33.) $\log_3 81^x$

$4x$

$3^{\square} = 81^x$

$3^{\square} = (3^4)^x$

31.) $10^{\log_{10} 8} = 8$

35.) $\log_2 32^x$

$2^{\square} = 32^x$

13.) $\log_{1/2} 8$

$$\left(\frac{1}{2}\right)^x = 8$$

$(2^3)^x$

55.) $\log_8 32$

$$8^{\boxed{x}} = 32$$

$$2^{3x} = 2^5$$

$x = 5/3$

55.) $\log_8 32$

$8^x = 32$

$(2^3)^x = 2^5$

$3x = 5$

$2^{3x} = 2^5$

54.) $\log_{27} 9$

$27^x = 9$

$3x = 2$

$3^{3x} = 3^2$

$x = 2/3$

29.) $\log_5 5^x = x$ $\quad \log_2 8 \quad 2^x = 8$

$5^{\square} = 5^x$

16.) $\log_{.14} 16 = -2$

$\frac{1}{4}^{\square} = 16$

$$\log_5 \frac{1}{5} < \log_5 \frac{1}{2} < \log_5 1$$

$$-1 < \log_5 \frac{1}{2} < 0$$

$$\log_3 3 < \log_3 8 < \log_3 9$$

$$1 < \log_3 8 < 2$$

4.5: Properties of Logs

Product Property: $\log(ab) = \log a + \log b$ $\quad x^2 \cdot x^3 = x^5$

Quotient Property: $\log(a/b) = \log a - \log b$ $\quad \frac{x^5}{x^2} = x^3$

Power Property: $\log a^b = b(\log a)$ $\quad (x^2)^3 = x^6$

The rules apply to a log with ANY valid base

examples: $\ln x$ $\quad \log_2 x$ $\quad \log_{1/3} x$

Expand the expression.

1 $\log_4\left(\frac{ab^2}{4c}\right) = \log_4 a + \log_4 b^2 - \log_4 4 - \log_4 c$

$= \log_4 a + 2\log_4 b - 1 - \log_4 c$

② $\ln\left(\frac{a^2 b^3}{e^4 d^5}\right)$ $\ln x = \log_e x$

$= 2\ln a + 3\ln b - 4\ln e - 5\ln d$

$2\ln a + 3\ln b - 4 - 5\ln d$

③ $\log\sqrt{\frac{1}{ab^4}}$

$\log\left(\frac{1}{ab^4}\right)^{1/2}$

$\frac{1}{2}\log\frac{1}{ab^4}$

$\frac{1}{2}\left(\log 1 - \log a - \log b^4\right)$

$\frac{1}{2}\left(0 - \log a - 4\log b\right) = -\frac{1}{2}\log a - 2\log b$

4 $\log_3(ab)$

$\log_3 a + \log_3 b$

5 $\log_3(a+b)$

already simplified

6 $\log_3\left(\frac{a+b}{a-b}\right)$

$\log_3(a+b) - \log_3(a-b)$

7 $\log_5(a^2-b^2)$

$\log_5((a+b)(a-b))$

$\log_5(a+b)+\log_5(a-b)$

8 $\log_5(a-b)^3 = 3\log_5(a-b)$

$(a-b)^3 \neq a^3-b^3$

Condense the expression.

(9) $2\log x + 4\log y - 7\log z$

$\log x^2 + \log y^4 - \log z^7$

$\log\left(\frac{x^2 y^4}{z^7}\right)$

10

$$\frac{1}{2}\log x - 3\log y + \log z$$

$$\log x^{1/2} - \log y^3 + \log z$$

$$\log\left(\frac{x^{1/2} z}{y^3}\right) = \log\left(\frac{\sqrt{x}\, z}{y^3}\right)$$

11 $\frac{1}{5}\ln a + \frac{2}{5}\ln b + \frac{3}{5}\ln c$

$\frac{1}{5}\left(\ln a + 2\ln b + 3\ln c\right)$

$\frac{1}{5}\ln\left(a \cdot b^2 \cdot c^3\right)$

$\ln\left(ab^2c^3\right)^{1/5} = \ln\sqrt[5]{ab^2c^3}$