4.6 Solving Logarithmic Equations

48.) $\log 3^{x+4} = \log 6^{2x-5}$

$(x+4)\log 3 = (2x-5)\log 6$

$x\log 3 + 4\log 3 = 2x\log 6 - 5\log 6$

$x\log 3 - 2x\log 6 = -4\log 3 - 5\log 6$

$x(\log 3 - 2\log 6) = \dfrac{-4\log 3 - 5\log 6}{\log 3 - \log 36}$

$\log 3 - \log 36 \rightarrow \log \frac{1}{12}$

$x = 5.374$

23) $\frac{3}{4}e^{2x} + \frac{7}{2} = 4 - \frac{7}{2}$

$\frac{\frac{3}{4}e^{2x}}{\frac{3}{4}} = \frac{\frac{1}{2}}{\frac{3}{4}}$

$\ln e^{2x} = \ln \frac{2}{3}$

$2x = \ln \frac{2}{3}$

$x = \frac{\ln \frac{2}{3}}{2}$

$-.203$

17) $4e^{-2x} = 17$

$\ln e^{-2x} = \ln \frac{17}{4}$

$-2x = \ln \frac{17}{4}$

$x = \frac{\ln \frac{17}{4}}{-2}$

REVIEW: Evaluate.

a) $\log_3\left(\frac{1}{9}\right) = -2$

b) $\frac{\log 36}{\log 6} = \log_6 36 = 2$

c) $\log_8(-8)$ $\emptyset$ (undefined)

d) $\log 0$

Domain of Logarithmic Functions

$$y = \log_b\left(f(x)\right)$$

Domain: $f(x) > 0$

ex: State the domain in set notation.

a) $y = \log_2(x-1)$

Set the "argument" of the log > 0

$x - 1 > 0$

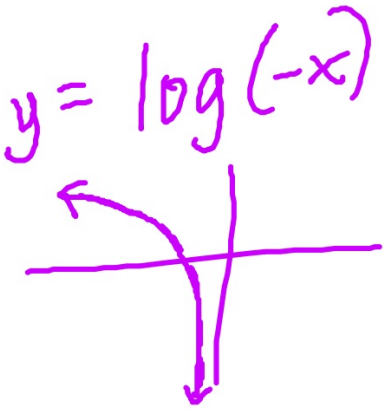
$\{x \mid x > 1\}$

ex: State the domain in set notation.

b) $y = 3 - \ln(-x)$

$-x > 0$

$\{x \mid x < 0\}$



c) $y = \log_9(x^2 + 9)$

$x^2 + 9 > 0$

$\{x \mid x \in R\}$

Solving Logarithmic Equations

Three Types:

1. 1 Logarithm

$$\text{ex: } 2-\log_2(x+1)=4$$

2. More than 1 Logarithm

$$\text{ex: } \log_3(x^2-3)=\log_3 2+\log_3 x$$

3. Quadratic Form

When solving type 1 or 2, rewrite the equation with ONE TERM on each side of the equation.

ex: Solve.

a) $\log_{36} x + \frac{1}{2} = 2$

$3\log_{36} x + \frac{3}{2} = 6$

$\log_{36} x = \frac{3}{2}$

$36^{3/2} = x$

$216 = x$

$36^{\log_{36} x} = 36^{\frac{3}{2}}$

$x = 36^{3/2}$

$x = 216$

ex: Solve.

b) $\log_{12}(x^2-4) = \log_{12}(3x)$

12 12

$x^2 - 4 = 3x$

$x^2 - 3x - 4 = 0$

$(x-4)(x+1) = 0$

$x = 4$ ~~$x = -1$~~

ex: Solve.

c) $\log_{31}(4x-5)-\log_{31}(2x-1)=0$

$\log_{31}(4x-5)=\log_{31}(2x-1)$

$31 \qquad 31$

$4x-5=2x-1$

$x=2$

ex: Solve.

d) $\log_2(4x-5)-\log_2(2x-1)=3$

condense

$\log_2 \frac{4x-5}{2x-1} = 3$

$2^{\log_2 \frac{4x-5}{2x-1}} = 2^3$

$\frac{4x-5}{2x-1} = \frac{8}{1}$ $\quad \emptyset$

$4x-5 = 16x-8$

$x = \frac{1}{4}$

ex: Solve.

e) $\log 18 - \log 3x = \log 2$

$$10^{\log \frac{18}{3x}} = 10^{\log 2}$$

$$\frac{18}{3x} = 2$$

$$6x = 18$$

$$x = 3$$

ex: Solve.

f) $\frac{\ln(-x)}{2}+4=5$

"e"

$e^{\ln(-x)}=e^{2}$

$-x=e^{2}$

$x=-e^{2}$

ex: Solve.

g) $2\log x = \log 2 + \log 8$

$\log x^2 = \log 16$

$x^2 = 16$

$x = \pm 4$

$x = 4$

ex: Solve.

h) $\log_5 \sqrt{x-2} = 1$

$5 \quad 5$

$\sqrt{x-2} = 5$

$x-2 = 25$

$x = 27$

$\frac{1}{2}\log_5(x-2) = 1$

$\log_5(x-2) = 2$

$5 \quad 5$

$x-2 = 25$

$x = 27$

ex: Solve.

i) $3\log_x 49 - 2 = 4$

base
x: $x > 1$ but $x \neq 1$

$\log_x 49 = 2$

$x^2 = 49$

$x = \pm 7$

$x = 7$

ex: Solve.

j) $\left(\log_2 x\right)^2 - 4\left(\log_2 x\right) - 5 = 0$

$u = \log_2 x$

$u^2 - 4u - 5 = 0$

$(u-5)(u+1) = 0$

$(\log_2 x - 5)(\log_2 x + 1) = 0$

$\log_2 x = 5$ $\quad$ $\log_2 x = -1$

$x = 32$ $\quad$ $2^{-1} = x$

$\frac{1}{2} = x$

ex: Solve.

k) $27^x \log_5(x-3) - 9\log_5(x-3) = 0$

$(\log_5(x-3))(27^x - 9) = 0$

$\log_5(x-3) = 0$

$5 \quad 5$

$x - 3 = 1$

$x = 4$

$27^x - 9 = 0$

$27^x = 9$

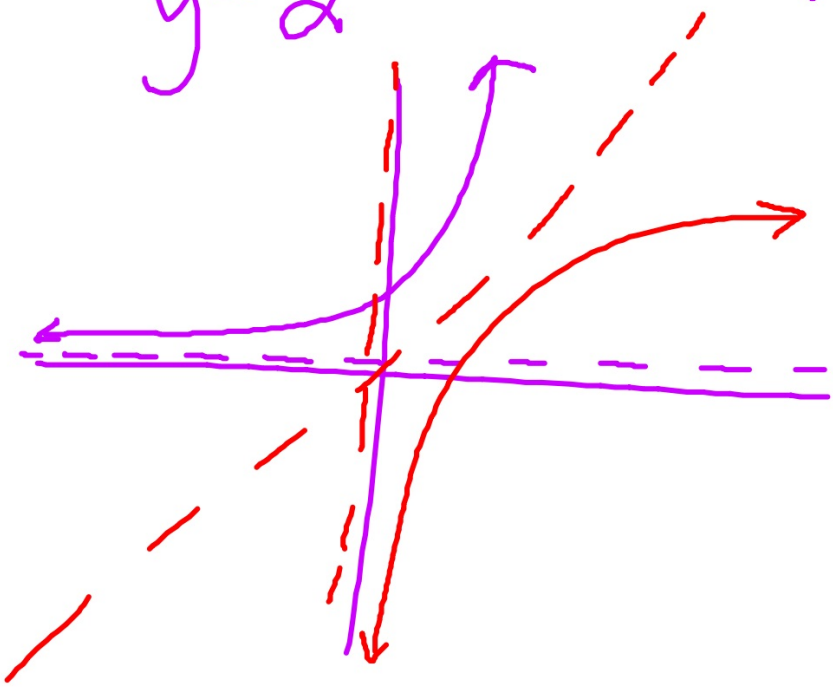
$x = \frac{2}{3}$ $3^{3x} = 3^2$

Find the inverse

$y = 2^x$ $\longrightarrow$ $\log_2 x = \log_2 2^y$

$\log_2 x = y$

$y^{-1} = \log_2 x$



a.) $f(x) = 2 \cdot 3^{x-1} + 1$

$x = 2 \cdot 3^{y-1} + 1$

$\log_3 \frac{x-1}{2} = \log_3 3^{y-1}$

$\log_3 \frac{x-1}{2} = y - 1$

$\log_3 \frac{x-1}{2} + 1 = f^{-1}(x)$

b.) $g(x) = 2\log_4(x+5) - 4$

$x = 2\log_4(y+5) - 4$

$4^{\frac{x+4}{2}} = 4^{\log_4(y+5)}$

$4^{\frac{x+4}{2}} - 5 = g^{-1}(x)$