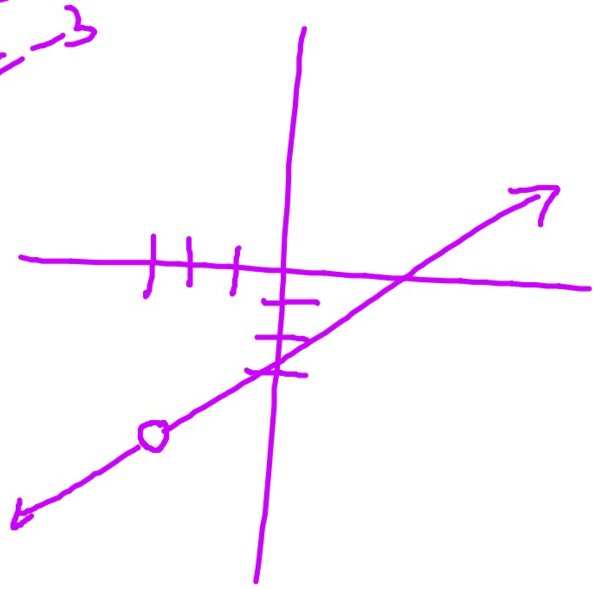
d) $y = \dfrac{x^2 - 9}{x+3}$

und. at $x = -3$

$y = \dfrac{(x-3)\cancel{(x+3)}}{\cancel{x+3}}$

$y = \dfrac{x-3}{1}$

SA: none



Finding Holes

To find holes:

1. Factor completely.
2. If the numerator and denominator share a common factor a hole exists.
3. The hole exists at the zero of the common factor.
4. To find the y-value, plug in x into the SIMPLIFIED version.

Rational functions can have more than one hole ex: Find all holes, if any.

a) $y = \dfrac{x^2 - 9}{x+3}$

ex: Find all holes, if any.

b) $y = \dfrac{x^2 + 4x - 5}{x^3 - 1}$

Hole: $(1, 2)$

$y = \dfrac{(x+5)\cancel{(x-1)}}{\cancel{(x-1)}(x^2+x+1)}$

$y = \dfrac{x+5}{x^2+x+1}$

$y(1) = \frac{6}{3} = 2$

ex: Find the x-intercepts, y-intercept, asymptotes and holes, if any.

$$f(x)=\frac{5x^2-9x-18}{x^2-3x}$$