

3.1 Evaluate nth Roots and Use Rational Exponents

3.2 Apply Properties of Rational Exponents

ex: Fill in the Chart...FAST!

x	x^2	x^3	x^4	x^5	x^6
1	1	1	1	1	1
2	4	8	16	32	64
3	9	27	81	243	729
4	16	64	256	1024	4096
5	25	125	625	3125	15625
6	36	216	1296	-----	-----

nth Roots

$$\sqrt[n]{a}$$

Where:

- a is called the radicand
- n is called the index (root)

*Square Roots: \sqrt{a} , $n = \underline{\hspace{2cm}}$

Domain of nth Roots

$$\sqrt[n]{a}$$

If:

- n is even: $a \geq 0$

- n is odd: all reals

$$\sqrt[3]{-27} = -3$$

ex: Evaluate. If no real value exists, write "nonreal."

a) $\sqrt{25} = 5$

b) $\sqrt[3]{8} = 2$

c) $\sqrt[3]{-125} = -5$

ex: Evaluate. If no real value exists, write "nonreal."

d) $\sqrt[5]{243} = 3$

e) $\sqrt[4]{-16}$ nonreal

f) $\sqrt{\frac{1}{9}} = \frac{1}{3}$

ex: Evaluate. If no real value exists, write "nonreal."

g) $\sqrt[5]{32} = 2$

h) $\sqrt[3]{\frac{125}{8}} = \frac{5}{2}$

i) $-5\sqrt[4]{16} = -5 \cdot 2 = -10$

ex: Evaluate. If no real value exists, write "nonreal."

g) $\sqrt[1]{1} = 1$

h) $\sqrt{27} = 3\sqrt{3}$

$$\sqrt{9 \cdot 3}$$

i) $\sqrt[4]{162}$

$$\sqrt[4]{81 \cdot 2} = 3\sqrt[4]{2}$$

ex: Evaluate. If no real value exists, write "nonreal."

j) $\sqrt[5]{-96} = -\sqrt[5]{96} = -\sqrt[5]{32 \cdot 3} = -2\sqrt[5]{3}$

k) $\sqrt[3]{32} = \sqrt[3]{8 \cdot 4} = 2\sqrt[3]{4}$

l) $\sqrt[6]{-64}$ nonreal

ex: Rationalize.

$$\text{a)} \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

$$\text{b)} \frac{2}{\sqrt[3]{3}} \cdot \frac{\sqrt[3]{9}}{\sqrt[3]{9}} = \frac{2\sqrt[3]{9}}{3}$$

ex: Rationalize.

$$c) \frac{1}{\sqrt[4]{2}} \cdot \frac{\sqrt[4]{8}}{\sqrt[4]{8}} = \frac{\sqrt[4]{8}}{2}$$

$$d) \frac{5}{\sqrt[3]{9}} \cdot \frac{5}{\sqrt[3]{9}} \cdot \frac{\sqrt[3]{81}}{\sqrt[3]{81}} = \frac{5\sqrt[3]{81}}{9} = \frac{5\sqrt[3]{81} \cdot \sqrt[3]{3}}{9\sqrt[3]{3}}$$
$$= \frac{5\sqrt[3]{3}}{3}$$

ex: Rationalize.

$$\sqrt[6]{64} = \sqrt[6]{2^6}$$

2

$$e) \frac{7}{\sqrt[6]{8}} \cdot \frac{\sqrt[6]{8}}{\sqrt[6]{8}} = \frac{7\sqrt[6]{8}}{2}$$

$$f) \frac{15}{\sqrt[5]{25}} \cdot \frac{\sqrt[5]{125}}{\sqrt[5]{125}} = \frac{15\sqrt[5]{125}}{5} = 3\sqrt[5]{125}$$

$$g) \frac{-8}{\sqrt[10]{16}}$$

Operations with nth Roots

$$\sqrt{5} + \sqrt{6}$$

- Addition/Subtraction - the radicals must be "like radicals"

$$\sqrt{x} + \sqrt{x} = 2\sqrt{x} \quad \sqrt[3]{5} + \sqrt{5}$$

ex: Perform the indicated operation.

*Like radicals have
Same radicand AND
Same index*

a) $7\sqrt[5]{8} + \sqrt[5]{8} = 8\sqrt[5]{8}$

b) $4\sqrt[3]{2} - 6\sqrt[5]{2}$ *A ready
simplified*

ex: Perform the indicated operation.

c) $\sqrt[4]{2} - \sqrt[3]{2}$

d) $\sqrt[3]{54} - \sqrt[3]{2}$

$(\sqrt[3]{27})\sqrt[3]{2} - \sqrt[3]{2}$
 $3\sqrt[3]{2} - \sqrt[3]{2} = 2\sqrt[3]{2}$

Operations with nth Roots

- Multiplication/Division - the radicals must have the same index

ex: Perform the indicated operation.

$$\text{a) } \frac{\sqrt[3]{250}}{\sqrt[3]{2}} = \sqrt[3]{125} = 5$$

ex: Perform the indicated operation.

$$\text{b) } \frac{\sqrt[4]{4}}{\sqrt[4]{36}} = \frac{1}{\sqrt[4]{9}} \cdot \frac{\sqrt[4]{9}}{\sqrt[4]{9}} = \frac{\sqrt[4]{9}}{\sqrt[4]{81}} = \frac{\sqrt[4]{9}}{3}$$

Rational Exponents

$$x^{\frac{e}{i}} = \sqrt[i]{x^e}$$

Where:

- e is called the exponent
- i is called the index

ex: Rewrite in radical form.

a) $4^{3/5} = \sqrt[5]{4^3}$

b) $27^{4/3} = \sqrt[3]{27^4}$

c) $9^{5/2}$

ex: Rewrite in radical form.

d) $-4^{1/2} = -| \cdot 4^{1/2} | = -| \cdot \sqrt{4} | \rightarrow -2$

e) $(-3)^{5/3}$ $\sqrt[3]{(-3)^5}$

f) $2 \cdot 7^{3/4}$

ex: Rewrite in exponential form.

$$\text{a) } \sqrt[7]{21} = 21^{\frac{1}{7}}$$

$$\text{b) } \sqrt{8^3} = 8^{3/2}$$

$$\text{c) } -\sqrt[3]{-9}$$

ex: Evaluate. If no real value exists, write "nonreal."

a) $9^{3/2}$

$$\sqrt[3]{9^3} \quad (3^2)^{3/2} \quad (8)^{2/3}$$
$$(\sqrt{9})^3 \quad 3^3 \quad (2^3)^{2/3}$$
$$27 \quad 27 \quad 4$$

b) $81^{3/4}$

$$\sqrt[4]{81^3} = (\sqrt[4]{81})^3 = 3^3 = 27$$

c) $4^{5/2}$

ex: Evaluate. If no real value exists, write "nonreal."

d) $8^{4/3}$

e) $1000^{2/3}$

f) $32^{3/5}$

ex: Evaluate. If no real value exists, write "nonreal."

g) $(-216)^{2/3} = (-6^2)^{1/3} = (-6)^2 = 36$

h) $16^{-3/4}$

$$\frac{1}{(\sqrt[4]{16})^3} = \frac{1}{2^3} = \frac{1}{8}$$

negative exponent
does not mean
neg. number.

i) $-4^{3/2}$

ex: Evaluate. If no real value exists, write "nonreal."

j) $-216^{-2/3}$

k) $2 \cdot 32^{3/5} = 2 \cdot 8 = 16$

\downarrow
 $(\sqrt[5]{32})^3$

$$\frac{x^m \cdot x^n}{x^{m+n}}$$

l) $64^{1/12} \cdot 64^{1/4} = (64)^{\frac{1}{12} + \frac{1}{4}} = 64^{1/3} = \sqrt[3]{64} = 4$

ex: Evaluate. If no real value exists, write "nonreal."

m) $\left(\frac{27}{8}\right)^{-4/3}$

n) $\frac{27^{-4/3}}{8}$

o) $(-9)^{5/2}$

p) $-(-8)^{-4/3}$

ex: Between which two consecutive integers does the expression lie?

a) $\sqrt{10}$



A horizontal number line with arrows at both ends. It has three tick marks. The first tick mark is labeled $\sqrt{9}$ below the line. The second tick mark is labeled $\sqrt{10}$ below the line. The third tick mark is labeled $\sqrt{16}$ below the line. Above the first tick mark is the number 3. Above the third tick mark is the number 4. There are vertical lines connecting each tick mark to its corresponding value above the line.

between
3 and 4

b) $\sqrt[5]{40}$



A horizontal number line with arrows at both ends. It has three tick marks. The first tick mark is labeled $\sqrt[5]{32}$ below the line. The second tick mark is labeled $\sqrt[5]{40}$ below the line. The third tick mark is labeled $\sqrt[5]{243}$ below the line. Above the first tick mark is the number 2. Above the third tick mark is the number 3. There are vertical lines connecting each tick mark to its corresponding value above the line.

ex: Between which two consecutive integers does the expression lie?

c) $\sqrt[3]{-7}$

d) $\sqrt[4]{5}$

e) $4^{3/4}$



ex: Evaluate on your calculator.

a) $\sqrt[5]{40}$

b) $\sqrt[3]{-7}$

c) $4^{3/4}$