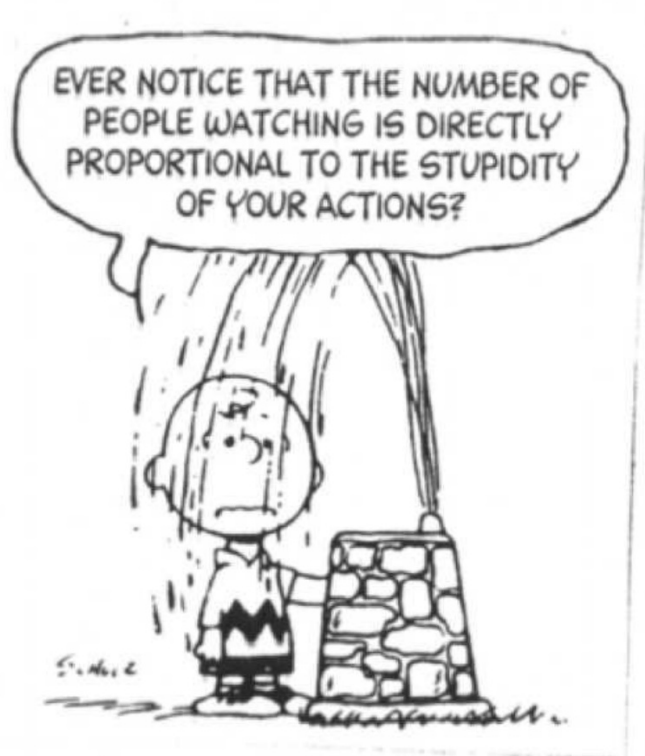


5.1 Variation Models



*See printout.

HW:

Variation Models

- Direct
- Inverse
- Joint
- Combined

Direct Variation

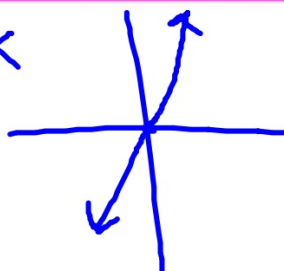
A direct variation is a relationship between two variables such that if one variable increases or decreases, then the other variable does the same.

In general, if a variable "y varies directly with a variable x", then we use the following variation function to represent this relationship:

$$y = kx$$

(where k is the constant of variation and $k \neq 0$)

$$y = 2x$$

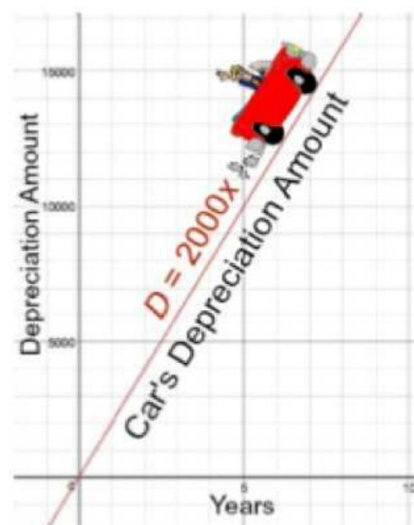


In this model we would say:

"D varies directly as x"

or

"D is (directly) proportional to x"



ex: y varies directly as x. If $y=45$ and $x=-5$, find y when $x=7$.

$$y = kx$$

$$45 = k(-5)$$

$$-9 = k$$

$$y = -9x$$

$$y = -9(7)$$

$$\boxed{-63}$$



ex: The dollar amount Jane earns is directly proportional to number of hours she works. If she earns \$116.25 after 15 hrs, how many hours did Jane work if she earned \$178.25?

$$d = kh$$

$$116.25 = k \cdot 15$$

$$7.75 = k$$

$$d = 7.75h$$

$$178.25 = 7.75h$$

$$23 \text{ hrs} = h$$

Inverse Variation

An inverse variation is a relationship between two variables such that if one variable increases or decreases, then the other variable does the opposite.

In general, if a variable "y varies inversely with a variable x", then we use the following variation function to represent this relationship:

$$y = \frac{k}{x}$$

(where k is the constant of variation and $k \neq 0$)



ex: a varies inversely as the cube of b. If $a=0.5$ and $b=2$, find a when $b=5$.

$$a = \frac{k}{b^3}$$

$$\frac{1}{2} = \frac{k}{8}$$

$$k = 4$$

$$a = \frac{4}{b^3}$$

$$a = \frac{4}{5^3}$$

$$a = \frac{4}{125}$$



ex: Determine whether each data set represents a direct variation, an inverse variation or neither.

a)

x	6.5	13	104
y	8	4	0.5

inverse
 $xy = 52$

$$y = kx$$
$$\frac{y}{x} = k$$

direct

$$y = \frac{k}{x}$$
$$yx = k$$

inverse



ex: Determine whether each data set represents a direct variation, an inverse variation or neither.

b)

x	3	6	8
y	5	14	21

neither



ex: Determine whether each data set represents a direct variation, an inverse variation or neither.

c)

x	5	8	12
y	30	48	72

$$\frac{y}{x} = k \quad \text{direct}$$

$$\frac{y}{x} = 6$$

Joint Variation

A joint variation (or jointly proportional variation) is a relationship between three variables.

In general, if a variable "y varies jointly with a variable x and z", then we use the following variation function to represent this relationship:

$$y = kxz$$

(where k is the constant of variation and $k \neq 0$)

ex: If l is jointly proportional to m and n , and $l=5$ when $m=3$ and $n=4$, then find l when $m=2$ and $n=3$.

$$l = kmn$$

$$5 = k(3)(4)$$

$$\frac{5}{12} = k$$

$$l = \frac{5}{12} \left(\frac{2}{1}\right) \left(\frac{3}{1}\right)$$

$$= \frac{30}{12}$$

$$= \frac{5}{2}$$

Combined Variation

A combined variation is a relationship that contains both direct (or joint) and inverse variation. Quantities that vary directly (or jointly) appear in the numerator and quantities that vary inversely appear in the denominator. The constant of variation, k , ALWAYS appears in the numerator.

In general, if a variable "y varies directly with a variable x and inversely with z", then we use the following variation function to represent this relationship:

$$y = \frac{kx}{z}$$

(where k is the constant of variation and $k \neq 0$)



ex: The volume of a gas varies inversely as the pressure and directly as the temperature. A certain gas has a volume of 10 liters, a temperature of 300 kelvins, and a pressure of 1.5 atmospheres. If the gas is compressed to a volume of 7.5 liters and is heated to 350 kelvins, what will the new pressure be?

$$V = \frac{K T}{P}$$

$$.05 = K$$

$$7.5 = \frac{(.05)(350)}{P} \cdot \bar{3}$$

$$7.5P = 17.5 \cdot \frac{7}{3}$$

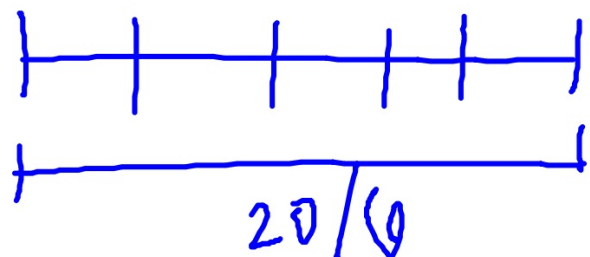
ex: Which of the following would best be represented as an inverse variation function?

~~a)~~ The distance traveled as a function of speed $d = r t$

~~b)~~ The total cost as a function of the number of items purchased

~~c)~~ The area of a circular swimming pool as a function of its radius $A = \pi r^2$

d) The number of posts in a 20ft fence as a function of distance between posts.



Mixed Practice

a) According to Hooke's Law, the force needed to stretch a spring is proportional to the amount the spring is stretched. If 50 lbs of force stretches a spring 5 in, how much will the spring be stretched by a force of 120 lbs?

12 in



Mixed Practice

b) Interest earned on a certificate of deposit (CD) at a certain rate varies jointly as the principal in dollars and the time in years. If Diane purchases a CD for \$2500 that earned \$12.50 interest in 3 months, how much interest would she earn in 6 months on a \$3000 CD purchased at the same bank?

\$ 30

Mixed Practice

c) Given that y varies inversely as the square of the difference of w and x , and that $y=6$ when $w=3$ and $x=1$, find an equation for y .

$$y = \frac{24}{(w-x)^2}$$



Mixed Practice

d) The centrifugal force of an object moving in a circle varies jointly with the radius of the circular path and the mass of the object and inversely as the square of the time it takes to move about one full circle. A 6 gram object moving in a circle with a radius of 75 cm at a rate of 1 revolution every 3 sec has a centrifugal force of 5,000 dynes. Find the centrifugal force of a 14 gram object moving in a circle with radius 125 cm at a rate of 1 revolution every 2 sec.

d) 43750

ex: Tell whether each statement is sometimes, always or never true.

a) Direct variation is a linear function. A

b) A linear function is a direct variation. S

c) An inverse variation is a linear function. N

d) The graph of an inverse variation passes through the origin. N

e) The graph of an direct variation passes through the origin. A