3 Types of Exponential Equations:

1. $a^x = b$, where a and b are integral powers of the same number

ex: $27^x = 9$ *powers of 3*

2. $a^x = b$, where a and b are NOT integral powers of the same number

ex: $3^x = 5$

3. quadratic form

ex: $3^{2x} + 3^x - 6 = 0$

Type 1

Property of Equality for Exponential Equations

If $a^x = a^y$, then $x = y$.

To solve these equations, use the property of equality to make the bases equal.

ex: Solve.

a) $3^x = 9^{x+2}$

$$3^x = 3^{2(x+2)}$$

set exponents equal

$$x = 2(x+2)$$
$$x = 2x + 4$$
$$-4 = x$$

$$3^{-4} = 9^{-2} \checkmark$$
$$\frac{1}{81} = \frac{1}{81}$$

ex: Solve.

b) $125^x = \left(\dfrac{1}{25}\right)^{x-1}$

$\left(5^3\right)^x = \left(5^{-2}\right)^{x-1}$

$3x = -2(x-1)$

$3x = -2x + 2$

$5x = 2$

$x = \dfrac{2}{5}$

ex: Solve.

c) $2^x \cdot 8^{x-1} = \left(\dfrac{1}{16}\right)^{2x-5}$

$x^2 \cdot x^3 = x^5$

$2^x \cdot 2^{3(x-1)} = 2^{-4(2x-5)}$

$x + 3x - 3 = -8x + 20$

$4x - 3 = -8x + 20$

$12x = 23$

$\boxed{x = 23/12}$

ex: Solve.

d) $3\left(4^{x-2}\right) - 5 = 1$

$\phantom{3\left(4^{x-2}\right)} +5 \; +5$

$$\frac{\cancel{3} \cdot 4^{x-2}}{\cancel{3}} = \frac{6}{3}$$

$$4^{x-2} = 2^1$$

$$\left(2^2\right)^{x-2} = 2^1$$

$$2(x-2) = 1$$

ex: Solve.

e) $16^{x^2} = \left(\dfrac{1}{4}\right)^{10x-12}$

$$\left(4^2\right)^{x^2} = 4^{-1(10x-12)}$$

$$2x^2 = -10x + 12$$
$$2x^2 + 10x - 12 = 0$$
$$2(x^2 + 5x - 6) = 0$$

$$2(x+6)(x-1) = 0$$

$$\boxed{x = -6, 1}$$

ex: Solve.

f) $\dfrac{5^{x-2}}{25^{x+3}} = \left(\dfrac{1}{125}\right)^{2x+1}$

$\dfrac{5^{x-2}}{5^{2(x+3)}} = 5^{-3(2x+1)}$

$5^{x-2-2(x+3)} = 5^{-3(2x+1)}$

$-x-8 = -6x-3$

$\dfrac{x^{10}}{x^3} = x^7$

$5x = 5$

$\boxed{x = 1}$