3.4 Inverse Functions - cont.
3.5 Graphs of Square Roots & Cube Roots

<u>Finding Inverses</u>

$-\frac{3}{2}x + \frac{5}{2}$

ex: Find the inverse, if possible.

1) switch the x's and y's
2) Solve for y
3) Replace y with the inverse notation

a) $f(x) = \dfrac{5-3x}{2}$

$y = \dfrac{5-3x}{2}$

$2 \cdot x = \dfrac{5-3y}{2} \cdot 2$

$2x = 5 - 3y$

$2x - 5 = -3y$

$-\frac{2}{3}x + \frac{5}{3} = y$

$-\frac{2}{3}x + \frac{5}{3} = f^{-1}(x)$

or

$f^{-1}(x) = \dfrac{2x-5}{-3}$

or

ex: Find the inverse, if possible.

$\sqrt[3]{x+8} \neq \sqrt[3]{x} + \sqrt[3]{8}$

b) $g(x) = \sqrt[3]{x+8}$

$y = \sqrt[3]{x+8}$

$(x)^3 = \left(\sqrt[3]{y+8}\right)^3$

$x^3 = y + 8$

$x^3 - 8 = y$

$\boxed{g^{-1}(x) = x^3 - 8}$

ex: Find the inverse, if possible.

c) $y = \dfrac{2}{x+1}$

$$\frac{x}{1} = \frac{2}{y+1}$$

$$x(y+1) = 2$$

$$y+1 = \frac{2}{x}$$

$$y = \frac{2}{x} - 1$$

$$y^{-1} = \frac{2}{x} - 1$$

$$xy + x = 2$$

$$xy = 2 - x$$

$$y = \frac{2}{x} - \frac{x}{x}$$

$$y^{-1} = \frac{2}{x} - 1$$

$$y^{-1} = \frac{-x+2}{x}$$

ex: Find the inverse, if possible.

d) $h(x) = x^2 - 5x + 1$

Not possible; h(x) fails the horizontal line test

ex: Given

$$\boxed{f(x) = x + 4} \qquad g(x) = \sqrt{x} - 5 \qquad h(x) = x^2 + x,\ x \geq -\frac{1}{2}$$

Find the indicated value.

a) $f^{-1}(0)$

y-coord. of f(x)

$f(x) = x + 4$

$0 = x + 4$

$-4 = x$

$$\boxed{f^{-1}(0) = -4}$$

ex: Given

$$f(x) = x + 4 \qquad g(x) = \sqrt{x} - 5 \qquad h(x) = x^2 + x,\ x \geq -\frac{1}{2}$$

Find the indicated value.

b) $g^{-1}(2)$

y-coord. for g(x)

$2 = \sqrt{x} - 5$

$7^2 = \sqrt{x}^2$

$49 = x$

$\boxed{g^{-1}(2) = 49}$

ex: Given

$$f(x) = x + 4 \qquad g(x) = \sqrt{x} - 5 \qquad h(x) = x^2 + x,\ x \geq -\frac{1}{2}$$

Find the indicated value.

c) $h^{-1}(2)$

$h^{-1}(2) = 1$

$2 = x^2 + x$

$0 = x^2 + x - 2$

$0 = (x+2)(x-1)$

$x = 1, \cancel{-2}$

ex: Given

$$f(x) = x + 4 \qquad g(x) = \sqrt{x} - 5 \qquad h(x) = x^2 + x,\ x \geq -\frac{1}{2}$$

Find the indicated value.

d) $\left(g^{-1} \circ f^{-1}\right)(-1) = \underline{0}$

$-1 = x + 4$

$-5 = x$

$-5 = \sqrt{x} - 5$

$0 = \sqrt{x}$

$0 = x$

ex: Given

$$f(x) = x + 4 \qquad g(x) = \sqrt{x} - 5 \qquad h(x) = x^2 + x,\ x \geq -\frac{1}{2}$$

Find the indicated value.

e) $\left(h \circ g^{-1}\right)(0)$ $= 650$

$0 = \sqrt{x} - 5$

$25 = x$

$h(25) =$

ex: Given

$$f(x) = x + 4 \qquad g(x) = \sqrt{x} - 5 \qquad h(x) = x^2 + x,\ x \geq -\frac{1}{2}$$

Find the indicated value.

f) $\left(f \circ f^{-1}\right)($ ☆ $)$

REVIEW

ex: Sketch and state the domain and range in interval notation.

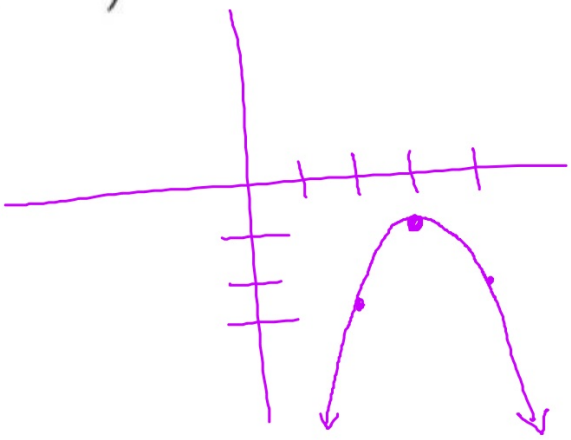
vertex: $(3,-1)$

$$y=-2(x-3)^2-1$$

x	y
2	-3
3	-1
4	-3

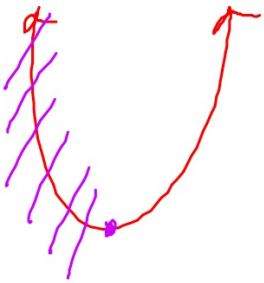
D: $(-\infty,\infty)$

R: $(-\infty,-1]$

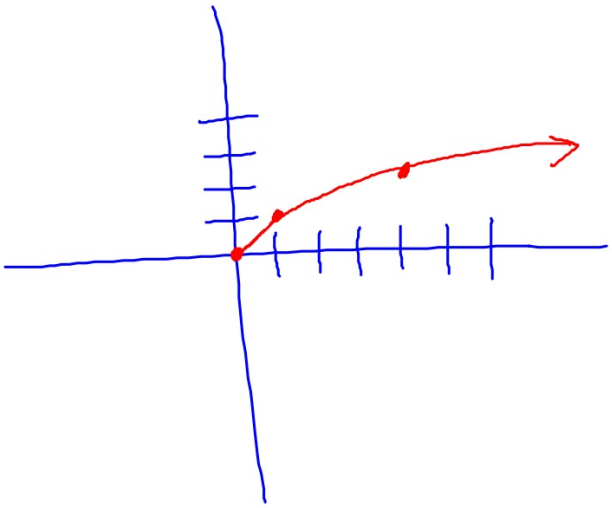


Square Root Graphs

Parent Function: $f(x) = \sqrt{x}$



Graph:



Domain:

Range:

ex: Sketch and state the domain and range in interval notation.

a) $y = 2\sqrt{x-1} - 1$

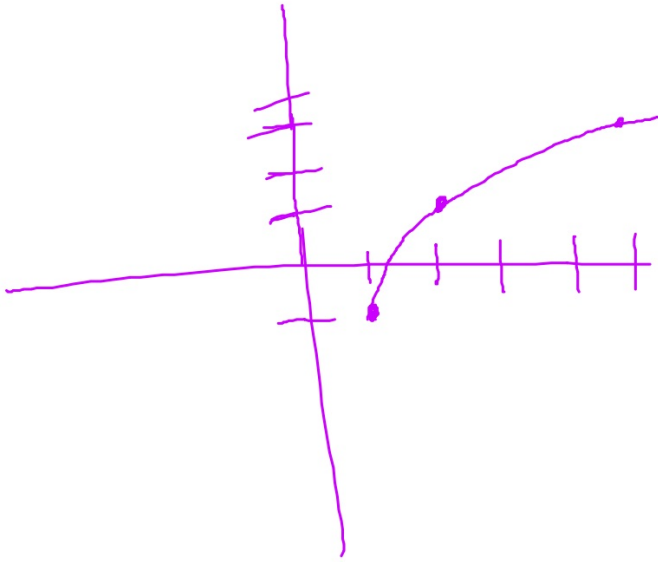
$x - 1 \geq 0$

D: $[1, \infty)$

R: $[-1, \infty)$

Key point: $(1, -1)$

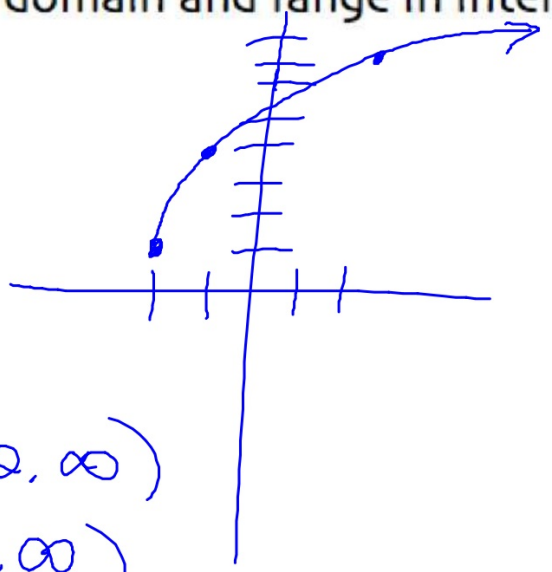
x	y
1	-1
2	1
5	3



ex: Sketch and state the domain and range in interval notation.

b) $y = 3\sqrt{x+2} + 1$

Keypoint
$(-2, 1)$



X	y
-2	1
-1	4
2	7

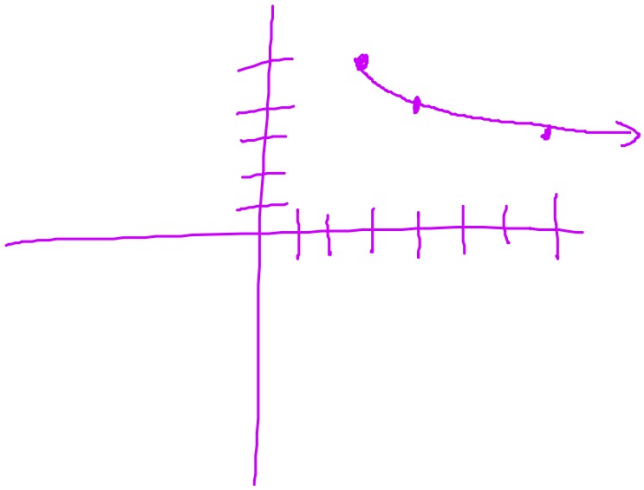
D: $[-2, \infty)$
R: $[1, \infty)$

ex: Sketch and state the domain and range in interval notation.

c) $y = -\sqrt{x-3} + 5$

D: $[3, \infty)$

R: $(-\infty, 5]$



x	y
3	5
4	4
7	3

ex: Sketch and state the domain and range in interval notation.

d) $y = 2 - \sqrt{1 - x}$

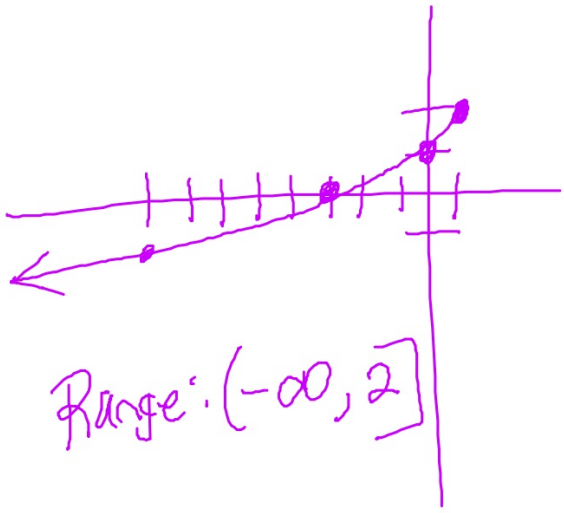
$y = -\sqrt{1-x} + 2$

Domain: $1 - x \geq 0$

$-x \geq -1$

$x \leq 1$

$(-\infty, 1]$



Range: $(-\infty, 2]$

Key point $(1,2)$

x	y
1	2
0	1
-3	0
-8	-1

ex: Sketch and state the domain and range in interval notation.

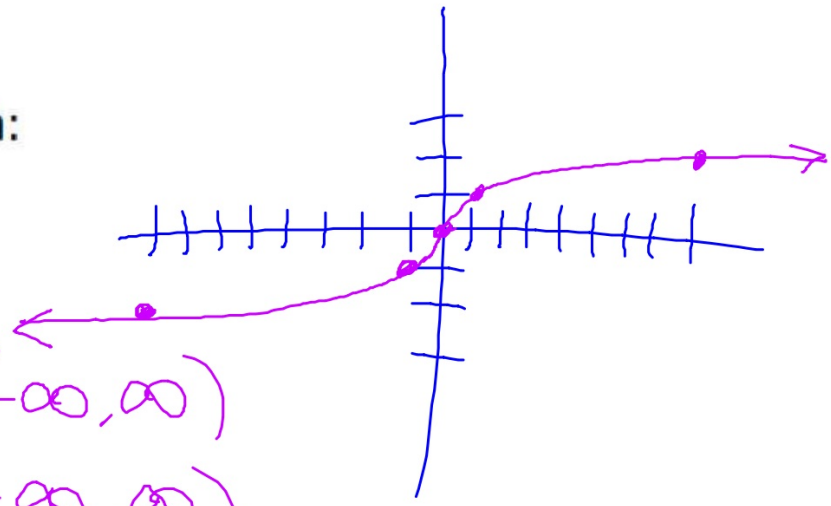
e) $y = \sqrt{3x+1} - 4$

Cube Root Graphs

Parent Function: $f(x) = \sqrt[3]{x}$

x	y
-8	-2
-1	-1
0	0
1	1
8	2

Graph:



D: $(-\infty, \infty)$

R: $(-\infty, \infty)$

Domain:

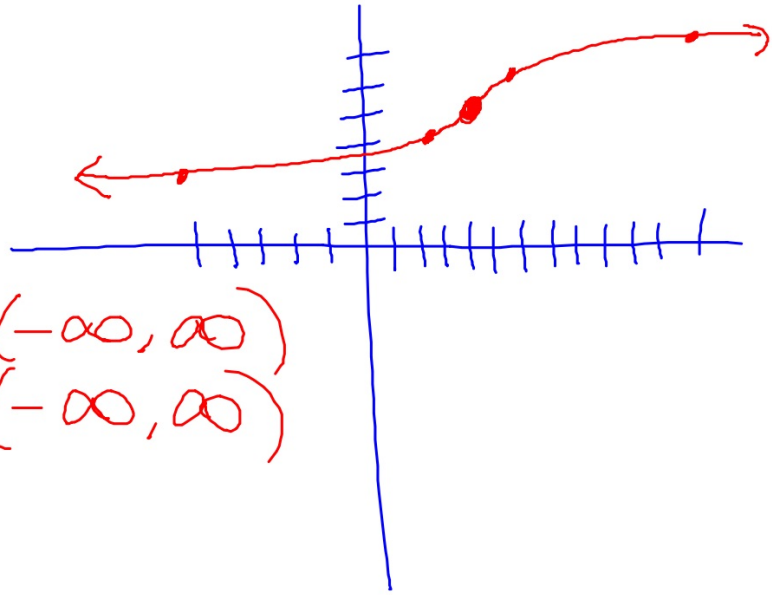
Range:

ex: Sketch and state the domain and range in interval notation.

f) $y = \sqrt[3]{x-4} + 5$

Key point (4, 5)

x	y
-4	3
3	4
4	5
5	6
12	7



D: $(-\infty, \infty)$

R: $(-\infty, \infty)$

ex: Sketch and state the domain and range in interval notation.

g) $y = \sqrt[3]{x+3} - 4$

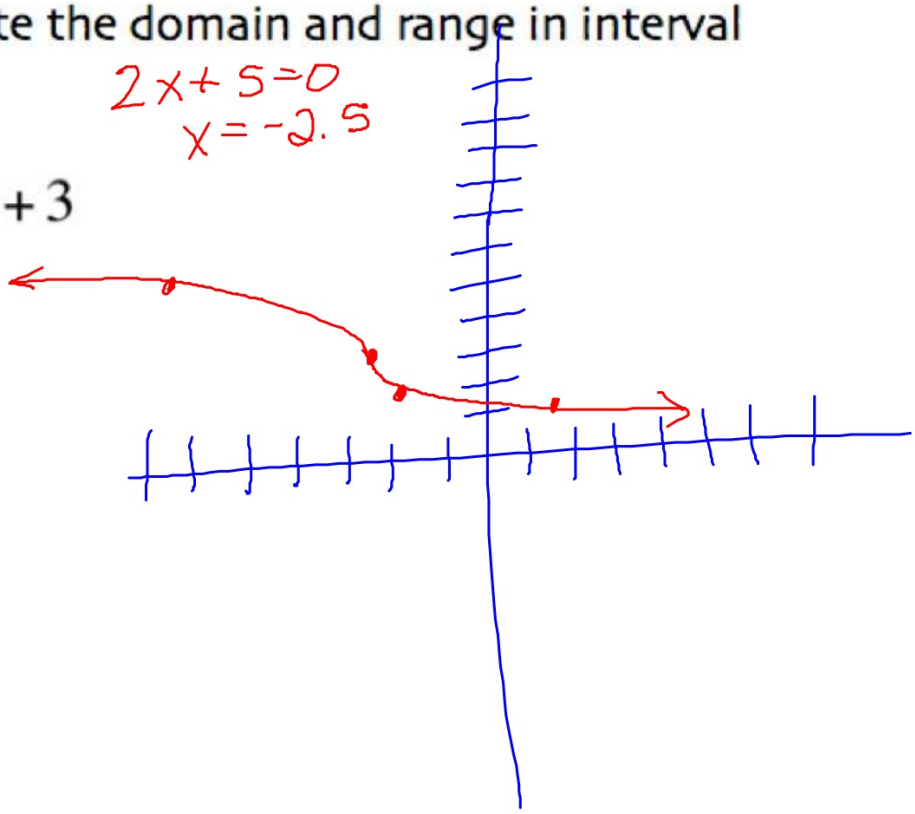
ex: Sketch and state the domain and range in interval notation.

$2x+5=0$

$x=-2.5$

h) $y=-\sqrt[3]{2x+5}+3$

X	y
$-13/2$	5
-3	4
-2.5	3
-2	2
$3/2$	1



REVIEW

ex: Are f(x) and g(x) inverse functions? Justify your answer algebraically.

$$f(x) = \sqrt[4]{2x+2} \qquad g(x) = \frac{x^4}{2} - 1$$

REVIEW

ex: Simplify.

$$\sqrt[6]{128x^6y^{13}z^9}$$

REVIEW

ex: Sketch the inverse function, if it exists.