

4.4 Graphs of Logarithmic Functions

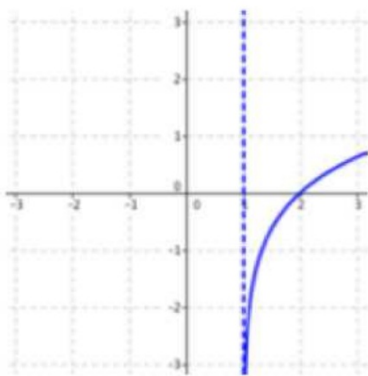
4.6 Solve Exponential and Logarithmic Equations.



HW:

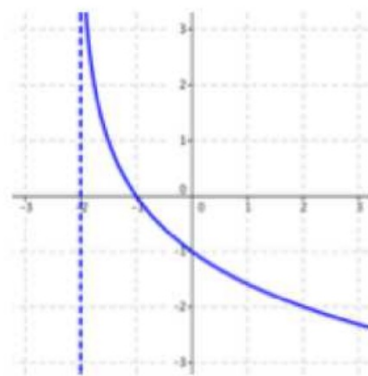
Graphs of Logarithmic Functions

$$f(x) = \log_b x$$



INCREASING

$$b > 1$$



DECREASING

$$0 < b < 1$$

$$f(x) = 2^x$$

$$D: (-\infty, \infty)$$

$$R: (0, \infty)$$

$$g(x) = \log_2 x$$

$$D: (0, \infty)$$

$$R: (-\infty, \infty)$$

$$(f \circ g)(x) = 2^{\log_2 x} = x$$

$$(g \circ f)(x) = \log_2 2^x = x$$

ex: Sketch. Then state the domain and range and determine if the graph is increasing or decreasing.

a) $y = \log_2(x+1) - 3$

$$y + 3 = \log_2(x+1)$$

$$x + 1 = 2^{y+3}$$

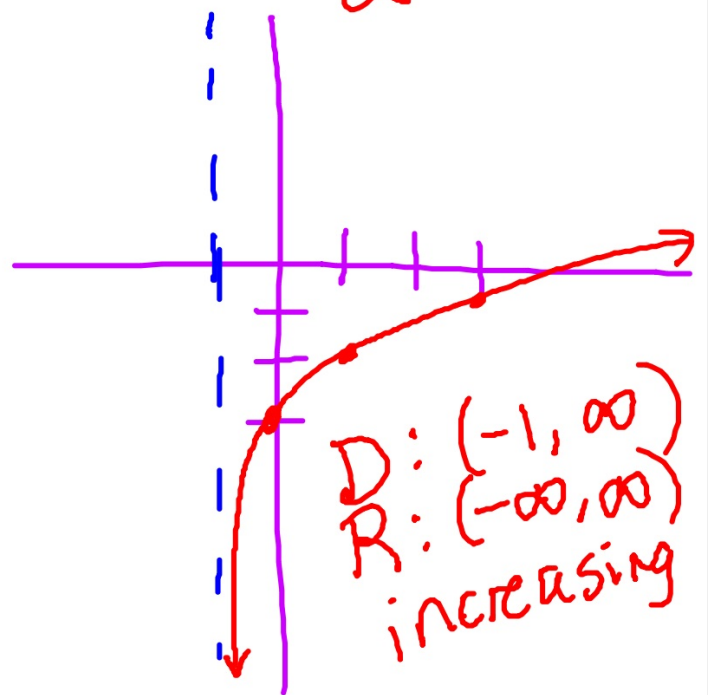
$$x = 2^{y+3} - 1$$

x	y
0	-3
1	-2
3	-1

vertical
asympt.
(VA)

$$\log_a b = y$$

$$a^y = b$$



ex: Sketch. Then state the domain and range and determine if the graph is increasing or decreasing.

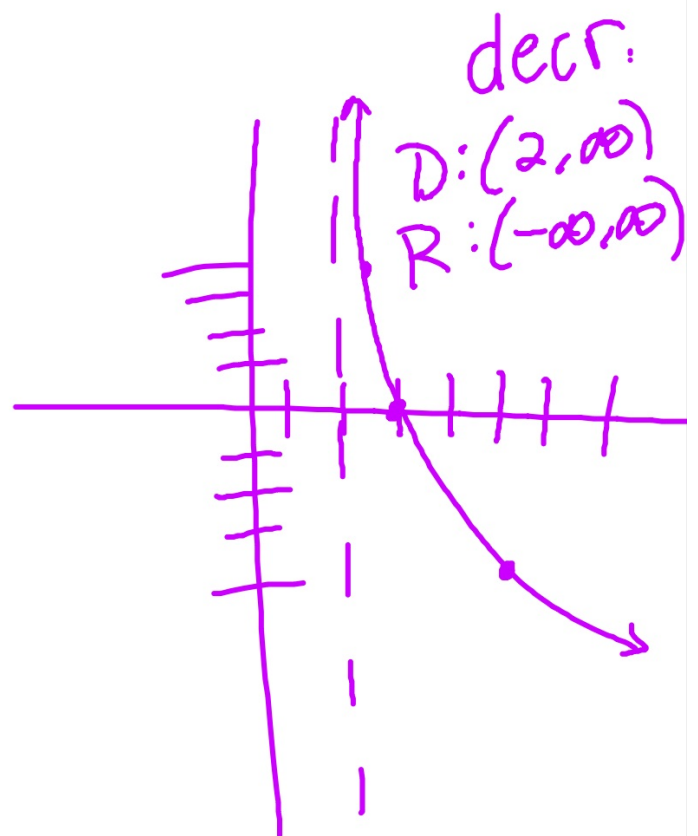
$$b) y = 4 \log_{\frac{1}{3}}(x-2)$$

$$\frac{y}{4} = \log_{\frac{1}{3}}(x-2)$$

$$\left(\frac{1}{3}\right)^{y/4} = x-2$$

$$\left(\frac{1}{3}\right)^{y/4} + 2 = x \quad \text{VA } x=2$$

x	y
3	0
5	-4
$2\frac{1}{3}$	4



ex: Sketch. Then state the domain and range and determine if the graph is increasing or decreasing.

c) $y = 3 - 2\log_5(x+6)$

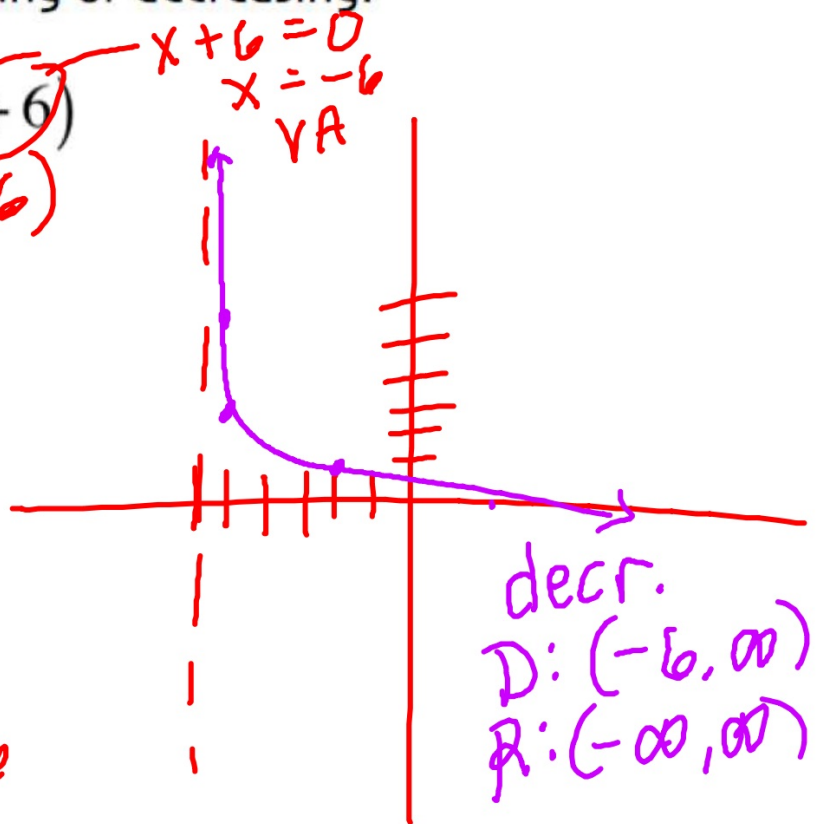
$$\frac{y-3}{-2} = \log_5(x+6)$$

$$5^{\frac{y-3}{-2}} = x+6$$

$$5^{\frac{y-3}{-2}} - 6 = x$$

x	y
-5	3
-6	5

VA $x = -6$



ex: Sketch. Then state the domain and range and determine if the graph is increasing or decreasing.

d) $y = 2 \log_3(2x + 1)$

ex: Sketch. Then state the domain and range and determine if the graph is increasing or decreasing.

e) $f(x) = -\log(-x)$

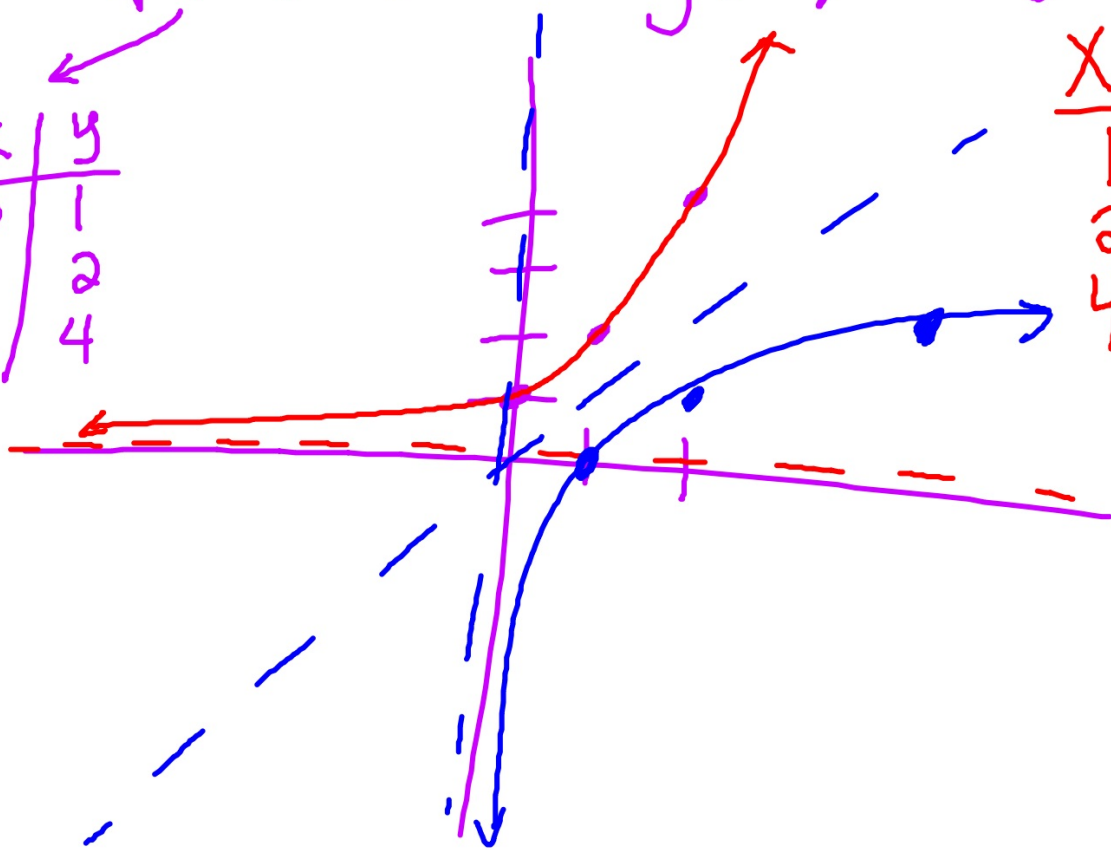
ex: Let $f(x) = -\log(-x)$. Sketch $f^{-1}(x)$. Then state the domain and range of $f^{-1}(x)$.

$$f(x) = 2^x$$

$$g(x) = \log_2 x$$

x	y
0	1
1	2
2	4

x	y
1	0
2	1
4	2



3 Types of Exponential Equations:

1. $a^x = b$, where a and b are integral powers of the same number

ex: $27^x = 9$

2. $a^x = b$, where a and b are NOT integral powers of the same number

ex: $3^x = 5$

3. quadratic form

ex: $3^{2x} + 3^x - 6 = 0$

Type 1

Property of Equality for Exponential Equations

$$\text{If } a^x = a^y, \text{ then } x = y.$$

To solve these equations, use the property of equality to make the bases equal.

ex: Solve.

a) $3^x = 9^{x+2}$

$$3^x = (3^2)^{(x+2)}$$

$$x = 2(x+2)$$

$$x = 2x + 4$$

$$-4 = x$$

check

$$3^{-4} = 9^{-2}$$

$$\frac{1}{81} = \frac{1}{81} \checkmark$$

ex: Solve.

$$b) 125^x = \left(\frac{1}{25}\right)^{x-1}$$

$$(X^2)^3 = X^6$$

$$5^{3x} = (5^{-2})^{x-1}$$

$$5^{3x} = 5^{-2(x-1)}$$

$$3x = -2x + 2$$

$$x = \frac{2}{5}$$

ex: Solve.

$$c) 2^x \cdot 8^{x-1} = \left(\frac{1}{16}\right)^{2x-5}$$

$$2^x \cdot 2^{3(x-1)} = 2^{-4(2x-5)}$$

$$2^{x+3x-3} = 2^{-8x+20}$$

$$4x-3 = -8x+20$$

$$12x = 23$$

$$x = \frac{23}{12}$$

ex: Solve.

$$d) 3 \cdot \underbrace{4^{x-2}} - 5 = 1$$

$$4^{x-2} = 2$$

$$2^{2(x-2)} = 2^1$$

$$2x - 4 = 1$$

$$x = \frac{5}{2}$$

ex: Solve.

$$e) 16^{x^2} = \left(\frac{1}{4}\right)^{10x-12}$$

$$2^{4x^2} = 2^{-2(10x-12)}$$

$$4x^2 = -20x + 24$$

$$4x^2 + 20x - 24 = 0$$

$$x^2 + 5x - 6 = 0$$

$$(x+6)(x-1) = 0$$

$$x = -6, 1$$

ex: Solve.

$$f) \frac{5^{x-2}}{25^{x+3}} = \left(\frac{1}{125}\right)^{2x+1}$$

$$\frac{5^{x-2}}{5^{2x+6}} = 5^{-3(2x+1)}$$

$$x-2-(2x+6) = -6x-3$$

$$-x-8 = -6x-3$$

$$5x = 5 \quad \text{X=1}$$

REVIEW

ex: Sketch. Then state the domain and range and classify as growth or decay.

$$f(x) = -3(2)^{x-5} + 1$$