2.8: Sketching Polynomial Functions

For each polynomial:

a) Find the real zeros. Determine if the graph will cross or bounce at the zeros.
b) Find the y-intercept
c) State the degree and max number of turning points.
d) State the end behavior
e) Sketch. Plot extra points when necessary.

1) $y = -2(x - 1)^2(x + 2)$

a.) $0 = -2(x-1)^2(x+2)$

$x = 1$ bounce

$x = -2$ cross

b.) y-int $(0, -4)$

c.) degree: 3

max turn: 2

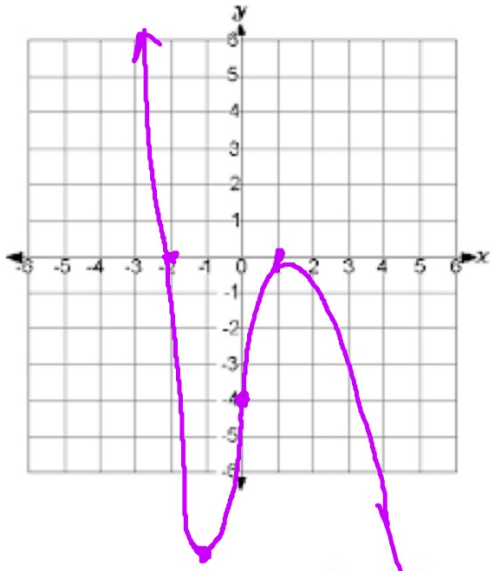
d.) odd

L.C.(−)

$x \to -\infty \quad y \to \infty$

$x \to \infty \quad y \to -\infty$

x	y
-1	-8



D: $(-\infty, \infty)$

R: $(-\infty, \infty)$

2) $y=x^4 - 9x^2$

a.) $0 = x^2(x^2-9)$

$0 = x^2(x+3)(x-3)$

$x=0$ bounce

$x=-3$ cross

$x=3$ cross

b.) y-int (0,0)

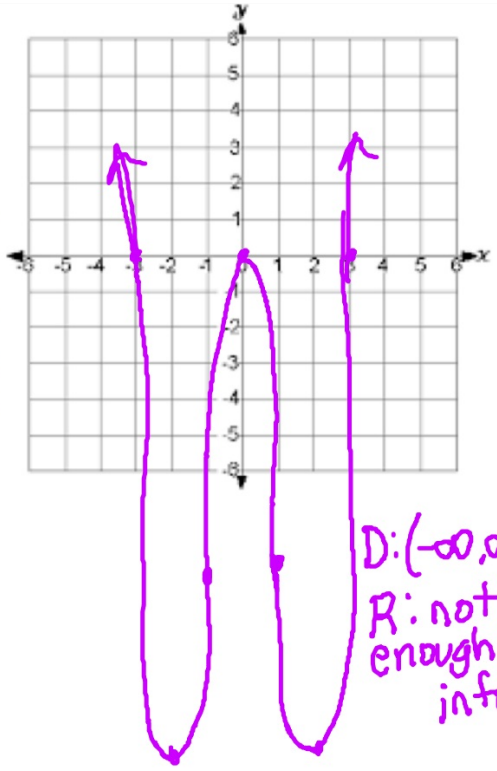
c.) degree 4

max turns 3

d.) even ↑↑ $x \to -\infty \quad y \to \infty$

Lc. (+) $x \to \infty \quad y \to \infty$

x	y
1	-8
-1	-8
2	-20
-2	-20



D: $(-\infty, \infty)$

R: not enough info.

3) $y = 4x^2 - 2x^3$

a.) $0 = 2x^2(2-x)$

$x = 0$ bounce

$x = 2$ cross

b.) y-int (0, 0)

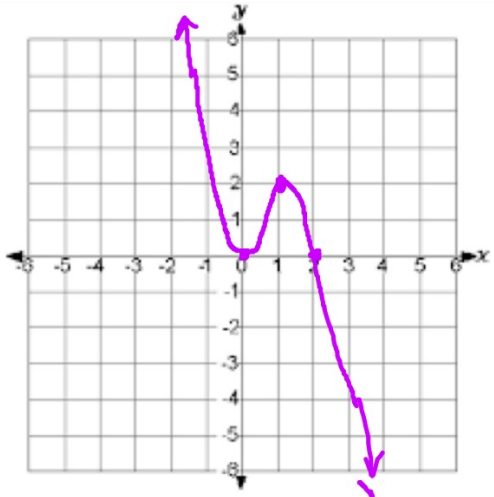
c.) degree 3

max turn : 2

d.) $x \to -\infty$ $y \to \infty$

$x \to \infty$ $y \to -\infty$

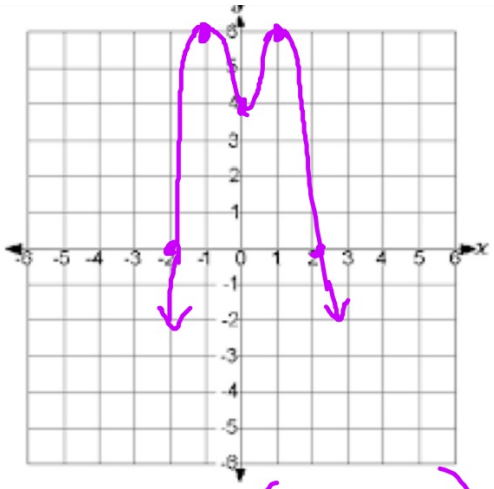
x	y
1	2



D: $(-\infty, \infty)$

R: $(-\infty, \infty)$

4) $y = -x^4 + 3x^2 + 4$



a.) $0 = -(x^4 - 3x^2 - 4)$

$0 = -(x^2 - 4)(x^2 + 1)$

$0 = -(x+2)(x-2)(x^2+1)$

$x = -2$ cross

$x = 2$ cross

b.) y-int $(0, 4)$

x	y
-1	6
1	6

c.) degree 4
max turn 3

d.) degree even
(-) L.C.

$x \to -\infty \; y \to -\infty$

$x \to \infty \; y \to -\infty$

D: $(-\infty, \infty)$

R: not enough info.

18.) $0 = 3(8x^3 + 27)$

$0 = 3(2x+3)(4x^2 - 6x + 9)$

$a = 2x$
$b = 3$

$$x = \frac{6 \pm \sqrt{36 - 4(4)(9)}}{2(4)}$$

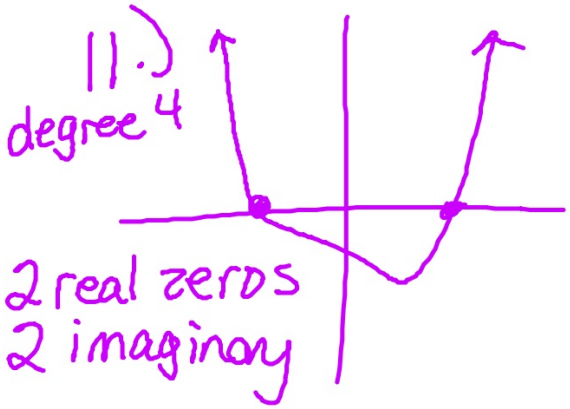
$$x = \frac{6 \pm \sqrt{-108}}{8} = \frac{6 \pm 6i\sqrt{3}}{8}$$

$$= \frac{3 \pm 3i\sqrt{3}}{4}$$

$$\frac{6}{8} \pm \frac{6\sqrt{3}}{8}i$$

11.)

degree 4



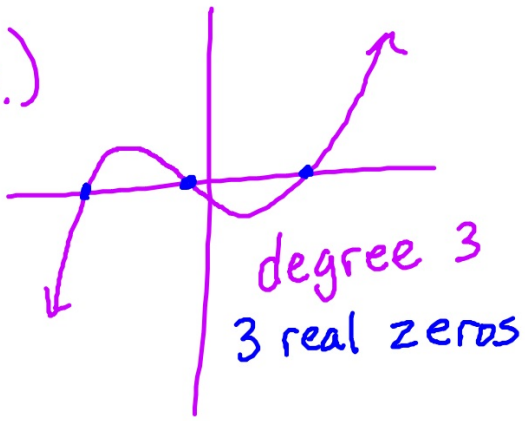
2 real zeros
2 imaginary

15.) $\dfrac{3(xy^{-2})^3}{(27x^{-1})(5y)^2}$

$\dfrac{x^1 3^1 x^3 y^{-6}}{9 \cancel{27}\cancel{x^{-1}} \cdot 25 y^2 y^6}$

$\dfrac{x^4}{225 y^8}$

10.)



degree 3

3 real zeros

16) $f(x) = 2x^4 - 11x^2 + 12$

$0 = (2x^2 - 3)(x^2 - 4)$

$2x^2 - 3 = 0$ $\quad x^2 - 4 = 0$

$\sqrt{x^2} = \sqrt{\frac{3}{2}}$ $\quad x = \pm 2$

$x = \pm \frac{\sqrt{6}}{2}$

17.) $f(x) = x^3 - 4x^2 - 11x + 30$

$$\begin{array}{r|rrrr} 2 & 1 & -4 & -11 & 30 \\ & & 2 & -4 & -30 \\ \hline & 1 & -2 & -15 & 0 \end{array}$$

$x^2 - 2x - 15 = 0$

$(x-5)(x+3) = 0$

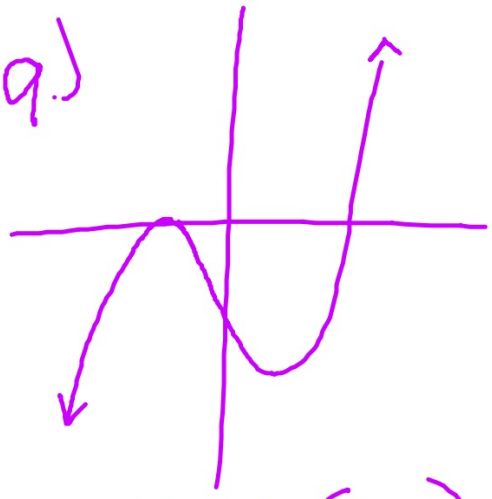
$x = 2, -3, 5$

14.) $(2^{-1}y^{-1}z^{-3})(64x^3y^9z^3)$

$$\frac{64x^3y^9z^3}{2^1y^1z^3}$$

$$32x^3y^8$$

9.)



$x = -1$ (even)

$x = 2$ (odd)

15) $\dfrac{3(x^3y^{-6})}{27x^{-1}\cdot 25y^2}$

$\dfrac{3x^3\cdot x^1}{9\,27\cdot 25y^2y^6}$

$\dfrac{x^4}{225y^8}$