3. $2 - \log_2(x+1) = 4$

$$\log_2(x+1) = -2$$

$$2^{\log_2(x+1)} = 2^{-2}$$

$$x+1 = \frac{1}{4}$$

$$x = -\frac{3}{4}$$

7) $\log_4(2x+2) - \log_4(x-2) = 1$

$$4^{\log_4 \frac{2x+2}{x-2}} = 4^1$$

$$\frac{2x+2}{x-2} = \frac{4}{1}$$

$$x = 5$$

17.) $\log_3 3^{x+5} = \log_3 7$

$x+5 = \log_3 7$

$x = (\log_3 7) - 5$

-3.229

21.)

$$3\sqrt{27} = 3^{4x}$$

$$\sqrt{3^3}$$

$$3^{3/2}$$

$$3^1 \cdot 3^{3/2} = 3^{4x}$$

$$1 + \frac{3}{2} = 4x$$

$$\frac{5}{2} = 4x$$

$$\frac{5}{8} = x$$

27.) $8^{x} \cdot 16^{x-1} = 4(32)^{2x+3}$

$$2^{3x} \cdot 2^{4(x-1)} = 2^{2} \cdot 2^{5(2x+3)}$$

$$3x + 4(x-1) = 2 + 5(2x+3)$$

$$x = -7$$

31.) $2^{\log_2 x^3} = 2^{\log_2 x}$

$$x^3 = x$$

$$x^3 - x = 0$$

$$x(x+1)(x-1) = 0$$

$$x = \cancel{0}, \cancel{-1}, 1$$

23.) $\frac{3000}{2+e^{2x}} = \frac{2}{1}$

$3000 = 4 + 2e^{2x}$

$\frac{2996}{2} = e^{2x}$

$\ln(1498) = \ln(e^{2x})$

$\ln(1498) = 2x$

$\frac{\ln 1498}{2} = x$

37.) $2x\ln\left(\frac{1}{x}\right) - x = 0$

$$x\left(2\ln\frac{1}{x} - 1\right) = 0$$

$\cancel{x = 0}$

$$2\ln\frac{1}{x} - 1 = 0$$

$$e^{\ln\frac{1}{x}} = e^{\frac{1}{2}}$$

$$\frac{1}{x} = \frac{e^{1/2}}{1}$$

$$x = e^{-1/2} = \frac{1}{e^{1/2}} = \frac{1}{\sqrt{e}}$$

35.) $\frac{e^x + 7e^{-x}}{-2} = -4$

$$e^x + 7e^{-x} = 8$$

$$\frac{e^x}{e^x} \cdot \frac{e^x}{1} + \frac{7}{e^x} = 8$$

$$\frac{e^{2x} + 7}{e^x} = \frac{8}{1}$$

$$e^{2x} + 7 = 8e^x$$

$$e^{2x} - 8e^x + 7 = 0$$

$$(e^x - 7)(e^x - 1) = 0$$

$e^x = 7$ $\quad$ $e^x = 1$

$x = \ln 7$ $\quad$ $x = 0$

33) $\frac{1}{4}\ln(x-1) = 1$

$$e^{\ln(x-1)} = e^{4}$$

$$x-1 = e^4$$

$$x = e^4 + 1$$

4.1, 4.2 - Graphs of Exponential Functions
4.4 - Graphs of Logarithmic Functions

ex: Find the inverse.

a) $f(x) = 2 \cdot 3^{x-1} + 1$

$x = 2 \cdot 3^{y-1} + 1$

$\log_3 \frac{x-1}{2} = \log_3 3^{y-1}$

$\log_3 \frac{x-1}{2} = y - 1$

$f^{-1}(x) = \log_3\left(\frac{x-1}{2}\right) + 1$

OR

$f^{-1}(x) = 1 + \log_3\left(\frac{x-1}{2}\right)$

ex: Find the inverse.

b) $g(x) = 2\log_4(x+5) - 4$

$x = 2\log_4(y+5) - 4$

$4^{\frac{x+4}{2}} = 4^{\log_4(y+5)}$

$4^{\frac{x+4}{2}} = y+5$

$4^{\frac{x+4}{2}} - 5 = g^{-1}(x)$

OR

$4^{\frac{1}{2}x+2} - 5 = g^{-1}(x)$

OR

$g^{-1}(x) = 2^{x+4} - 5 = g^{-1}(x)$

Exponential Functions

$$f(x) = ab^x$$

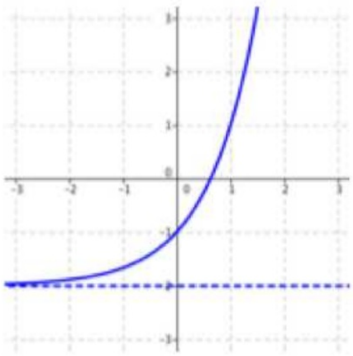
$$a \neq 0, \quad b > 0, \quad b \neq 1$$

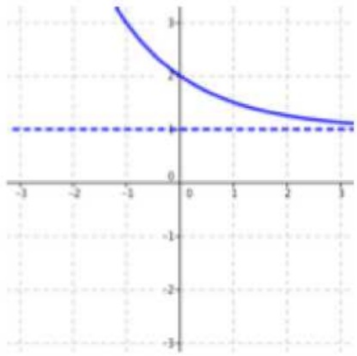
b is called the growth or decay factor

b: base

Graphs of Exponential Functions

$$f(x) = ab^x$$





<u>Exponential Growth</u>

$$b > 1$$

*the RIGHT side of the graph moves AWAY from the asymptote

<u>Exponential Decay</u>

$$0 < b < 1$$

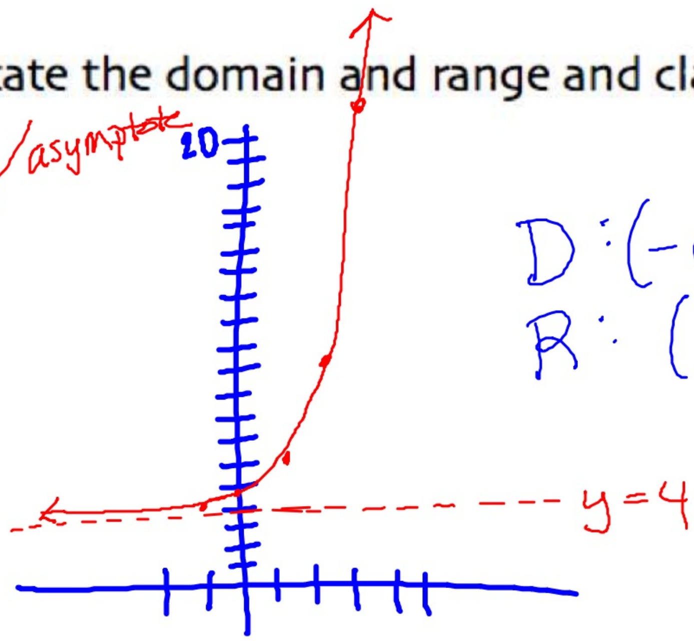
*the RIGHT side of the graph moves TOWARDS the asymptote

ex: Sketch. Then state the domain and range and classify as growth or decay.

b>1

a) $y = 2 \cdot 3^{x-1} + 4$

x	y
-1	$4\,2/9 = 4.2$
0	$4\,2/3 = 4.67$
1	6
2	10
3	22



D: $(-\infty, \infty)$

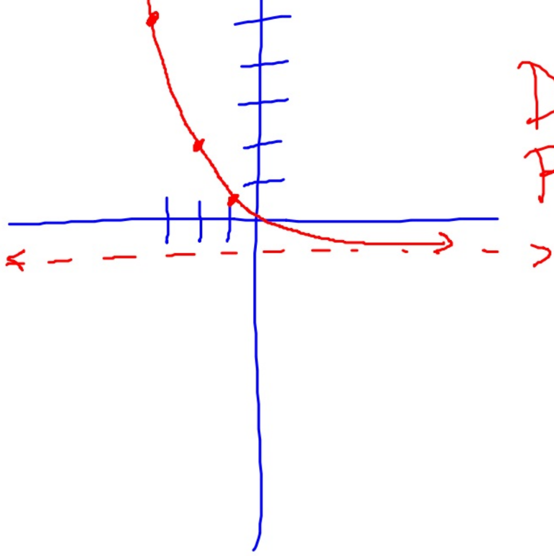
R: $(4, \infty)$

ex: Sketch. Then state the domain and range and classify as growth or decay.

$0 < b < 1$

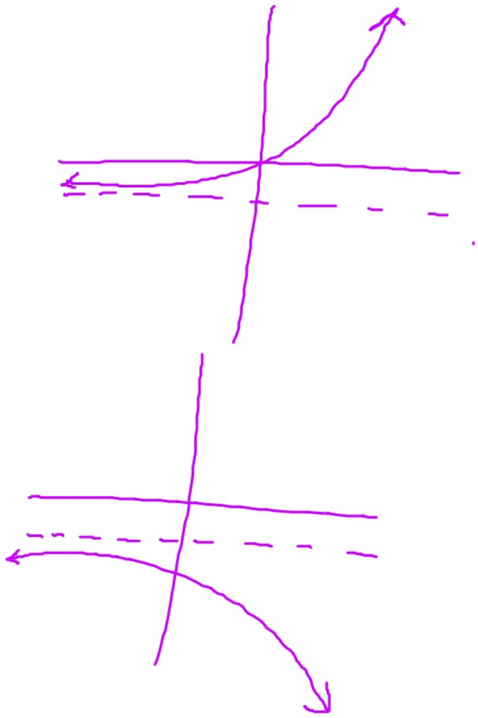
b) $y = 3\left(\frac{1}{2}\right)^{x+2} - 1$

x	y
-1	1/2
-2	2
-3	5

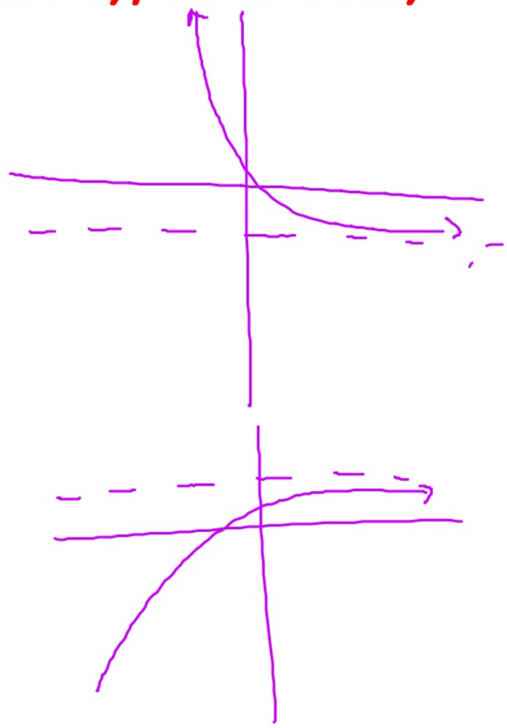


D: $(-\infty, \infty)$
R: $(-1, \infty)$

Two types of growth



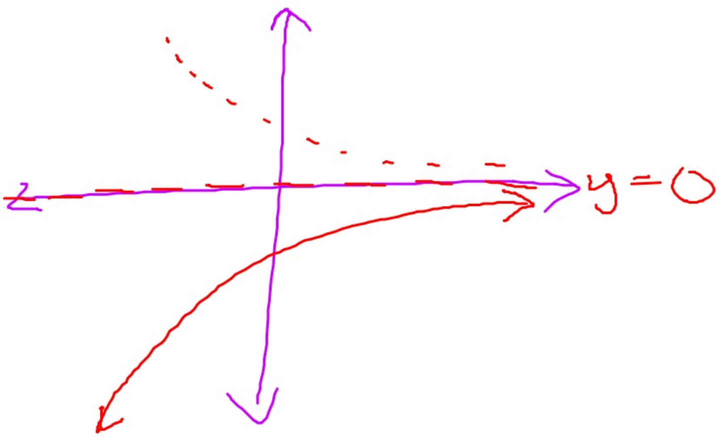
Two types of decay



ex: Sketch. Then state the domain and range and classify as growth or decay.

Decay function;
$0 < b < 1$

c) $y = -\left(\frac{2}{3}\right)^x$



ex: Sketch. Then state the domain and range and classify as growth or decay.

e) $y = -e^{x+3} + 1$

ex: WITHOUT graphing determine if the function represents growth or decay, then state the growth or decay factor.

a) $y=\frac{1}{2}\cdot 3^{x-4}+5$

$b = 3$ (growth factor)

growth

ex: WITHOUT graphing determine if the function represents growth or decay, then state the growth or decay factor.

b) $y = -\left(\frac{4}{5}\right)^{x+1}$

$b = \frac{4}{5}$ (decay factor)

decay

$y = 2^x$

growth

$y = 2^{-x}$

$y = \left(\frac{1}{2}\right)^x$

decay

ex: WITHOUT graphing determine if the function represents growth or decay, then state the growth or decay factor.

c) $y = 5^{4-x} + 2$ $\longrightarrow$ $y = 5^{-x+4} + 2$

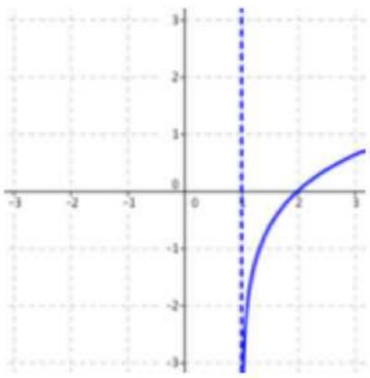
decay:

decay factor: $\frac{1}{5}$

Graphs of Logarithmic Functions

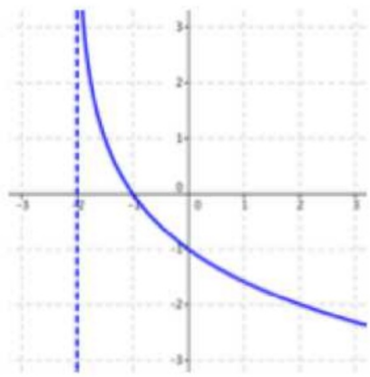
$$f(x) = \log_b x$$

$b > 0, b \neq 1$



<u>INCREASING</u>

$b > 1$



<u>DECREASING</u>

$0 < b < 1$

ex: Sketch. Then state the domain and range and determind if the graph is increasing or decreasing.

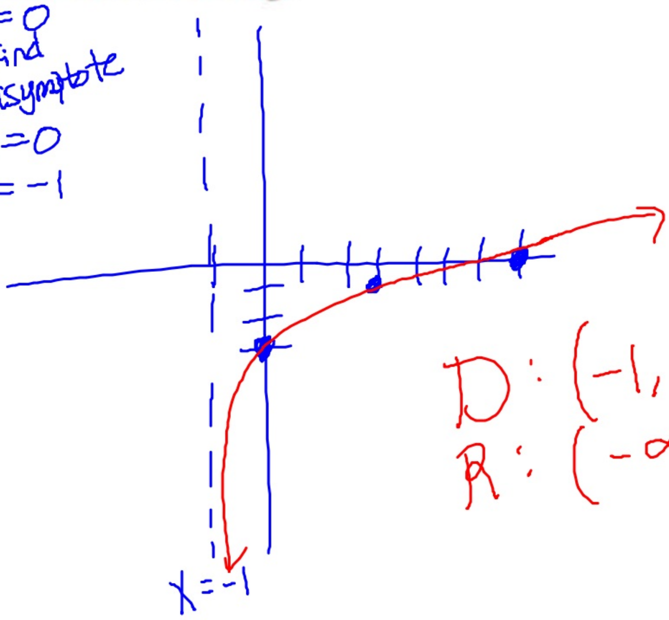
a) $y = \log_2(x+1) - 3$

Set = 0 (find asymptote

$x+1=0$

$x=-1$

x	y
0	-3
3	-1
7	0



D: $(-1, \infty)$

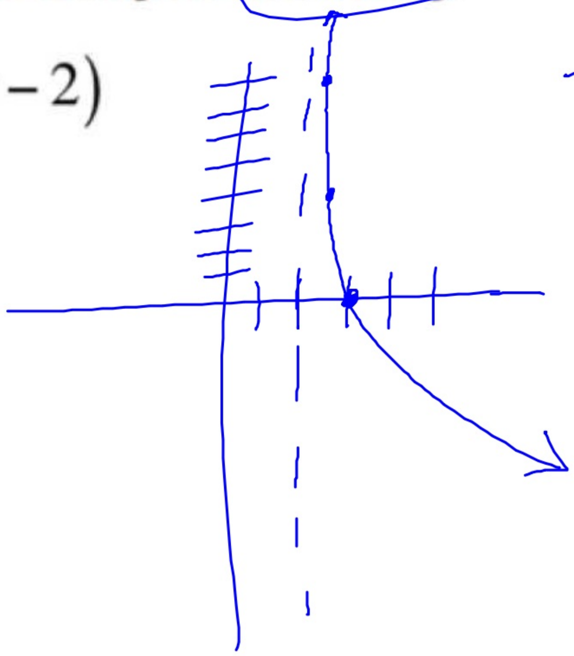
R: $(-\infty, \infty)$

ex: Sketch. Then state the domain and range and determind if the graph is increasing or decreasing.

b) $y = 4\log_{\frac{1}{3}}(x-2)$

$\left(\frac{1}{3}\right)^{y/4} + 2 = x$

x	y
3	0
$2\frac{1}{3}$	4
$2\frac{1}{9}$	8



D: $(2, \infty)$

R: $(-\infty, \infty)$

ex: Sketch. Then state the domain and range and determind if the graph is increasing or decreasing.

$$\log_a b = \frac{\log b}{\log a}$$

$$y = \log_3(x-2)+1$$

asymptote
$x = 2$

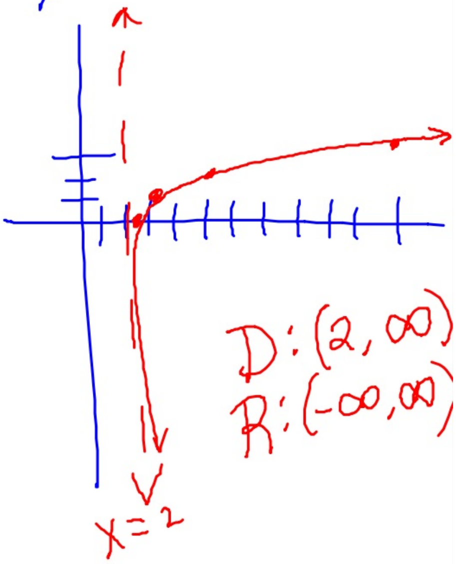
solve for x

$$y-1 = \log_3(x-2)$$

$$3^{y-1} + 2 = x$$

x	y
3	1
11	3
5	2

x	y
$2\frac{1}{3}$	0
3	1
5	2



D: $(2, \infty)$
R: $(-\infty, \infty)$