4.6 Solving Exponential Equations

3 Types of Exponential Equations:

1. $a^x = b$, where a and b are integral powers of the same number

$$\text{ex: } 27^x = 9$$

2. $a^x = b$, where a and b are NOT integral powers of the same number

$$\text{ex: } 3^x = 5$$

3. quadratic form

$$\text{ex: } 3^{2x} + 3^x - 6 = 0$$

Type 1

Property of Equality for Exponential Equations

$$\text{If } a^x = a^y, \text{ then } x = y.$$

To solve these equations, use the property of equality to make the bases equal.

ex: Solve.

a) $3^x = 9^{x+2}$

$3^x = (3^2)^{x+2}$

$3^x = 3^{2x+4}$

$x = 2x + 4 \; ; \; -x = 4$

$x = -4$

Check

$3^{-4} = 9^{-4+2}$

$\frac{1}{81} = \frac{1}{81}$ ✓

ex: Solve.

b) $125^x = \left(\frac{1}{25}\right)^{x-1}$

$(a^m)^n = a^{mn}$

$(5)^{3x} = 5^{-2(x-1)}$

$3x = -2x + 2$

$x = \frac{2}{5}$

ex: Solve.

$x^m \cdot x^n = x^{m+n}$

c) $2^x \cdot 8^{x-1} = \left(\frac{1}{16}\right)^{2x-5}$

$2^x \cdot 2^{3(x-1)} = 2^{-4(2x-5)}$

$2^{x+3x-3} = 2^{-8x+20}$

$4x-3 = -8x+20$

$12x = 23$

$x = \frac{23}{12}$

Type 2

ex: Solve.

d) $3^x = 5$

Method 1

Re-write as a log.

$$\log_3 5 = x$$

$$\frac{\log 5}{\log 3} = x$$

$$1.465 = x$$

Method 2

take the log of both sides.

$$\log 3^x = \log 5$$

$$x \cdot \log 3 = \log 5$$

$$x = \frac{\log 5}{\log 3}$$

ex: Solve.

e) $e^{x+1} = 10$

Method 1

$\log_e 10 = x+1$

$\ln 10 = x+1$

$(\ln 10) - 1 = x$

$1.303 = x$

Method 2

$\ln e^{x+1} = \ln 10$

$(x+1) \cdot \cancel{\ln e} = \ln 10$

$x+1 = \ln 10$

ex: Solve.

f) $2 \cdot 10^{x-3} - 3 = 37$

$$10^{x-3} = 20$$

$$\log_{10} 20 = x - 3$$

$$(\log 20) + 3 = x$$

$$4.301 = x$$

ex: Solve.

g) $2 - 5^{x-2} = 3$

ex: Solve.

h) $5^{x-1} = 3^{x+2}$

$$(\log_5 5)^{x-1} = \log_5 3^{x+2}$$

$$x - 1 = (x+2)\log_5 3$$

$$x - 1 = (\log_5 3)x + (\log_5 3)\cdot 2$$

$$x - (\log_5 3)x = 1 + (\log_5 3)\cdot 2$$

$$x(1 - \log_5 3) = 1 + 2\log_5 3$$

$$x = \frac{1 + \log_5 9}{-\log_5 3 + 1} =$$

$$x = 7.452$$

$$32341 \approx 32341$$

Type 3

ex: Solve.

$a = 2^x$

$a^2 = 2^{2x}$

i) $2^{2x} - 5 \cdot 2^x - 24 = 0$

$a^2 - 5a - 24 = 0$

$(a-8)(a+3) = 0$

$a = 8 \quad a = -3$

$2^x = 8 \quad \cancel{2^x = -3}$

$x = 3$

ex: Solve.

$a = 4^x$

j) $4^{2x} - 7 \cdot 4^x + 12 = 0$

$a^2 - 7a + 12 = 0$

$(a-4)(a-3) = 0$

$a = 4 \qquad a = 3$

$4^x = 4 \qquad 4^x = 3$

$x = 1 \qquad \log_4 3 = x$

MIXED PRACTICE

ex: Solve.

a) $5 \cdot 2^x - 3 = 157$

b) $2 \cdot 3^{2x} - 4 \cdot 2^x - 12 = 0$

c) $8^{x+1} = 4^{x-3}$

d) $\dfrac{81^{3-x}}{3^{x+1}} = \left(\dfrac{1}{3}\right)^{6x-5}$

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b) $2 \cdot 3^{2x} - 4 \cdot 2^{x} - 12 = 0$

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