4.5 Properties of Logarithms - Cont.

REVIEW - Evaluate.

$\ln e = 1$

a) $\log_5 125$ 3

$\log_e e = 1$

e) $\log_5 1$ 0

b) $\log_{27} 3$ 1/3

$\log_5 5$

1

f) $\log_4\left(\frac{1}{8}\right)$ $-3/2$

c) $\ln e^8$ 8

g) $7^{\log_7 15}$ 15

d) $\log 1000$ 3

h) $3(\log 0.01) - 2\ln e + (\log 1)^{10}$

$-6 - 2 = -8$

ex: Between which two consecutive integers does each value lie?

a) $\log_2 7$

$\log_2 2$ $\quad$ $\log_2 4$ $\quad$ $\log_2 8$

1 $\quad$ 2 and 3

b) $\log 5$

$\log 1$ $\quad$ $\log 10$

0 and 1

REVIEW - Expand.

b) $\log_3 \sqrt{x^2+y^2} = \log_3 \left(\sqrt{x^2+y^2}\right)^{1/2} = \frac{1}{2}\log_3(x^2+y^2)$

a) $\ln\left(\frac{x^5}{10y^4z^3}\right) = 5\ln x - \ln 10 - 4\ln y - 3\ln z$

c) $\frac{1}{2}\left(\log_3(x+3)(x-3)\right) = \frac{1}{2}\log_3(x+3) + \frac{1}{2}\log_3(x-3)$

REVIEW - Condense.

a) $3\ln x - 3\ln(y+z) + 4\ln 20$

$$\ln\left(\frac{x^3 \cdot 20^4}{(y+z)^5}\right)$$

b) $\log_5 a - 4\log_5 b - 2\log_5 25$

$$\log_5\left(\frac{a}{b^4 \cdot 25}\right)$$

$\log_5 25$

2

ex: Evaluate.

a) $\log_3 54 - \log_3 2$

$\log_3\left(\frac{54}{2}\right) = \log_3 27 = 3$

$4^{3/2} = 8$

b) $\log_4\left(\frac{32}{3}\right) + \log_4\left(\frac{3}{4}\right) = \log_4\left(\frac{32}{3} \cdot \frac{3}{4}\right) = \log_4 8$

$= 3/2$

ex: Evaluate.

c) $9^{\log_9 20 - \log_9 4}$

$9^{\log_9 5} = 5$

d) $9^{\log_3 5} = 3^{2\log_3 5}$

$= 3^{\log_3 25} = 25$

e) $2^{\log_2 15 + \log_4 9}$

ex: Let

$\log 10 = 1$

$x = \log 2 \qquad y = \log 3 \qquad z = \log 7$

Rewrite each expression in terms of x, y and z.

$3^5 = 243$

a) $\log 14$

$\log(2 \cdot 7)$

$\log 2 + \log 7$

$x + z$

ex: Let

$\log 10 = 1$

$x = \log 2$ $\quad$ $y = \log 3$ $\quad$ $z = \log 7$

Rewrite each expression in terms of x, y and z.

b) $\log 243$

$\log 3^5$

$5(\log 3)$

$5y$

c) $\log 30$

$\log(3 \cdot 10)$

$\log 3 + \log 10$

$y + 1$

d) $\log 5$

$\log\left(\frac{10}{2}\right)$

$\log 10 - \log 2$

$1 - x$

ex: Let $\log_2 2 = 1$ $\log_2 4 = 2$

$\log_2 3 \approx 1.585$ $\log_2 5 \approx 2.322$

Approximate each expression.

a) $\log_2 15$

$\log_2(3 \cdot 5)$

$\log_2 3 + \log_2 5$

$1.585 + 2.322$

3.907

b) $\log_2 20 = \log_2(4 \cdot 5)$

$\log_2 4 + \log_2 5$

$2 + 2.322$

4.322

Change of Base Formula

$$\log_b x = \frac{\log x}{\log b} \text{ or } \frac{\ln x}{\ln b}$$

ex: Rewrite using common or natural logarithms. Then evaluate on your calculator.

a) $\log_2 11 = \frac{\log 11}{\log 2} = 3.459$

$\frac{\ln 11}{\ln 2} =$

b) $\log_3 25 = 2.93$

ex: Evaluate.

a) $\dfrac{\ln 8}{\ln 2} = \log_2 8 = 3$

b) $\dfrac{\log 64}{\log 2} = \log_2 64 = 6$

c) $\dfrac{\log_2 5}{\log_2 125} = \log_{125} 5 = \frac{1}{3}$