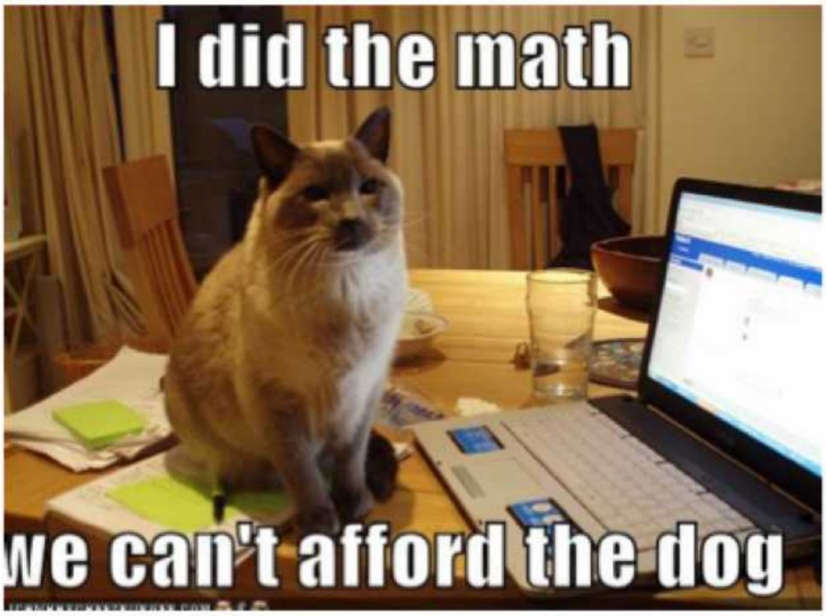
4.4 Evaluating Logarithms
4.5 Properties of Logarithms



ex: Evaluate.

a) 2^5 32

b) $81^{3/4}$ 27

c) $9^{-5/2}$ $\frac{1}{243}$

d) $-16^{5/4}$ -32

ex: Solve.

a) $2^x = 16$ $x = 4$

b) $3^x = \frac{1}{3}$ $x = -1$

c) $71^x = 1$ $x = 0$

d) $25^x = 5$ $x = \frac{1}{2}$

e) $27^x = 9$

$27^{2/3} = 9$ $x = 2/3$

Definition of a Logarithm

Let b and y be positive numbers with $b \neq 1$. The **logarithm of y with base b** is denoted by $\log_b y$ and is defined as follows:

$$\log_b y = x \quad \text{if and only if} \quad b^x = y$$

The expression $\log_b y$ is read as "log base b of y."

ex: Rewrite in exponential form.

a) $\log_3 9 = 2$ $\qquad 3^2 = 9$

exponent

b) $\log_{22} 1 = 0$ $\qquad 22^0 = 1$

ex: Rewrite in logarithmic form.

a) $3^5 = 243$ $\log_3 243 = 5$

b) $27^{-2/3} = \frac{1}{9}$ $\log_{27} \frac{1}{9} = -\frac{2}{3}$

ex: Evaluate.

a) $\log_4 64$ $= 3$

$4^{\square} = 64$

b) $\log_3 81$ $= 4$

c) $\log_5 25$ $= 2$

d) $\log_7\left(\frac{1}{7}\right)$ $= -1$

e) $\log_{13} 1$ $= 0$

ex: Evaluate.

f) $\log_{25} 5 = 1/2$

i) $\log_2(-4)$ undefined

g) $\log_5\left(\dfrac{1}{125}\right) = -3$

j) $\log_{25}\left(\dfrac{1}{5}\right) = -\dfrac{1}{2}$

$\log_5 5^{50} = 50$

h) $\log_{81} 27 = 3/4$

$81^{\square} = 27$

k) $\log_{\bigstar}\left(\bigstar^{100}\right) = 100$

$\bigstar > 0, \bigstar \neq 1$

Special Logarithms

SPECIAL LOGARITHMS A **common logarithm** is a logarithm with base 10. It is denoted by $\log_{10}$ or simply by log. A **natural logarithm** is a logarithm with base e. It can be denoted by $\log_e$, but is more often denoted by ln.

Common Logarithm	Natural Logarithm
$\log_{10} x = \log x$	$\log_e x = \ln x$

Most calculators have keys for evaluating common and natural logarithms.

base 10

$\log 10 = 1$; $\log_{10} 10$

$\ln e = 1$; $\log_e e = 1$

$\log_{10} 1 = 0$ $\qquad$ $\log 10$

$\log_{10} 10$

$e^{\boxed{0}} = 1$ $\qquad$ $10^{\boxed{0}} = 1$

$\log_7 1$

$\ln 1 = 0$

$\log_e 1$

ex: Evaluate.

a) $\log_{10} 100 = 2$

b) $\log\left(\frac{1}{10}\right) = \log_{10} 10^{-1} = -1$

c) $\log .001 = \log \frac{1}{1000} = -3$ $\qquad 10^{\square} = \frac{1}{1000}$

ex: Evaluate.

d) $\ln 1 = 0$

$7^{\square} = 1$

e) $\ln\left(\frac{1}{e}\right) = \ln e^{-1} = \log_e e^{-1} = -1$

f) $\ln e^2 = 2$

$\log_4 4 = 1$

$\log_8 8 = 1$

g) $\ln e = 1$

$\log_e e = 1$

 ex: Evaluate on your calculator.

a) $\log 16 = 1.204$

b) $\ln 7 = 1.946$

Logarithms and Exponentials are INVERSES!

$f(x) = \log_b x \qquad g(x) = b^x$

ex: Evaluate.

a) $(f \circ g)(x) = \log_b b^x = x$

b) $(g \circ f)(x) = b^{\log_b x} = x$

$2^{\log_2 8}$

2^3

8

ex: Evaluate.

a) $7^{\log_7 x}$ $= x$

b) $\log_{62} 62^x$ $= x$

c) $\log 10^x$ $= x$

ex: Evaluate.

d) $e^{\ln 7} = 7$

$\log_{64} 4 = \frac{1}{3}$

e) $\log_5 25^x = 2x$

$\log_5 5^{2x}$

f) $\log_{64} 4^y = \frac{1}{3} y$

$64^{\square} = 4^y$

REVIEW - Exponent Properties

$$\boxed{\begin{aligned} b^m \cdot b^n &= b^{m+n} \\ \frac{b^m}{b^n} &= b^{m-n} \end{aligned}}$$

Logarithm Properties

$(b^m)^n = b^{mn}$

Let b, m, and n be positive numbers such that $b \neq 1$.

Product Property $\log_b (mn) = \log_b m + \log_b n$

Quotient Property $\log_b \frac{m}{n} = \log_b m - \log_b n$

Power Property $\log_b m^n = n \log_b m$

Logarithm properties are used to EXPAND and CONDENSE logarithmic expressions.

ex: Expand. and simplify

a) $\log_3\left(\frac{abc}{9d}\right) = \log_3 a + \log_3 b + \log_3 c - \log_3 9 - \log_3 d$

$= \log_3 a + \log_3 b + \log_3 c - 2 - \log_3 d$

ex: Expand.

b) $\log_5\left(\frac{a^2b^3}{c^4}\right) = \log_5 a^2 + \log_5 b^3 - \log_5 c^4$

$= 2\log_5 a + 3\log_5 b - 4\log_5 c$

c) $\log\left(\frac{100a^2}{b^3c}\right) = \underbrace{\log 100}_{} + \log a^2 - \log b^3 - \log c$

$= 2 + 2\log a - 3\log b - \log c$

ex: Expand.

d) $\ln\left(\frac{1}{ab^2c^3}\right) = -\ln a - \ln b^2 - \ln c^3$

$= -\ln a - 2\ln b - 3\ln c$

e) $\log_3\left(a+b^2\right) = \log_3(a+b^2)$

ex: Expand.

f) $\log_4\left(\dfrac{a+b}{a^2-b^2}\right)$ $\quad \log_4 \dfrac{1}{(a-b)}$

$\log_4 \dfrac{\cancel{(a+b)}}{\cancel{(a+b)}(a-b)}$ $\quad -\log_4(a-b)$

g) $\log_2\left(a^2-b^2\right) = \log_2(a+b)(a-b)$

$\log_2(a+b)+\log_2(a-b)$

ex: Expand.

h) $\log_3(a-b)^7 = 7\log_3(a-b)$

i) $\ln\left(\sqrt{\dfrac{y^3+z}{x^3(a+1)^5}}\right)^{1/2} = \frac{1}{2}\ln\left(\dfrac{y^3+z}{x^3(a+1)^5}\right)$

$= \frac{1}{2}\ln(y^3+z) - \frac{3}{2}\ln x - \frac{5}{2}\ln(a+1)$

ex: Condense.

a) $2\log_5 a - 3\log_5 b + 4\log_5(c+d)$

$$\log_5 a^2 - \log_5 b^3 + \log_5 (c+d)^4$$

$$\log_5\left(\frac{a^2(c+d)^4}{b^3}\right)$$

ex: Condense.

b) $\frac{1}{2}\log x+\frac{3}{2}\log y-10\log z$

$$\log\left(\frac{x^{1/2}\,y^{3/2}}{z^{10}}\right)$$

ex: Condense.

c) $-3\log x - 4\log y - \frac{2}{3}\log z = -\log x^3 - \log y^4 - \log z^{2/3}$

$$\log\left(\frac{1}{x^3 y^4 z^{2/3}}\right)$$