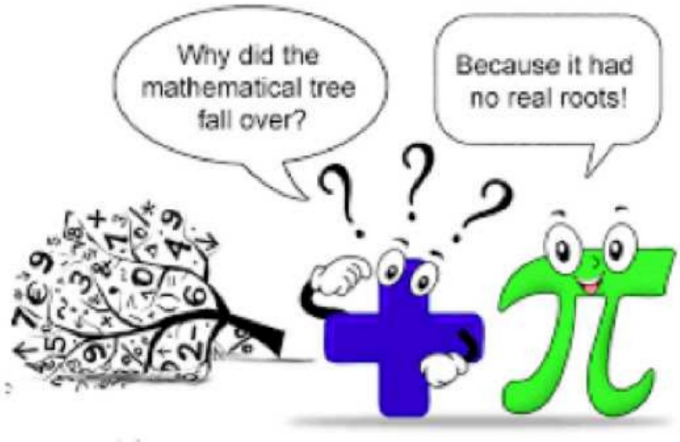
3.4 Inverse Functions



*See printout.

f and g are inverse functions if

$$(f \circ g)(x) = x$$

AND

$$(g \circ f)(x) = x$$

Verifying Inverse Functions

1. Algebraically

Show: $(f \circ g)(x) = x$ AND $(g \circ f)(x) = x$

ex 1: Show $f(x) = 4x + 9$ and $g(x) = \frac{x-9}{4}$

are inverses, algebraically.

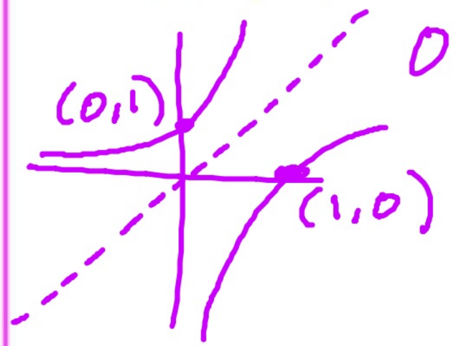
$(f \circ g)(x) = 4\left(\frac{x-9}{4}\right) + 9 = x - 9 + 9 = x$

$(g \circ f)(x) = \frac{4x + 9 - 9}{4} = \frac{4x}{4} = x$

Verifying Inverse Functions

2. Graphically

Show: f (x) and g(x) reflect over y = x



Verifying Inverse Functions

3. Numerically (NOT A PROOF)

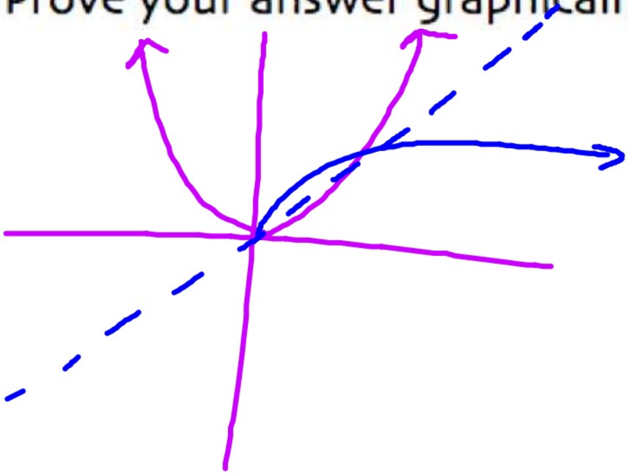
Show: for $f(x)$ (a,b) $g(x)$ (b,a)

for all coordinates

ex 2: If f and g are inverse functions and f contains the point (2, -3) then g must contain the point (-3, 2).

ex 3: Are $f(x) = x^2$ and $g(x) = \sqrt{x}$ inverses?

a) Prove your answer graphically.



No

ex 3: Are $f(x) = x^2$ and $g(x) = \sqrt{x}$ inverses?

b) Prove your answer algebraically.

$(f \circ g)(x) = (\sqrt{x})^2 = x$

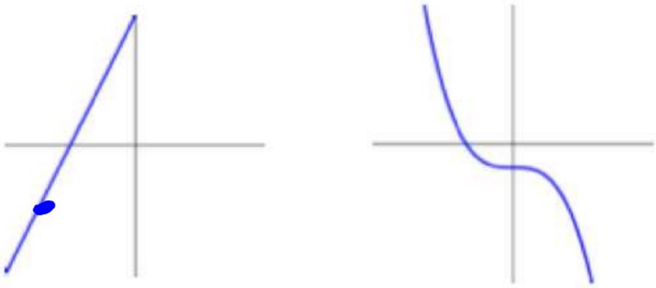
$(g \circ f)(x) = \sqrt{x^2} = |x|$

No.

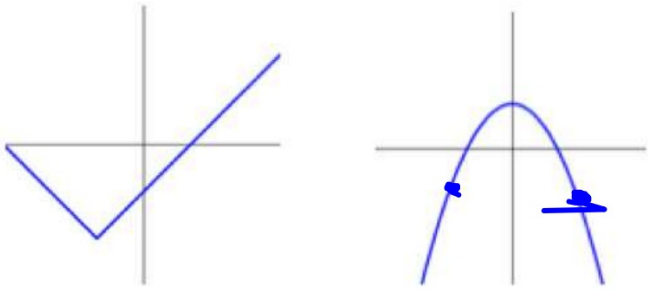
$(f \circ g)(x) \neq (g \circ f)(x) \therefore$ not inverses

The Existence of an Inverse

Examples of functions that DO have inverses:



Examples of functions that DO NOT have inverse functions.



The Existence of an Inverse

A function has an inverse function if it passes BOTH the vertical line test (VLT) and horizontal line test(HLT).

always increasing or
always decreasing

One-To-One

A function is one-to-one if it passes BOTH the vertical line test (VLT) and horizontal line test(HLT).

Inverse Notation

$$\boxed{f^{-1}(x)}$$

$x^{-1} = \frac{1}{x}$

NOTE: $f^{-1}(x) \neq [f(x)]^{-1}$ or $\frac{1}{f(x)}$

ex 4: Determine whether each function has an inverse function.

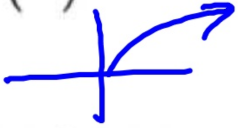
a) $f(x) = x+1$ Yes

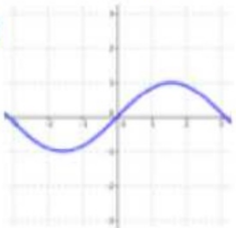
b) $f(x) = x^4 - x^2 + 7$ no

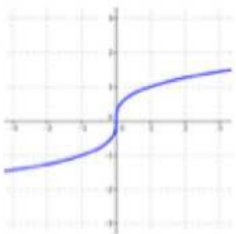
c) $f(x) = -3x^2 + 4x + 5$ no

ex 5: Determine whether each function has an inverse function.

a) $f(x) = \sqrt{x}$ Yes

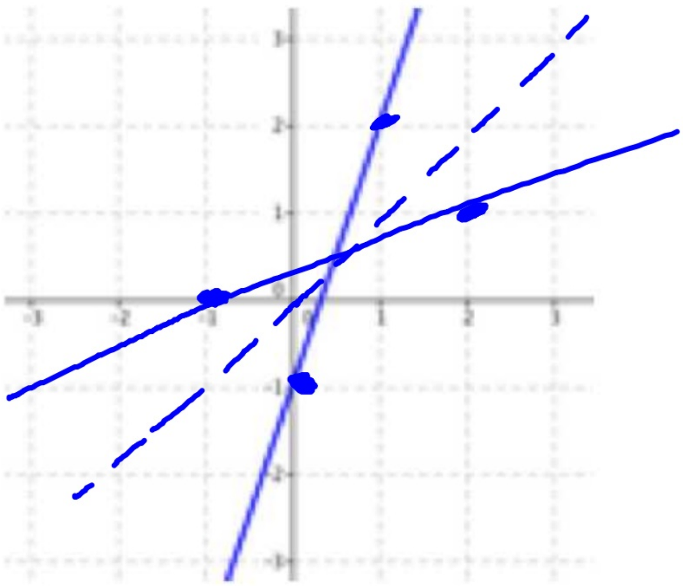


b)  no

c)  Yes

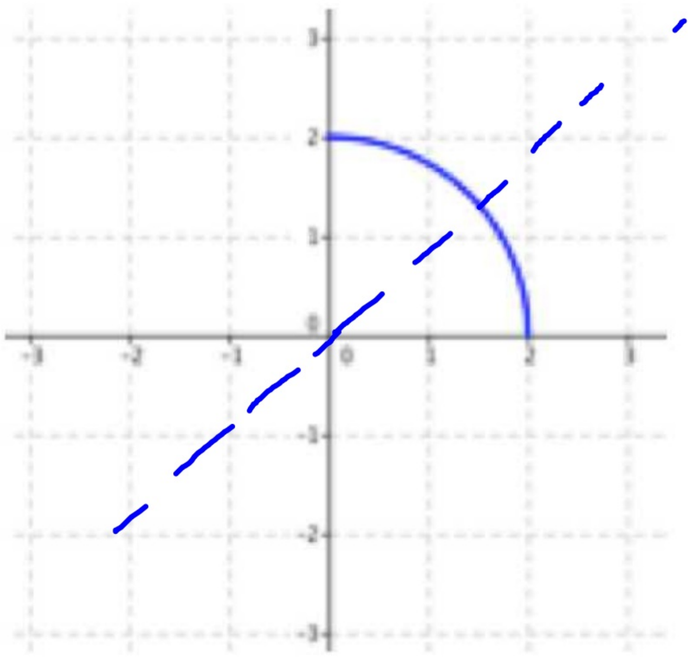
ex 6: Sketch the inverse function, if it exists.

a)



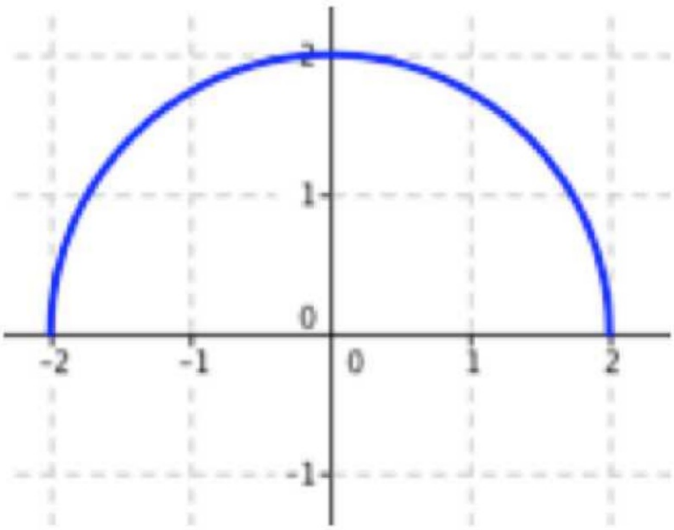
ex 6: Sketch the inverse function, if it exists.

b)



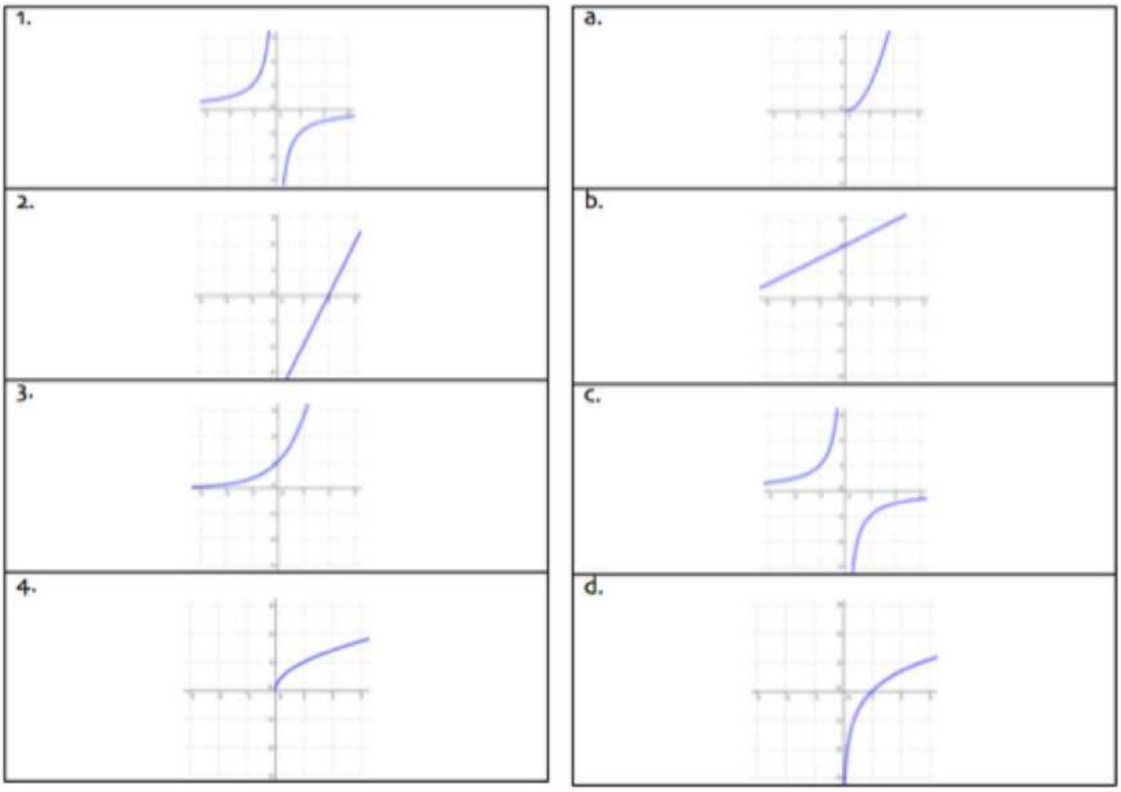
ex 6: Sketch the inverse function, if it exists.

c)

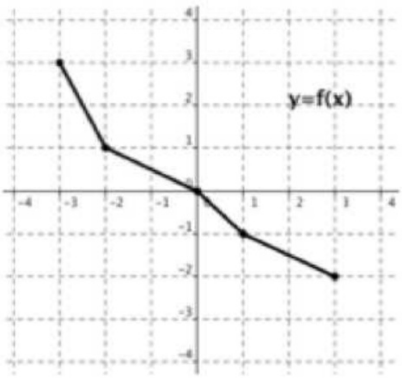


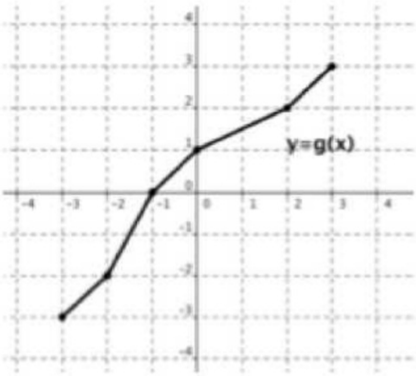
does not exist

ex 7: Match each graph with the graph of its inverse.



ex 8: Find the indicated values, if possible.





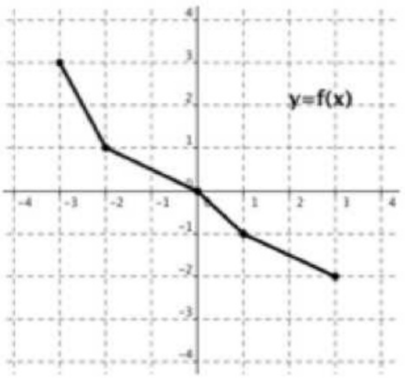
a) $f^{-1}(3) = -3$

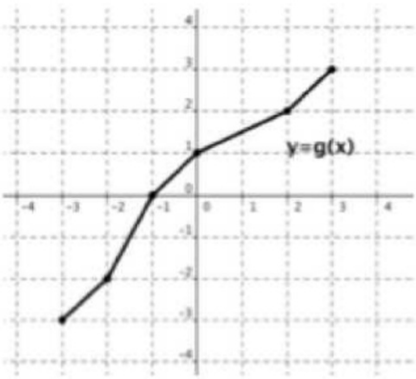
b) $g^{-1}(0) = -1$

c) $g^{-1}(1) = 0$

d) $f^{-1}(0) = 0$

ex 8: Find the indicated values, if possible.





e) $(f \circ g)(-1) = 0$

0

f) $(g \circ f)(-2) = 1.5$

1

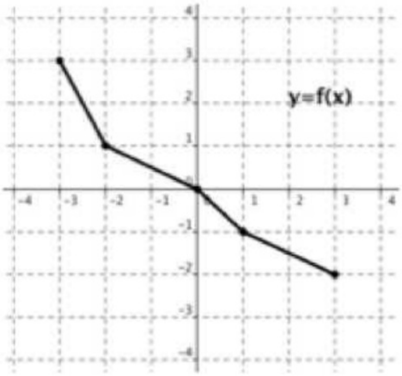
g) $(f \circ g^{-1})(1) = 0$

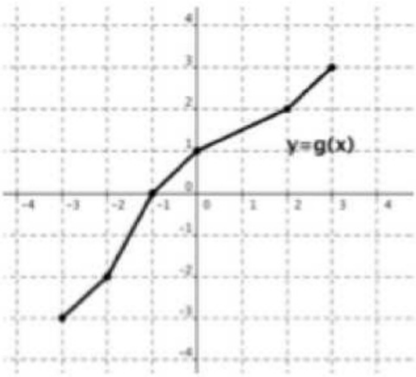
0

h) $(g \circ f^{-1})(-1) = 1.5$

1

ex 8: Find the indicated values, if possible.





i) $\left(f \circ f^{-1}\right)(2)$ = 2

j) $\left(g^{-1} \circ f^{-1}\right)(2)$ = −2.5

−2.5

Finding Inverses

ex 9: Find the inverse, if possible.

1) switch the x and y's
2) Solve for y
3) Set inverse equal to f^{-1}

a) $f(x) = \dfrac{5-3x}{2}$

$$\frac{x}{1} = \frac{5-3y}{2}$$

$$2x = 5-3y$$

$$2x-5 = -3y$$

$$\frac{2x-5}{-3} = y$$

$$\frac{2x-5}{-3} = f^{-1}(x)$$

ex 9: Find the inverse, if possible.

b) $g(x) = \sqrt[3]{x+8}$

$x^3 = \left(\sqrt[3]{y+8}\right)^3$

$x^3 = y+8$

$x^3 - 8 = y$

$x^3 - 8 = g^{-1}(x)$

ex 9: Find the inverse, if possible.

c) $y = \dfrac{2}{x+1}$

$\dfrac{x}{1} = \dfrac{2}{y+1}$

$xy + x = 2$

$xy = 2 - x$

$y = \dfrac{2-x}{x}$

$y^{-1} = \dfrac{2-x}{x}$

$y^{-1} = \dfrac{2}{x} - 1$

$$y = \frac{x+3}{x-1}$$

$$y^{-1} = \frac{x+3}{x-1}$$

$$\frac{x}{1} = \frac{y+3}{y-1}$$

$$xy - x = y + 3$$

$$xy - y = x + 3$$

$$\frac{y\cancel{(x-1)}}{\cancel{x-1}} = \frac{x+3}{x-1}$$

ex 9: Find the inverse, if possible.

d) $h(x) = x^2 - 5x + 1$

$x = y^2 - 5y + 1$

ex 10: Given

$$f(x) = x + 4 \qquad g(x) = \sqrt{x} - 5 \qquad h(x) = x^2 + x,\ x \geq -\frac{1}{2}$$

Find the indicated value.

a) $f^{-1}(0) = -4$

$0 = x + 4$

$-4 = x$

ex 10: Given

$$f(x) = x + 4 \qquad g(x) = \sqrt{x} - 5 \qquad h(x) = x^2 + x,\ x \geq -\frac{1}{2}$$

Find the indicated value.

b) $g^{-1}(2)$

$2 = \sqrt{x} - 5$

$7^2 = \sqrt{x}^2$

$49 = x$

49

ex 10: Given

$$f(x) = x + 4 \qquad g(x) = \sqrt{x} - 5 \qquad h(x) = x^2 + x,\ x \geq -\frac{1}{2}$$

Find the indicated value.

c) $h^{-1}(2) = 1$

$2 = x^2 + x$

$0 = x^2 + x - 2$

$0 = (x+2)(x-1)$

$\cancel{-2}, 1$

ex 10: Given

$$f(x) = x + 4 \qquad g(x) = \sqrt{x} - 5 \qquad h(x) = x^2 + x,\ x \geq -\frac{1}{2}$$

Find the indicated value.

d) $\left(g^{-1} \circ f^{-1}\right)(-1) = 0$

$-1 = x + 4$

$-5 = x$

$-5 = \sqrt{x} - 5$

$0 = \sqrt{x}$

$0 = x$

ex 10: Given

$$f(x) = x + 4 \qquad g(x) = \sqrt{x} - 5 \qquad h(x) = x^2 + x,\ x \geq -\frac{1}{2}$$

Find the indicated value.

e) $(h \circ g^{-1})(0)$

$0 = \sqrt{x} - 5$

$25 = x$

$h(25) = 25^2 + 25 = 650$

ex 10: Given

$$f(x) = x + 4 \qquad g(x) = \sqrt{x} - 5 \qquad h(x) = x^2 + x,\ x \geq -\frac{1}{2}$$

Find the indicated value.

f) $\left(f \circ f^{-1}\right)(6) = 6$