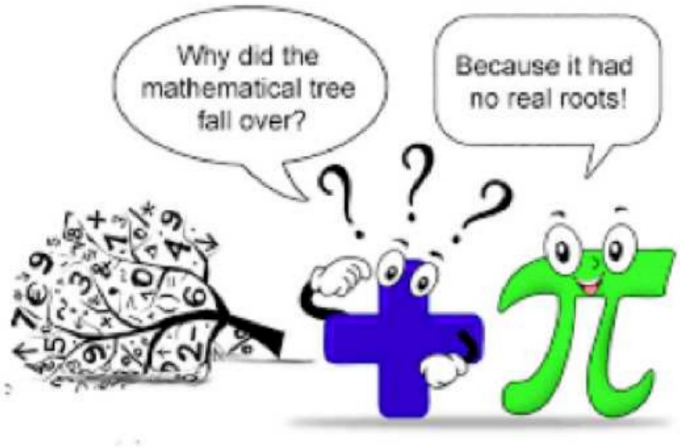
3.4 Inverse Functions



*See printout.

f and g are inverse functions if

$$(f \circ g)(x) = \quad X$$

AND

$$(g \circ f)(x) = \quad X$$

Verifying Inverse Functions

1. Algebraically

Show: $(f \circ g)(x) = x$ AND $(g \circ f)(x) = x$

ex 1: Show $f(x) = 4x + 9$ and $g(x) = \dfrac{x-9}{4}$

are inverses, algebraically.

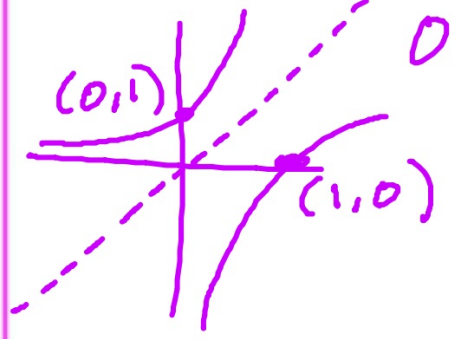
$(f \circ g)(x) = \cancel{4}\left(\dfrac{x-9}{\cancel{4}}\right) + 9 = x - 9 + 9 = x$

$(g \circ f)(x) = \dfrac{4x + 9 - 9}{4} = \dfrac{4x}{4} = x$

Verifying Inverse Functions

2. Graphically

Show: $f(x)$ and $g(x)$ reflect over $y = x$



Verifying Inverse Functions

3. Numerically (NOT A PROOF)

Show: for $f(x)$ $g(x)$

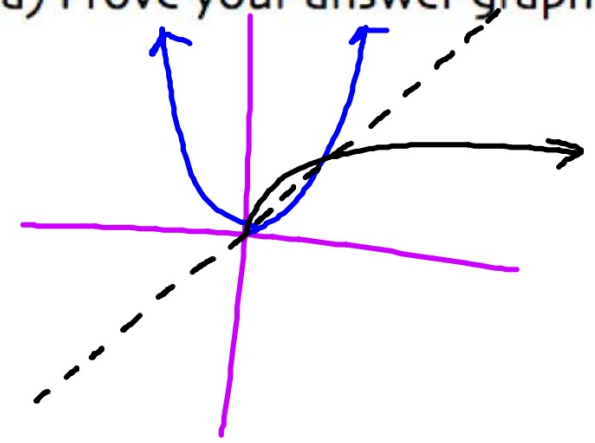
(a,b) (b,a)

for all coordinates

ex 2: If f and g are inverse functions and f contains the point (2, -3) then g must contain the point (-3, 2).

ex 3: Are $f(x) = x^2$ and $g(x) = \sqrt{x}$ inverses?

a) Prove your answer graphically.



No

ex 3: Are $f(x) = x^2$ and $g(x) = \sqrt{x}$ inverses?

b) Prove your answer algebraically.

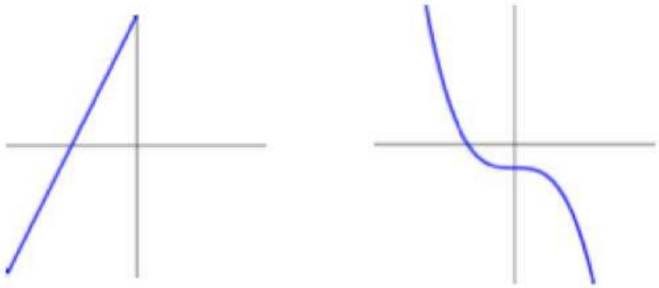
$(f \circ g)(x) = (\sqrt{x})^2 = x$

$(g \circ f)(x) = \sqrt{x^2} = |x|$

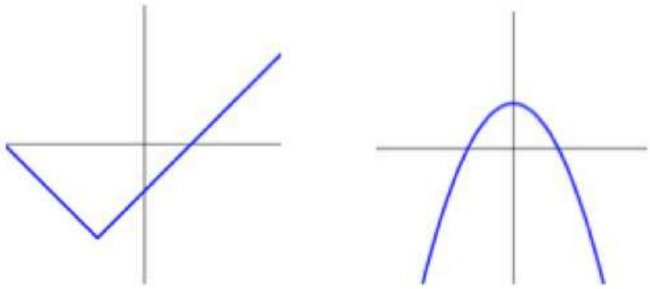
No.

The Existence of an Inverse

Examples of functions that DO have inverses:



Examples of functions that DO NOT have inverse functions.



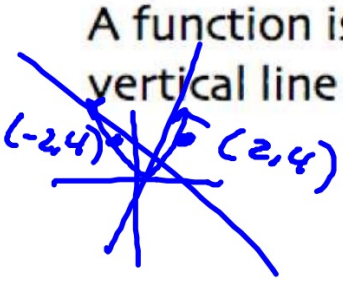
The Existence of an Inverse

A function has an inverse function if it passes BOTH the vertical line test (VLT) and horizontal line test(HLT).

always increasing or
always decreasing

One-To-One

A function is one-to-one if it passes BOTH the vertical line test (VLT) and horizontal line test(HLT).



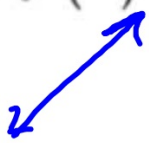
Inverse Notation

$$\boxed{f^{-1}(x)}$$

NOTE: $f^{-1}(x) \neq [f(x)]^{-1}$ or $\dfrac{1}{f(x)}$

ex 4: Determine whether each function has an inverse function.

a) $f(x) = x + 1$



Yes

b) $f(x) = x^4 - x^2 + 7$

NO

c) $f(x) = -3x^2 + 4x + 5$

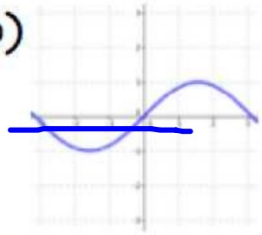
no

ex 5: Determine whether each function has an inverse function.

a) $f(x) = \sqrt{x}$

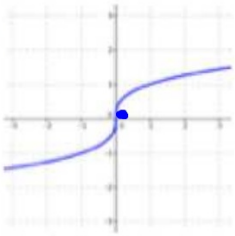
yes

b)



no

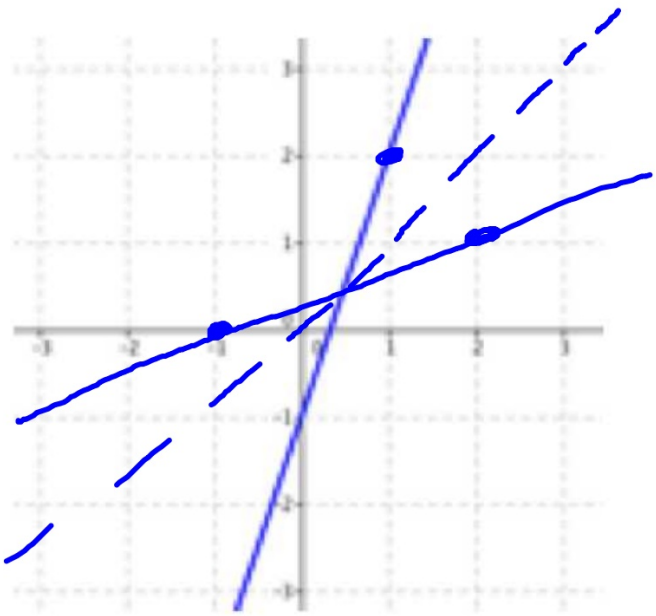
c)



yes

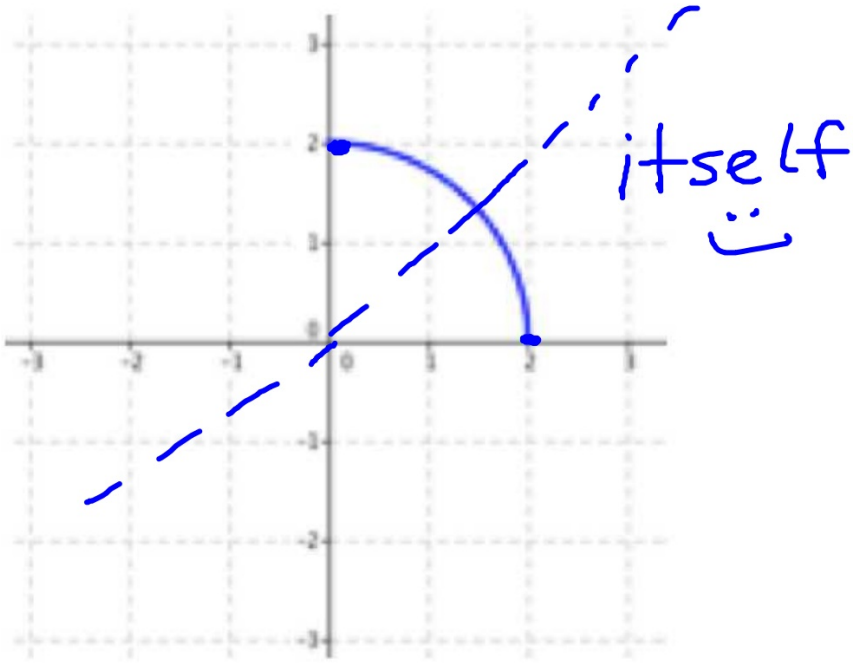
ex 6: Sketch the inverse function, if it exists.

a)



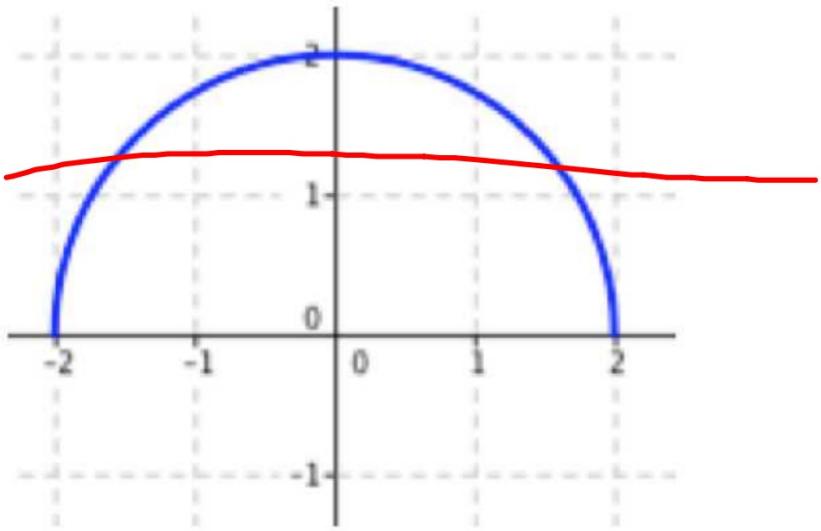
ex 6: Sketch the inverse function, if it exists.

b)



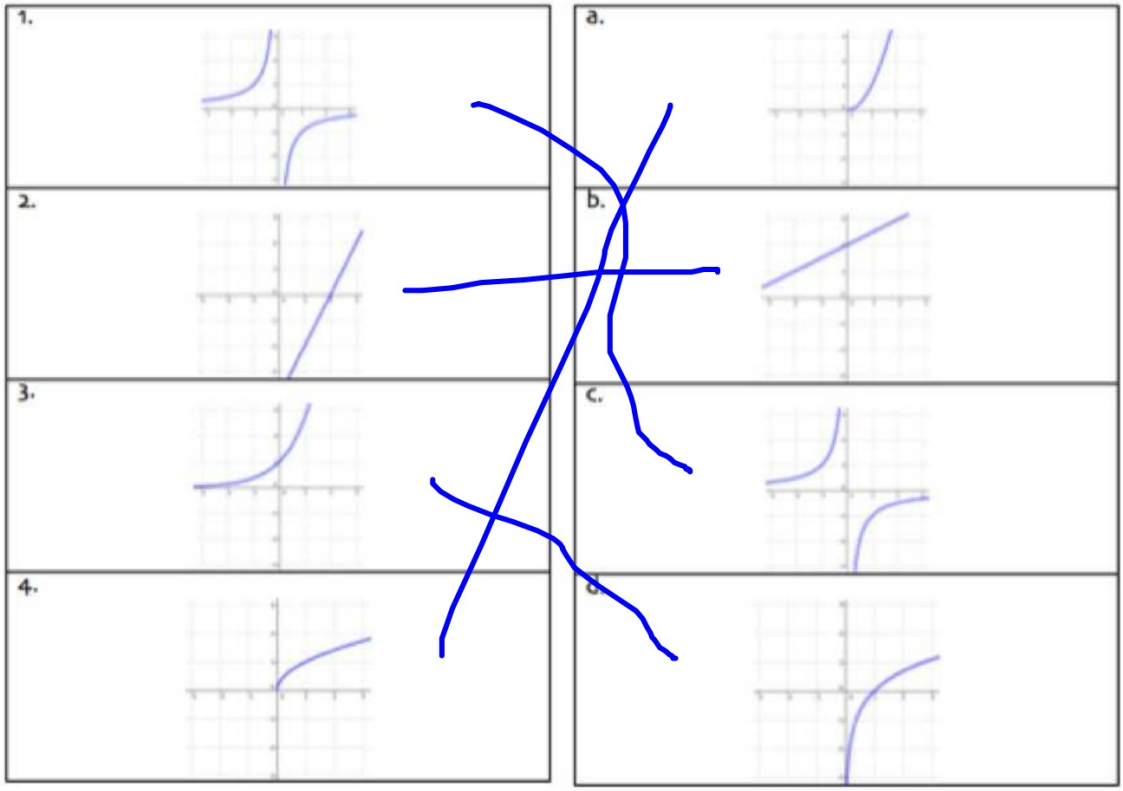
ex 6: Sketch the inverse function, if it exists.

c)

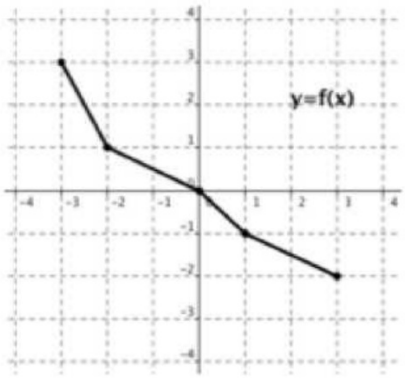


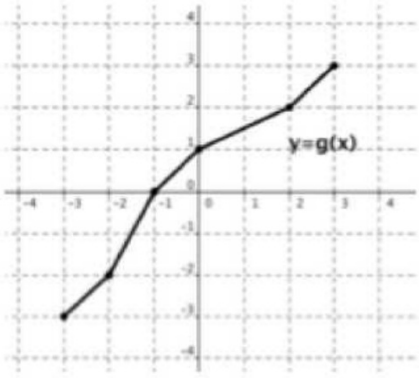
dne

ex 7: Match each graph with the graph of its inverse.



ex 8: Find the indicated values, if possible.





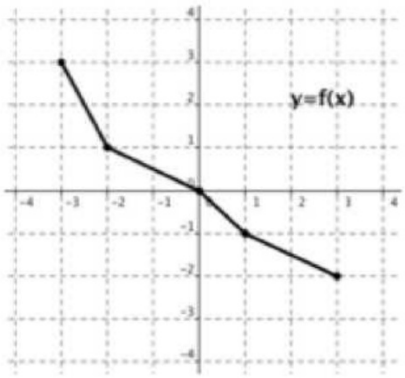
a) $f^{-1}(3)$

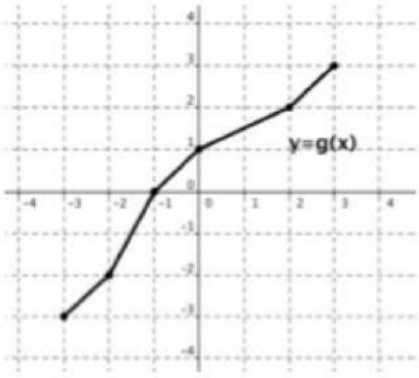
b) $g^{-1}(0)$

c) $g^{-1}(1)$

d) $f^{-1}(0)$

ex 8: Find the indicated values, if possible.





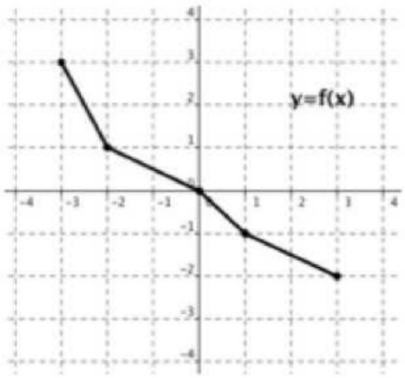
e) $(f \circ g)(-1)$

f) $(g \circ f)(-2)$

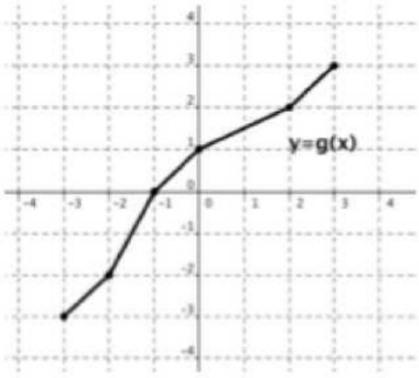
g) $(f \circ g^{-1})(1)$

h) $(g \circ f^{-1})(-1)$

ex 8: Find the indicated values, if possible.



i) $\left(f \circ f^{-1}\right)(2)$



j) $\left(g^{-1} \circ f^{-1}\right)(2)$

Finding Inverses

ex 9: Find the inverse, if possible.

a) $f(x) = \dfrac{5-3x}{2}$

ex 9: Find the inverse, if possible.

b) $g(x) = \sqrt[3]{x+8}$

ex 9: Find the inverse, if possible.

c) $y = \dfrac{2}{x+1}$

ex 9: Find the inverse, if possible.

d) $h(x) = x^2 - 5x + 1$

ex 10: Given

$$f(x) = x + 4 \qquad g(x) = \sqrt{x} - 5 \qquad h(x) = x^2 + x,\ x \geq -\frac{1}{2}$$

Find the indicated value.

a) $f^{-1}(0)$

ex 10: Given

$$f(x) = x + 4 \qquad g(x) = \sqrt{x} - 5 \qquad h(x) = x^2 + x,\ x \geq -\frac{1}{2}$$

Find the indicated value.

b) $g^{-1}(2)$

ex 10: Given

$$f(x) = x + 4 \qquad g(x) = \sqrt{x} - 5 \qquad h(x) = x^2 + x,\ x \geq -\frac{1}{2}$$

Find the indicated value.

c) $h^{-1}(2)$

ex 10: Given

$$f(x) = x + 4 \qquad g(x) = \sqrt{x} - 5 \qquad h(x) = x^2 + x,\ x \geq -\frac{1}{2}$$

Find the indicated value.

d) $\left(g^{-1} \circ f^{-1}\right)(-1)$

ex 10: Given

$$f(x) = x + 4 \qquad g(x) = \sqrt{x} - 5 \qquad h(x) = x^2 + x,\ x \geq -\frac{1}{2}$$

Find the indicated value.

e) $\left(h \circ g^{-1}\right)(0)$

ex 10: Given

$$f(x) = x + 4 \qquad g(x) = \sqrt{x} - 5 \qquad h(x) = x^2 + x,\ x \geq -\frac{1}{2}$$

Find the indicated value.

f) $\left(f \circ f^{-1}\right)(\ \star\ )$