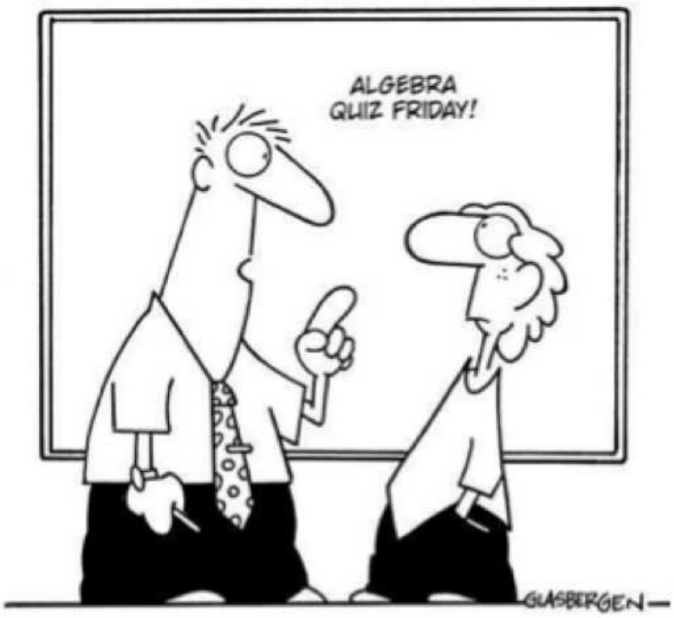
3.3 Function Operations & Compositions

© Randy Glasbergen / glasbergen.com



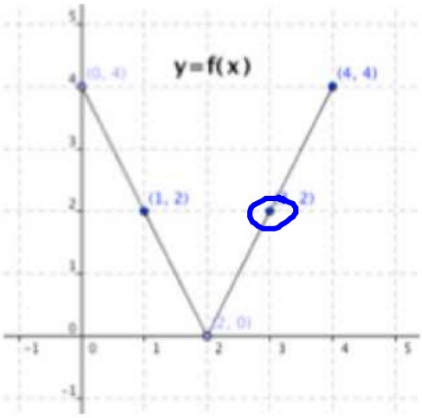
"It's important to learn math because someday you might accidentally buy a phone without a calculator."

HW:

Function Operations

Addition	$f(x)+g(x)=(f+g)(x)$
Subtraction	$f(x)-g(x)=(f-g)(x)$
Multiplication	$f(x)g(x)=(fg)(x)$
Division	$\frac{f(x)}{g(x)}=\left(\frac{f}{g}\right)(x)$

ex: Evaluate.



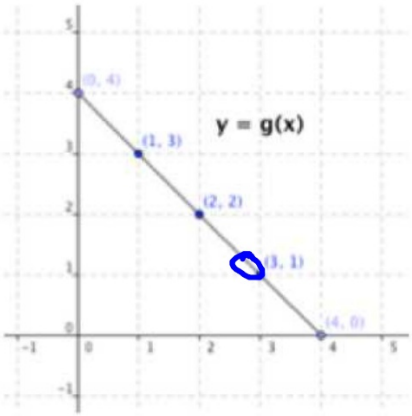
a) $(f+g)(3)$

$f(3)+g(3)$

$2+1$

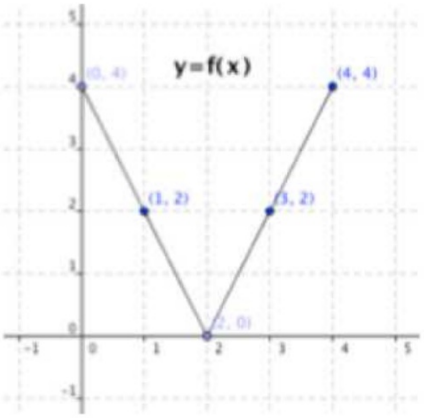
3

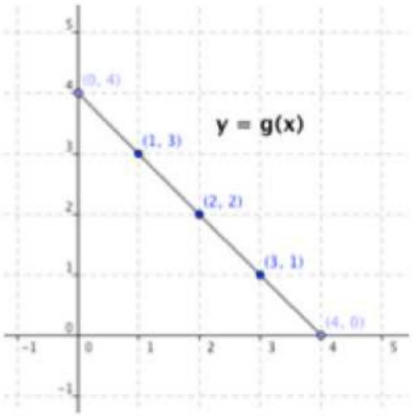
b) $(f-g)(1) = -1$



c) $(fg)(4)$ 0

ex: Evaluate.





d) $(fg)(0)$

e) $\left(\frac{g}{f}\right)(1)$

f) $\left(\frac{f}{g}\right)(3)$

g) $-5(fg)(2)$

ex: Evaluate each expression given the functions below.

$f(x) = x^{2/3}$ $\quad g(x) = \sqrt[4]{x}$ $\quad h(x) = x + 5$

a) $(f+h)(27)$

$f(27) + h(27)$

$27^{2/3} + 32 = 9 + 32 = 41$

b) $(fg)(1) = 1$

c) $\left(\frac{h}{g}\right)(64) = \frac{69}{\sqrt[4]{64}} = \frac{69}{2\sqrt[4]{4}} \cdot \frac{\sqrt[4]{4}}{\sqrt[4]{4}} = \frac{69\sqrt[4]{4}}{4}$

$2 \cdot 2 \cdot 2 \cdot 2$

Finding Domain Algebraically

Polynomials and Odd Roots

D: $\{x \mid x \in \mathbb{R}\}$

ex: Find the domain. State the answer in set notation.

a) $y = x^4 - 2x + 1$ $\{x \mid x \in \mathbb{R}\}$

b) $y = \sqrt[3]{x^2 + 2x - 3}$ $\{x \mid x \in \mathbb{R}\}$

Even Roots

D: $\{x \mid x \geq 0\}$

$\sqrt[4]{-1}$

ex: Find the domain. State the answer in set notation.

c) $y = \sqrt[4]{x+1}$

Set the radicand ≥ 0

$x + 1 \geq 0$

$\{x \mid x \geq -1\}$

ex: Find the domain. State the answer in set notation.

d) $y = \sqrt{-x}$

$-x \geq 0$

$\{x \mid x \leq 0\}$

Fractions: $y = \frac{f(x)}{g(x)}$ ← polynomial

← polynomial or radical

D: $g(x) \neq 0$ (for poly)

$g(x) > 0$ (for radicals)

ex: Find the domain. State the answer in set notation.

e) $y = \frac{x+1}{2x-5}$

$2x - 5 \neq 0$

$x \neq \frac{5}{2}$

$\{x \mid x \neq \frac{5}{2}\}$

ex: Find the domain. State the answer in set notation.

f) $y = \dfrac{x^3 - 7x^2 + 1}{x^2 - 9}$

$x^2 - 9 \neq 0$

$(x+3)(x-3) \neq 0$

$\{x \mid x \neq \pm 3\}$

ex: Find the domain. State the answer in set notation.

g) $y = \dfrac{x^3 - 7x^2 + 1}{x^2 + 9}$

$\{x \mid x \in R\}$

$x^2 + 9 \neq 0$

↓

imaginary...

ex: Find the domain. State the answer in set notation.

h) $y = \dfrac{6}{x^2 - 3x + 2}$

$\{x \mid x \neq 1, 2\}$

$x^2 - 3x + 2 \neq 0$

$(x-2)(x-1) \neq 0$

$x \neq 2, 1$

ex: Find the domain. State the answer in set notation.

i) $y = \dfrac{3x-1}{\sqrt{x+9}}$

$x+9 > 0$

$\{x \mid x > -9\}$

ex: Find the domain. State the answer in set notation.

j) $y = \dfrac{\sqrt{x+9}}{3x-1}$

ex: Find the domain. State the answer in set notation.

k) $y = \dfrac{\sqrt[4]{x+2}}{\sqrt[3]{x-1}}$

ex: Find the domain. State the answer in set notation.

l) $y = 5x^{3/4} - 2$

even root
index is 4

$y = 5\sqrt[4]{x^3} - 2$

$x^3 \geq 0$

$\{x \mid x \geq 0\}$

ex: Find the domain. State the answer in set notation.

m) $y = \sqrt[3]{x} - \sqrt{x}$

$\{x \mid x \geq 0\}$

Function Compositions

$(f \circ g)(x) =$ $f(g(x))$

$(g \circ f)(x) =$ $g(f(x))$

ex: Let

$$f(x)=\sqrt{x} \qquad m(x)=x^2$$

$$g(x)=\sqrt[5]{x} \qquad n(x)=x+5$$

$$h(x)=\frac{1}{x} \qquad p(x)=x^2-10x+25$$

Find the composition, if possible.

a) $(f\circ g)(1)$ $= f(g(1)) = f(1) = 1$

ex: Let

$$f(x)=\sqrt{x} \qquad m(x)=x^2$$

$$g(x)=\sqrt[5]{x} \qquad n(x)=x+5$$

$$h(x)=\frac{1}{x} \qquad p(x)=x^2-10x+25$$

Find the composition, if possible.

b) $(n\circ m)(0)=$ $n(m(0))=n(0)=5$

ex: Let

$$f(x)=\sqrt{x} \qquad m(x)=x^2$$

$$g(x)=\sqrt[5]{x} \qquad n(x)=x+5$$

$$h(x)=\frac{1}{x} \qquad p(x)=x^2-10x+25$$

$\frac{1}{\frac{1}{32}} = 32$

Find the composition, if possible.

c) $(f\circ h)\left(\frac{1}{32}\right) = f\left(h\left(\frac{1}{32}\right)\right) = f(32) = \sqrt{32} = 4\sqrt{2}$

ex: Let

$$f(x)=\sqrt{x} \qquad m(x)=x^2$$

$$g(x)=\sqrt[5]{x} \qquad n(x)=x+5$$

$$h(x)=\frac{1}{x} \qquad p(x)=x^2-10x+25$$

Find the composition, if possible.

d) $(f\circ n)(-6)$ $f(-1)=\sqrt{-1}$ nonreal

ex: Let

$$f(x)=\sqrt{x} \qquad m(x)=x^2$$

$$g(x)=\sqrt[5]{x} \qquad n(x)=x+5$$

$$h(x)=\frac{1}{x} \qquad p(x)=x^2-10x+25$$

Find the composition, if possible.

e) $(p\circ h)(0)$ undefined

f.) $(p\circ p)(5)$ 25

g) $h(n(3))$ $h(8)=\frac{1}{8}$

ex: Let

$$f(x)=\sqrt{x} \qquad m(x)=x^2$$

$$g(x)=\sqrt[5]{x} \qquad n(x)=x+5$$

$$h(x)=\frac{1}{x} \qquad p(x)=x^2-10x+25$$

Find the composition and state the domain.

Consider the domain of the function plugged in AND the resulting function.

a) $(f\circ n)(x)$ $f(n(x))=f(x+5)=\sqrt{x+5}$

$\{x \mid x \geq -5\}$

ex: Let

$$f(x)=\sqrt{x} \qquad m(x)=x^2$$

$$g(x)=\sqrt[5]{x} \qquad n(x)=x+5$$

$$h(x)=\frac{1}{x} \qquad p(x)=(x^2-10x+25)$$

Find the composition and state the domain.

b) $(h\circ p)(x)=h(p(x))=\dfrac{1}{x^2-10x+25}$

$x^2-10x+25\neq 0$

$(x-5)^2\neq 0$

$\{x \mid x\neq 5\}$

ex: Let

$$f(x)=\sqrt{x} \qquad m(x)=x^2$$

$$g(x)=\sqrt[5]{x} \qquad n(x)=x+5$$

$$h(x)=\frac{1}{x} \qquad p(x)=x^2-10x+25$$

Find the composition and state the domain.

c) $(f\circ m)(x)$ $= f(m(x)) = f(x^2) = \sqrt{x^2} = |x|$

$\{x \mid x \in R\}$

ex: Let

$$f(x)=\sqrt{x} \qquad m(x)=x^2$$

$$g(x)=\sqrt[5]{x} \qquad n(x)=x+5$$

$$h(x)=\frac{1}{x} \qquad p(x)=x^2-10x+25$$

Find the composition and state the domain.

d) $(p\circ n)(x)$ $= p(n(x)) = p(x+5) = (x+5)^2-10(x+5)+25$

$= x^2$

$\{x \mid x \in R\}$

ex: Let

$$f(x)=\sqrt{x} \qquad m(x)=x^2$$

$$g(x)=\sqrt[5]{x} \qquad n(x)=x+5$$

$$h(x)=\frac{1}{x} \qquad p(x)=x^2-10x+25$$

Find the composition and state the domain.

e) $(n\circ h)(x)=\frac{1}{x}+5$ $\{x\mid x\neq 0\}$

f) $(f\circ g)(x)=\sqrt{\sqrt[5]{x}}=x^{1/10}$ $\{x\mid x\geq 0\}$

g) $(m\circ f)(x)=\left(\sqrt{x}\right)^2=x$ $\{x\mid x\geq 0\}$

ex: Let

$$f(x)=\sqrt{x} \qquad m(x)=x^2$$

$$g(x)=\sqrt[5]{x} \qquad n(x)=x+5$$

$$h(x)=\frac{1}{x} \qquad p(x)=x^2-10x+25$$

$1 \cdot \frac{x}{1}$

Find the composition and state the domain.

f) ~~$(f \circ g)(x)$~~ $(h \circ h)(x)$ $\frac{1}{\frac{1}{x}}=x$ $\{x \mid x \neq 0\}$

ex: Let

$$f(x)=\sqrt{x} \qquad m(x)=x^2$$

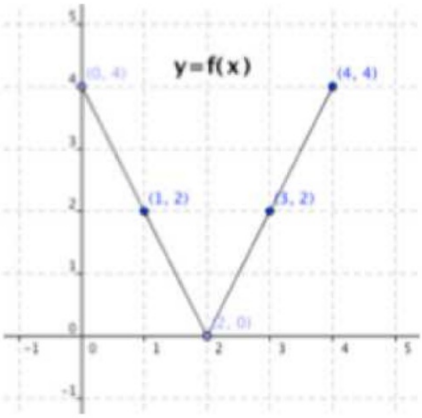
$$g(x)=\sqrt[5]{x} \qquad n(x)=x+5$$

$$h(x)=\frac{1}{x} \qquad p(x)=x^2-10x+25$$

Find the composition and state the domain.

g) $(m\circ f)(x)$

ex: Evaluate.



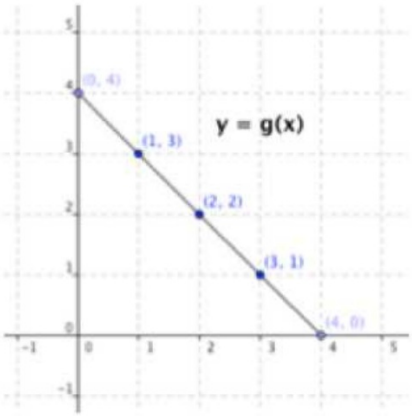
a) $(f \circ g)(2)$

$f(g(2)) =$

$f(2) = 0$

b) $(g \circ f)(3)$

$g(f(3)) = 2$



c) $(f \circ f)(1) = 0$

d.) 3

e.) 0