3.6 Radical Equations

Square Root Equations - 1 Root

ex: Solve. (REAL SOLUTIONS ONLY)

a) $\sqrt{x-5}+2=7$

1.) Isolate radical
2.) take a power of both sides and solve.
3.) Check!

$(\sqrt{x-5})^2=(5)^2$

$x-5=25$

$x=30$

$\sqrt{30-5}+2=7$

$5+2=7$ ✓

ex: Solve.

b) $3\sqrt{x+2}-4=-10$

$\sqrt{x+2} = -2$

$\left(\sqrt{x+2}\right)^2 = (-2)^2$

$x+2=4$

$x=2$

$3\sqrt{2+2}-4=-10$

$2 \neq -10$

NO solution

ex: Solve.

c) $1-2\sqrt{x^2-5x+15}=-5$

$$\left(\sqrt{x^2-5x+15}\right)^2 = 3^2$$

$$x^2-5x+15=9$$

$$x^2-5x+6=0$$

$$(x-2)(x-3)=0$$

$$x=2,3$$

Check

$x=2$ ✓

$\sqrt{x^2-5x+15}=3$

$\sqrt{9}=3$

$x=3$

$\sqrt{9-15+15}=3$

$\sqrt{9}$ ✓

Square Root Equations - More Than 1 Root

ex: Solve.

d) $\sqrt{5x+6} = 3\sqrt{x-1}$

$(\sqrt{5x+6})^2 = (3\sqrt{x-1})^2$

$5x+6 = 9(x-1)$

$\frac{15}{4} = x$

$\sqrt{5\cdot\frac{15}{4}+6} = 3\sqrt{\frac{15}{4}-1}$

$\sqrt{\frac{75}{4}+6} = 3\sqrt{\frac{11}{4}}$

$\sqrt{\frac{75+24}{4}}$ // $\sqrt{}$

$\sqrt{\frac{99}{4}}$

$3\sqrt{\frac{11}{4}}$

ex: Solve.

e) $\left(\sqrt{x-3}\right)^2 = \left(\sqrt{x+4}-1\right)^2$

$x-3 = (\sqrt{x+4}-1)(\sqrt{x+4}-1)$

$x-3 = x+4-2\sqrt{x+4}+1$

-5

$-8 = -2\sqrt{x+4}$

$4 = \sqrt{x+4}^2$

$16 = x+4$

$12 = x$

Check

$\sqrt{12-3} = \sqrt{12+4}-1$

✓

ex: Solve.

f) $\sqrt{x-10}+\sqrt{x}=1$

$\left(\sqrt{x-10}\right)^2=\left(1-\sqrt{x}\right)^2$

$x-10=(1-\sqrt{x})(1-\sqrt{x})$

$x-10=1-2\sqrt{x}+x$

$\frac{11}{2}=\sqrt{x}$

$\frac{121}{4}=x$

No solution

extraneous solution

check

$\sqrt{\frac{121}{4}-10}+\sqrt{\frac{121}{4}}=1$

$(+)\quad +\frac{11}{2}\neq 1$

ex: Solve.

g) $\sqrt{2x+9} - \sqrt{x+1} = \sqrt{x+4}$ $\sqrt{3x+1} - \sqrt{x+4} = 1$

$$\left(\sqrt{3x+1}\right)^2 = \left(1 + \sqrt{x+4}\right)^2$$

$$3x+1 = (1+\sqrt{x+4})(1+\sqrt{x+4})$$

$$3x+1 = 1 + 2\sqrt{x+4} + x + 4$$

$$\frac{2x-4}{2} = \sqrt{x+4}$$

$$(x-2)^2 = \sqrt{x+4}^2$$

$$x^2 - 4x + 4 = x + 4$$

$$x^2 - 5x = 0$$

$$x(x-5) = 0$$

$$x = \cancel{0}, 5$$

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Nth Root Equations

ex: Solve.

h) $\sqrt[3]{2x+1}-4=1$

$\left(\sqrt[3]{2x+1}\right)^3 = 5^3$

$2x+1=125$

$\boxed{x=62}$

ex: Solve.

i) $\left(\sqrt[4]{5x^2-4}\right)^4 = x^4$

$5x^2-4 = x^4$

$0 = x^4-5x^2+4$

$0 = (x^2-4)(x^2-1)$

$\pm 2 \quad \pm 1$

1, 2

ex: Solve.

j) $\sqrt{x} = \sqrt[4]{x}$

$$\left(x^{1/2}\right)^4 = \left(x^{1/4}\right)^4$$

$$x^2 = x$$

$$x^2 - x = 0$$

$$x(x-1) = 0$$

$$0, 1$$

ex: Solve.

k) $3\sqrt{2x^3-4x^2-4x+24}-7=3x-1$

ex: Solve.

l) $\sqrt{x^2} = \sqrt{4}$

$|x| = 2$

$x = \pm 2$

m) $\sqrt[3]{x^3} = \sqrt[3]{-8}$

$x = -2$

Rational Exponent Equations

ex: Solve.

n) $(x^{3/4})^{4/3} = (8)^{4/3}$ — odd index; reciprocal power

$x^1 = 8^{4/3} = (\sqrt[3]{8})^4$

$x = 16$

o) $(x^{2/3})^{3/2} = (9)^{3/2}$ → even index

$|x| = 9^{3/2}$

$x = \pm 27$

ex: Solve.

p) $3x^{3/2} - 2 = 373$

$(x^{3/2})^{2/3} = (125)^{2/3}$ ← odd (not ±)

$x = 25$

q) $4x^{2/3} + 5 = 41$

$(x^{2/3})^{3/2} = (9)^{3/2}$ ← even (need ±)

$|x| = 27$

$x = \pm 27$

ex: Solve.

a) $2(x+4)^{2/3}+1=19$

$((x+4)^{2\cdot\frac{1}{3}})^{3/2}=(9)^{3/2}$ ← even ± !

$|x+4|=27$

$x+4=27$ → 23

$x+4=-27$ → -31

ex: Solve.

~~4)~~ $5-3(2x-1)^{2/3}=32$

5) $\left((2x-1)^{2/3}\right)^{3/2}=(-9)^{3/2}$

2 → even root!

$\emptyset$

ex: Solve.

b) $\frac{1}{2}(x+3)^{5/2} - 4 = 12$

$\left((x+3)^{5/2}\right)^{2/5} = (32)^{2/5}$ ← odd no ±

$x+3 = 4$

$x = 1$

ex: Solve.

$3\left(x^2-5x-5\right)^{7/6}-4=-1$

$\left(\left(x^2-5x-5\right)^{7/6}\right)^{6/7}=(1)^{6/7}$

$x^2-5x-5=1$

$x^2-5x-6=0$

$(x-6)(x+1)=0$

$x=6,-1$

ex: Solve.

t) $\left(x^{3/2}\right)^2 = (x)^2$

$x^3 = x^2$

$x^3 - x^2 = 0$

$x^2(x-1) = 0$

$x = 0, 1$

REVIEW

ex: Sketch and state the domain and range in set notation.

$$y = -2\sqrt{5 - x}$$

REVIEW

ex: Sketch and state the domain and range in set notation.

$$y = 3\sqrt[3]{4x+1} - 2$$

REVIEW

ex: Solve.

$$\sqrt{5x+6}+3=\sqrt{3x+3}+4$$