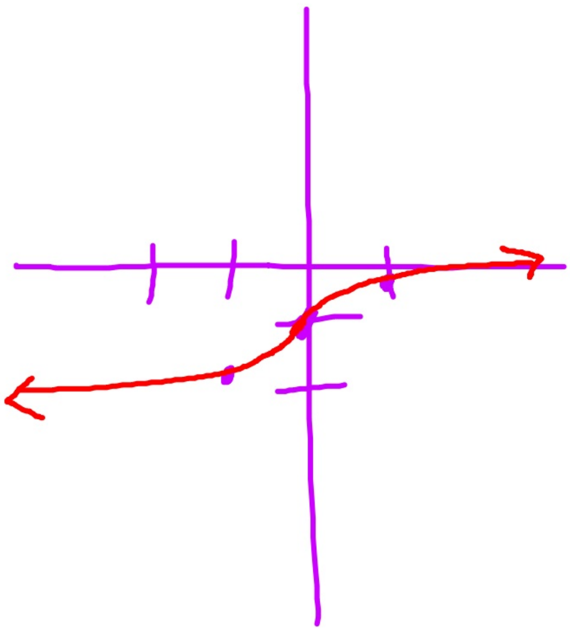
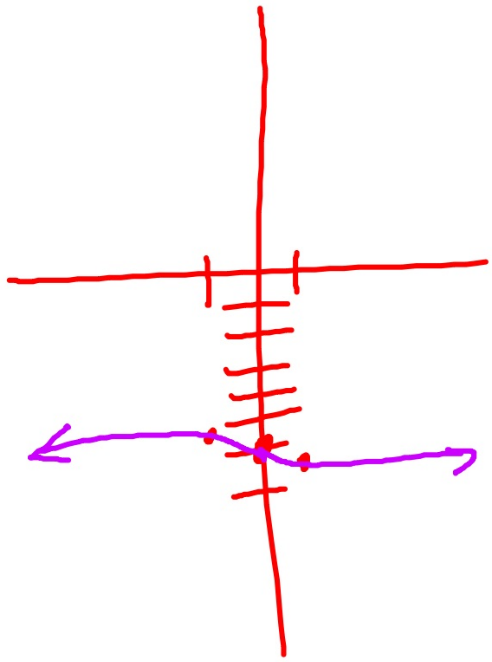
19.) $y = \frac{3}{4}x^{1/3} - 1$

x	y
-1	$-\frac{3}{4} - 1 = -1\frac{3}{4}$
0	-1
1	$\frac{3}{4} - 1 = \frac{-1}{4}$



$$g(x) = -\frac{1}{3}\sqrt[3]{x} - 6$$

x	y
-1	$\frac{1}{3} - 6 = -5\frac{2}{3}$
0	-6
1	$\frac{-1}{3} - 6 = -6\frac{1}{3}$



33.) $f(x) = \frac{1}{4}\sqrt{x-3} + 6$

$(3, 6)$

D: $[3, \infty)$

R: $[6, \infty)$

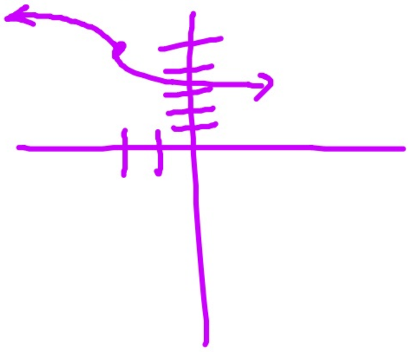
b.) $y = \underline{\underline{5}} - \sqrt[3]{5x+10} = -\sqrt[3]{5x+10} + 5$

$= -\sqrt[3]{5(x \underline{\underline{+2}})} + 5$

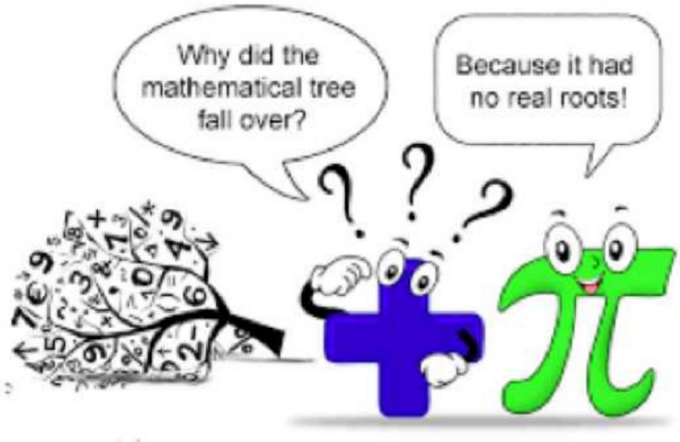
$(-2,\ 5)$

x	y
$-11/5$	6
-2	5
$-9/5$	4

$5x + 10 = {}^{-}1$



3.4 Inverse Functions



*See printout.

$\boxed{\begin{array}{c}\text{f and g are inverse functions if} \\ (f \circ g)(x) = x \\ \text{AND} \\ (g \circ f)(x) = x\end{array}}$

Verifying Inverse Functions

1. Algebraically

Show: $\boxed{(f \circ g)(x) = x}$ AND $\boxed{(g \circ f)(x) = x}$

ex 1: Show $f(x) = 4x + 9$ and $g(x) = \dfrac{x-9}{4}$

are inverses, algebraically.

$$(f \circ g)(x) = 4\left(\frac{x-9}{4}\right) + 9 = x - 9 + 9 = x$$

$$(g \circ f)(x) = \frac{4x + \cancel{9} - \cancel{9}}{4} = \frac{4x}{4} = x$$

Verifying Inverse Functions

2. Graphically

Show: *Sketch each function and show that they graphs are symmetrical with the line $y = x$*

Verifying Inverse Functions

3. Numerically (NOT A PROOF)

Show: $f(a) = b$ (a, b)

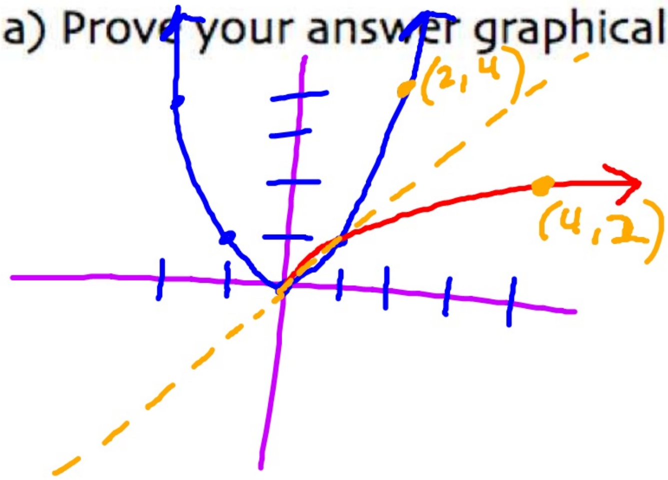
and

$g(b) = a$ (b, a)

ex 2: If f and g are inverse functions and f contains the point (2, -3) then g must contain the point (-3, 2).

ex 3: Are $f(x) = x^2$ and $g(x) = \sqrt{x}$ inverses?

a) Prove your answer graphically.



No...

ex 3: Are $f(x) = x^2$ and $g(x) = \sqrt{x}$ inverses?

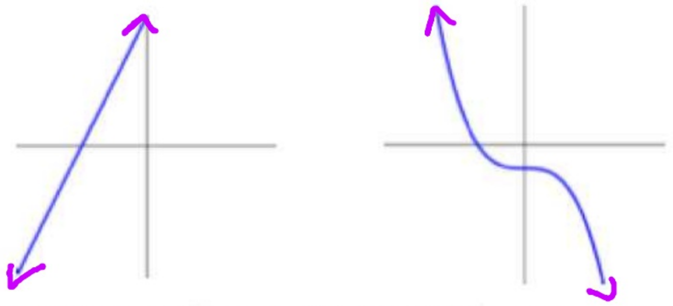
b) Prove your answer algebraically.

$(f \circ g)(x) = (\sqrt{x})^2 = x$ NO

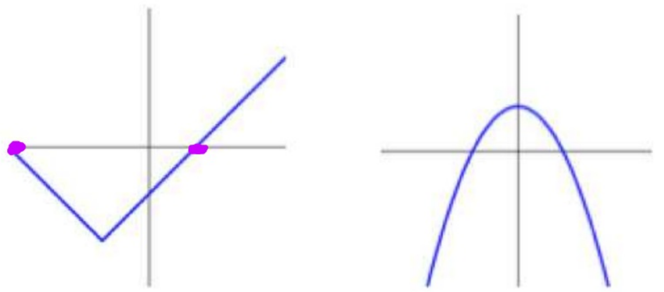
$(g \circ f)(x) = \sqrt{x^2} = |x|$ $x \neq |x|$

The Existence of an Inverse

Examples of functions that DO have inverses:



Examples of functions that DO NOT have inverse functions.



The Existence of an Inverse

A function has an inverse function if it passes BOTH the vertical line test (VLT) and horizontal line test(HLT).

One-To-One

A function is one-to-one if it passes BOTH the vertical line test (VLT) and horizontal line test(HLT).

One-to-one functions are either always increasing OR always decreasing

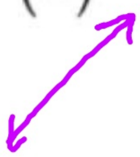
Inverse Notation

$$\boxed{f^{-1}(x)}$$

NOTE: $f^{-1}(x) \neq \underbrace{\left[f(x)\right]^{-1} \text{ or } \frac{1}{f(x)}}_{\text{not } f^{-1}(x)}$

ex 4: Determine whether each function has an inverse function.

a) $f(x) = x + 1$

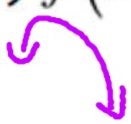


Yes

b) $f(x) = x^4 - x^2 + 7$

No

c) $f(x) = -3x^2 + 4x + 5$

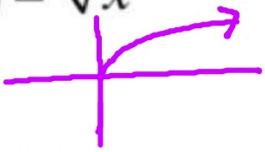


no

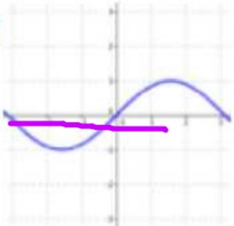
ex 5: Determine whether each function has an inverse function.

a) $f(x) = \sqrt{x}$

Yes

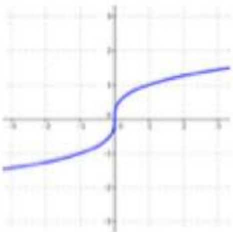


b)



no

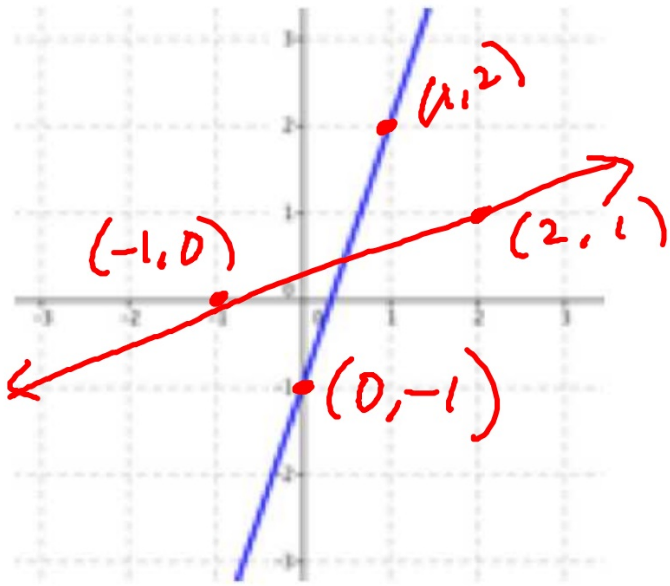
c)



Yes

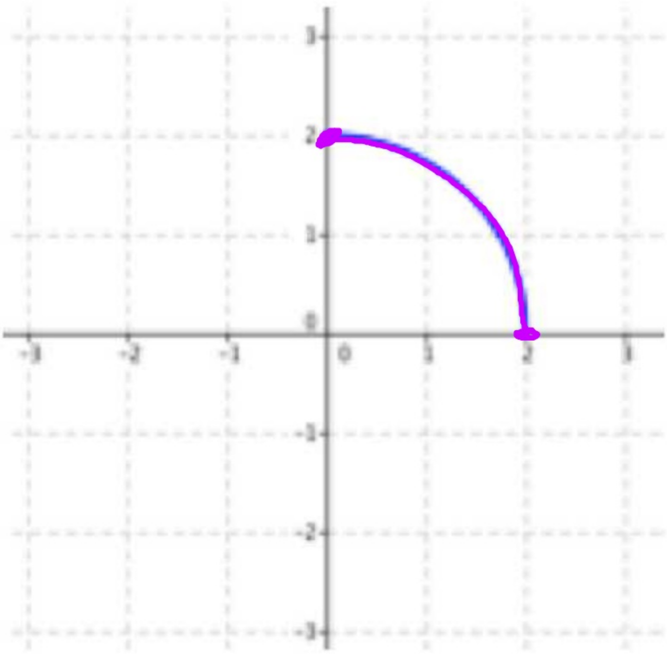
ex 6: Sketch the inverse function, if it exists.

a)



ex 6: Sketch the inverse function, if it exists.

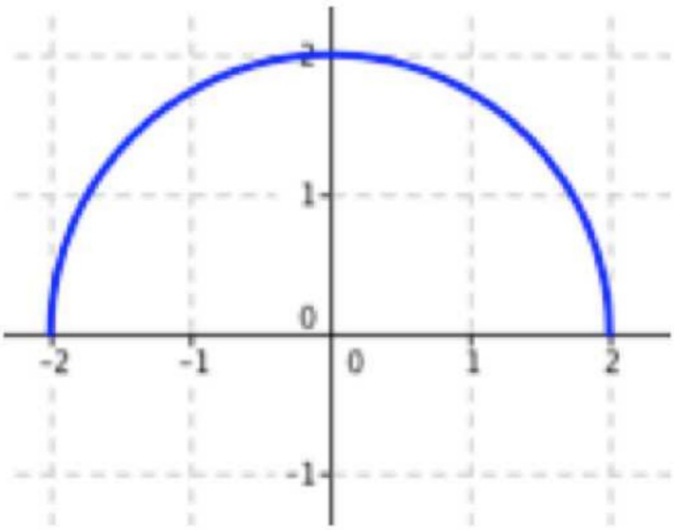
b)



The inverse is itself

ex 6: Sketch the inverse function, if it exists.

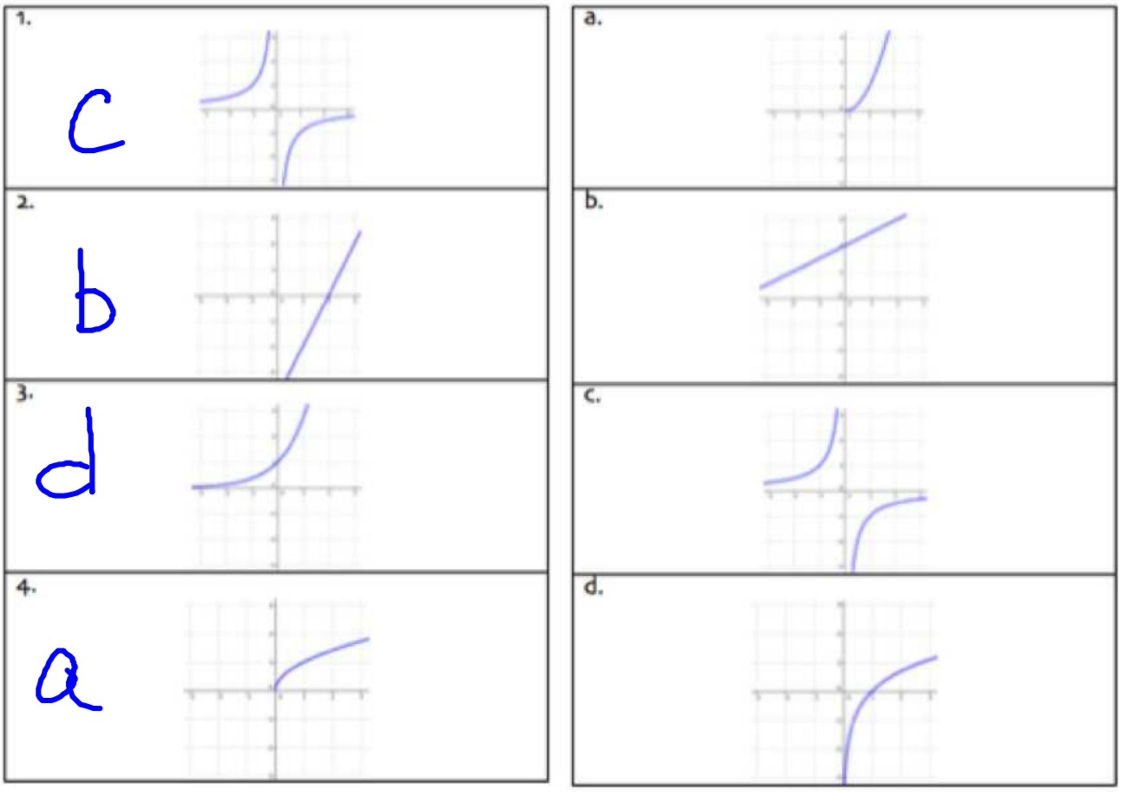
c)



Will not have an
inverse funciton

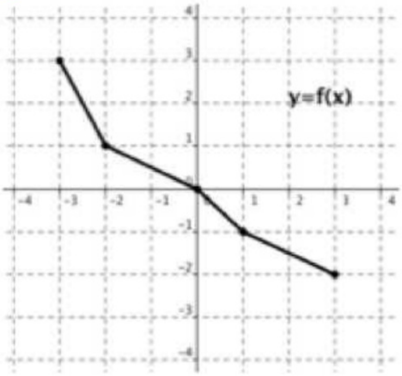
the graph fails
the HLT

ex 7: Match each graph with the graph of its inverse.



ex 8: Find the indicated values, if possible.

$f:(2,3)$ $f^{-1}(3,2)$

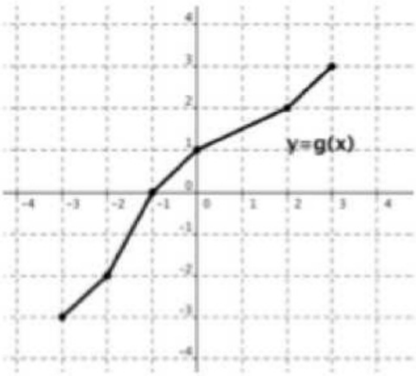


a) $f^{-1}(3)$ y-coord for $f(x)$

-3

b) $g^{-1}(0)$ y coord. for $g(x)$

-1



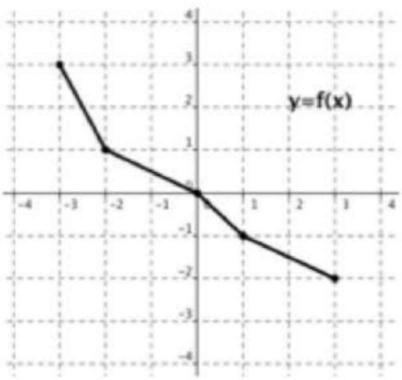
c) $g^{-1}(1)$

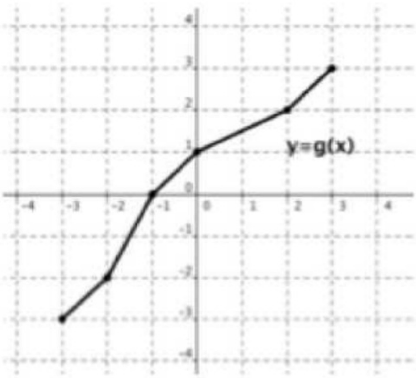
0

d) $f^{-1}(0)$

0

ex 8: Find the indicated values, if possible.





e) $(f \circ g)(-1) = 0$

f) $(g \circ f)(-2) = 1.5$

(1)

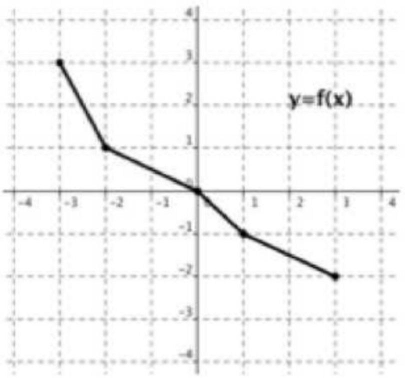
g) $(f \circ g^{-1})(1) = 0$

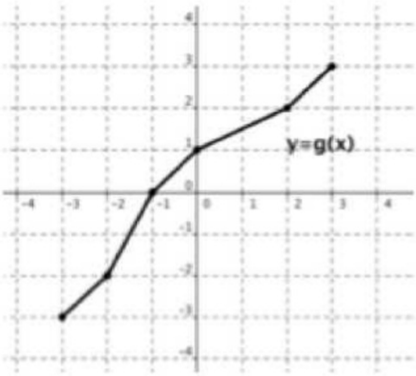
0

h) $(g \circ f^{-1})(-1) = 1.5$

1

ex 8: Find the indicated values, if possible.





i) $\left(f \circ f^{-1}\right)(2) = 2$

$\left(f \circ f^{-1}\right)(x) = x$

j) $\left(g^{-1} \circ f^{-1}\right)(2)$

$\boxed{-2.5}$

Finding Inverses

ex 9: Find the inverse, if possible.

a) $f\left(x\right)=\frac{5-3x}{2}$

Switch the x's and y's in the equation.

$$x = \frac{5-3y}{2}$$

$$2x = 5 - 3y$$

$$2x - 5 = -3y$$

$$\frac{2x-5}{-3} = y$$

Solve for y.

$$\boxed{f^{-1}(x) = \frac{2x-5}{-3}}$$

ex 9: Find the inverse, if possible.

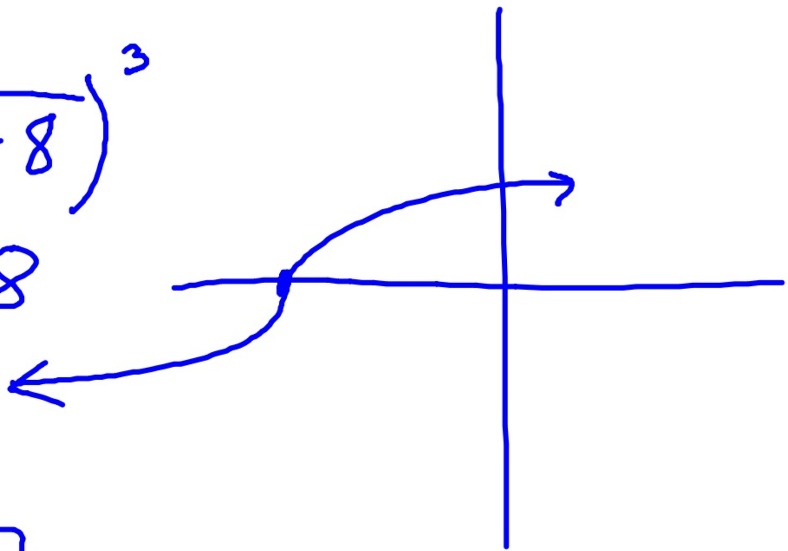
b) $g(x)=\sqrt[3]{x+8}$

$$x^3=\left(\sqrt[3]{y+8}\right)^3$$

$$x^3=y+8$$

$$x^3-8=y$$

$$\boxed{g^{-1}(x)=x^3-8}$$



ex 9: Find the inverse, if possible.

c) $y = \dfrac{2}{x+1}$

$$\frac{x}{1} = \frac{2}{y+1}$$

$$xy + x = 2$$

$$xy = 2 - x$$

$$y = \frac{2-x}{x}$$

$$\frac{1}{x} = \frac{y+1}{2}$$

$$\frac{2}{x} = y + 1$$

$$\frac{2}{x} - 1 = y$$

$$\boxed{y^{-1} = \frac{2}{x} - 1}$$

ex 9: Find the inverse, if possible.

d) $h(x) = x^2 - 5x + 1$

Not possible. Does not pass the HLT.

ex 10: Given

$$f(x) = x + 4 \qquad g(x) = \sqrt{x} - 5 \qquad h(x) = x^2 + x,\ x \geq -\frac{1}{2}$$

Find the indicated value.

a) $\boxed{f^{-1}(0) = -4}$

$0 = x + 4$

$-4 = x$

ex 10: Given

$$f(x) = x + 4 \qquad g(x) = \sqrt{x} - 5 \qquad h(x) = x^2 + x,\ x \geq -\frac{1}{2}$$

Find the indicated value.

b) $\boxed{g^{-1}(2) = 49}$

$g(x) = \sqrt{x} - 5$

$2 = \sqrt{x} - 5$

$49 = x$

ex 10: Given

$$f(x) = x + 4 \qquad g(x) = \sqrt{x} - 5 \qquad h(x) = x^2 + x,\ x \geq -\frac{1}{2}$$

Find the indicated value.

c) $h^{-1}(2) = 1$

$2 = x^2 + x$

$0 = x^2 + x - 2$

$(x+2)(x-1)$

ex 10: Given

$$f(x) = x + 4 \qquad g(x) = \sqrt{x} - 5 \qquad h(x) = x^2 + x,\ x \geq -\frac{1}{2}$$

Find the indicated value.

d) $\left(g^{-1} \circ f^{-1}\right)(-1) = 0$

$-1 = x + 4$ $\qquad$ $-5 = \sqrt{x} - 5$

$-5 = x$

ex 10: Given

$$f(x) = x + 4 \qquad g(x) = \sqrt{x} - 5 \qquad h(x) = x^2 + x,\ x \geq -\frac{1}{2}$$

Find the indicated value.

e) $\left(h \circ g^{-1}\right)(0) = 650$

$0 = \sqrt{x} - 5$

$25 = x$

$25^2 + 25$

ex 10: Given

$$f(x) = x + 4 \qquad g(x) = \sqrt{x} - 5 \qquad h(x) = x^2 + x,\ x \geq -\frac{1}{2}$$

Find the indicated value.

f) $\left(f \circ f^{-1}\right)(\bigstar) = \bigstar$