11.) $25^{10x+8} = \left(\frac{1}{125}\right)^{4-2x}$

$$\left(5^{2}\right)^{10x+8} = \left(5^{-3}\right)^{4-2x}$$

$$20x+16 = -12+6x$$

$$14x = -28$$

$$x = -2$$

17.)

$$4e^{-2x} = 17$$

$$\ln e^{-2x} = \ln \frac{17}{4}$$

$$-2x = \ln \frac{17}{4}$$

$$x = \frac{\ln \frac{17}{4}}{-2} = -.723$$

19.) $-3e^{2x} + 16 = 5$

$$e^{2x} = \frac{-11}{-3}$$

$$e^{2x} = \frac{11}{3}$$

$$\frac{\ln \frac{11}{3}}{2} = \cancel{2}x$$

21.) $\frac{1}{3}(6)^{-4x} + 1 = 6$

$\log_{10} \frac{1}{3}$

$\log_6 6^{-4x} = \log_6 15$

$-4x = \log_6 15$

$x = \frac{\log_6 15}{-4}$

$-.378$

$\log 6^{-4x} = \log 15$

$-4x \cdot \log 6 = \log 15$

$x = \frac{\log 15}{(-4 \log 6)}$

$= \frac{\log 15}{\log 6} \div -4$

53.) $5^{2x} + 20 \cdot 5^{x} - 125 = 0$

$(5^{x} - 5)(5^{x} + 25) = 0$

$5^{x} - 5 = 0$ $\quad$ $5^{x} + 25 = 0$

$5^{x} = 5$ $\quad$ $\cancel{\log 5^{x} = \log(-25)}$

$x = 1$

48.) $\log_3 3^{x+4} = \log_3 6^{2x-5}$

$(x+4) = (2x-5)\log_3 6$

$x+4 = 2x\log_3 6 - 5\log_3 6$

$x - 2x\log_3 6 = -4 - 5\log_3 6$

$x(1-2\log_3 6) = -4 - 5\log_3 6$

$$x = \frac{-4-5\log_3 6}{1-2\log_3 6}$$

4.6 Solving Logarithmic Equations

REVIEW: Condense.

$$\log(x-2)+2\log x-3\log(x-1)$$

$$\log\left(\frac{x^2(x-2)}{(x-1)^3}\right)$$

REVIEW: Evaluate.

a) $\log_3\left(\frac{1}{9}\right) = -2$

b) $\frac{\log 36}{\log 6} = \log_6 36 = 2$

c) $\log_8(-8)$ $\emptyset$

d) $\log 0$ $\emptyset$

Domain of Logarithmic Functions

$$y = \log_b\left(f(x)\right)$$

Domain: $f(x) > 0$

positive

ex: State the domain in set notation.

a) $y = \log_2(x-1)$

$x - 1 > 0$

$x > 1$

$\{x \mid x > 1\}$

ex: State the domain in set notation.

b) $y = 3 - \ln(-x)$

$-x > 0$

$x < 0$

$\{x \mid x < 0\}$

c) $y = \log_9(x^2 + 9)$

$x^2 + 9 > 0$

$\{x \mid x \in \mathbb{R}\}$

Solving Logarithmic Equations

Two Types:

1. 1 Logarithm

$$\text{ex: } 2 - \log_2(x+1) = 4$$

2. More than 1 Logarithm

$$\text{ex: } \log_3(x^2 - 3) = \log_3 2 + \log_3 x$$

3. Quadratic Form

When solving type 1 or 2, rewrite the equation with ONE TERM on each side of the equation.

ex: Solve.

a) $\log_{36} x + \frac{1}{2} = 2$

$\log_{36} x = \frac{3}{2}$

Method #1
rewrite as an exponential

$36^{3/2} = x$

$216 = x$

Method #2 Take a "base" of both sides

$\log_{36} x = \frac{3}{2}$

$36^{\log_{36} x} = 36^{\frac{3}{2}}$

$x = 36^{3/2}$

$x = 216$

ex: Solve.

b) $\log_{12}\left(x^2-4\right)=\log_{12}\left(3x\right)$

12 12

$x^2-4=3x$

$x^2-3x-4=0$

$(x-4)(x+1)=0$

$4, \cancel{-1}$

ex: Solve.

c) $\log_{31}(4x-5) = \log_{31}(2x-1) \cancel{= 0}$

31 31

$4x - 5 = 2x - 1$

$x = 2$

ex: Solve.

d) $\underbrace{\log_2(4x-5)-\log_2(2x-1)}=3$

$$2^{\log_2 \frac{4x-5}{2x-1}} = 2^3$$

$$\frac{4x-5}{2x-1} = 8 \; ; \; 4x-5 = 16x-8$$

$$\cancel{\frac{1}{4} = x}$$

$$\emptyset$$

ex: Solve.

e) $\underbrace{\log 18 - \log 3x}$ $= \log 2$

$\log 18 - \log 9$

$\log \frac{18}{9}$

$10^{\log \frac{18}{3x}} = 10^{\log 2}$

$\frac{18}{3x} = 2$

$3 = x$

ex: Solve.

f) $\dfrac{\ln(-x)}{2} + 4 = 5$

$e \approx 2.718$

$e^{\ln(-x)} = e^{2}$

$-x = e^{2}$

$x = -e^{2}$

ex: Solve.

g) $2\log x = \log 2 + \log 8$

$\log_{10} x^2 = \log 16$

$x^2 = 16$

$x = \pm 4$

$x = 4$

ex: Solve.

h) $\log_5 \sqrt{x-2} = 1$

$5^{\log_5 \sqrt{x-2}} = 5^1$

$\sqrt{(x-2)}^2 = 5^2$

$x-2 = 25$

$x = 27$

$\frac{1}{2}\log_5(x-2) = 1$

$5^{\log_5(x-2)} = 5^2$

$x-2 = 25$

$x = 27$

ex: Solve.

i) $3\log_x 49 - 2 = 4$

$$x^{\log_x 49} = x^2$$

$$49 = x^2$$

$$\pm 7 = x$$

$$7 = x$$

ex: Solve.

j) $(\log_2 x)^2 - 4(\log_2 x) - 5 = 0$

$(\log_2 x - 5)(\log_2 x + 1) = 0$

$\log_2 x = 5$

$x = 32$

$\log_2 x = -1$

$x = \frac{1}{2}$

$a^2 - 4a - 5$

$(a - 5)(a + 1)$

ex: Solve.

k) $27^x \log_5(x-3) - 9\log_5(x-3) = 0$

Review

ex: Let

$$x = \log 7 \qquad y = \log 14$$

Rewrite in terms of x and y.

$$\log 2$$

Review

ex: Solve.

$$9^{3x^2-6x} = 3^{x+5}$$

$$2(3x^2-6x) = x+5$$

$$6x^2-13x-5=0$$

$$(2x-5)(3x+1)=$$

$$5/2, -1/3$$

Review

ex: Solve.

$$6^{3-6x} = 3^{x+5}$$

Review

ex: Expand.

$$\log_5 \sqrt{\frac{x-2}{25y^3x^7}}$$