$$\begin{array}{c|ccccc}
\sqrt{2} & 1 & 0 & -6 & 0 & 8 \\
 & & \sqrt{2} & 2 & -4\sqrt{2} & -8 \\
\hline
(x-\sqrt{2}) & 1 & \sqrt{2} & -4 & -4\sqrt{2} & 0
\end{array}$$

$$x^3+\sqrt{2}x^2-4x-4\sqrt{2}$$

2 a RZT

b factor

c RZT

d factor

e factor

f RZT

g factor

$$x^3(x+1)-8(x+1)$$

$$(x^3-8)(x+1)=0$$

$$(x-2)(x^2+2x+4)(x+1)=0$$

$$2 \qquad -1$$

$$(x^2+2x+1)-1+4=0$$

$$\sqrt{(x+1)^2}=\sqrt{-3}$$

$$|x+1|=i\sqrt{3}$$

$$x=-1\pm i\sqrt{3}$$

c.) $f(x) = x^4 - x^3 - x^2 - x - 2$

$\pm 1, \pm 2$

$$\begin{array}{c|ccccc} 2 & 1 & -1 & -1 & -1 & -2 \\ & & 2 & 2 & 2 & 2 \\ \hline & 1 & 1 & 1 & 1 & 0 \end{array}$$

$x^3 + x^2 + x + 1 = 0$

$x^2(x+1) + 1(x+1) = 0$

$(x^2+1)(x+1) = 0$

$$\boxed{2, -1, \pm i}$$

$$x^2 + 2x + 1 - \frac{3x+5}{x^2-2}$$

$$x^2 - 2 \,\overline{\big)\, \cancel{x^4} + 2x^3 - x^2 - 7x - 7}$$

$$-\cancel{x^4} \qquad + 2x^2$$

$$\cancel{2x^3} + x^2 - 7x \qquad x^2 + 2x + 1 + \frac{-3x-5}{x^2-2}$$

$$-\cancel{2x^3} \qquad + 4x$$

$$x^2 - 3x - 7$$

$$-x^2 \qquad + 2$$

$$-3x - 5$$

2.)

$$2x^2 - 3x + 6 + \frac{-8x+3}{x^2+x-1}$$

$$x^2 + x - 1 \,\overline{\big)\, \cancel{2x^4} - x^3 + x^2 + x - 3}$$

$$-\cancel{2x^4} \mp 2x^3 + 2x^2$$

$$-3x^3 + 3x^2 + x$$

$$+3x^3 + 3x^2 + 3x$$

$$6x^2 - 2x - 3$$

$$-\;6x^2 \mp 6x + 6$$

$$-8x + 3$$

$$2x^2 - 3x + 6 - \frac{8x-3}{x^2+x-1}$$

e.) $x^2(x+5)-4(x+5)$

$(x^2-4)(x+5)=0$

b.) $(9x^2+1)(9x^2-1)$

g.) $0=(3x^2+1)(5x^2-2)$

$$\sqrt{x^2}=\sqrt{\frac{2}{5}}$$

$$|x|=\frac{\sqrt{10}}{5}$$

$$x=\pm\frac{\sqrt{10}}{5}$$

f.) $y = x^4 + x^3 + 2x^2 + 4x - 8$

± 1
± 2
± 4
± 8

$$\begin{array}{c|ccccc} 1 & 1 & 1 & 2 & 4 & -8 \\ & & 1 & 2 & 4 & 8 \\ \hline & 1 & 2 & 4 & 8 & 0 \end{array}$$

$$x^3 + 2x^2 + 4x + 8 = 0$$

$$x^2(x+2) + 4(x+2) = 0$$

$$(x^2+4)(x+2) = 0$$

$$\boxed{1, \pm 2i; -2}$$

a.) $h(x) = x^3 + 3x^2 - 25x + 21$

$\pm 1, \pm 3, \pm 7, \pm 21$

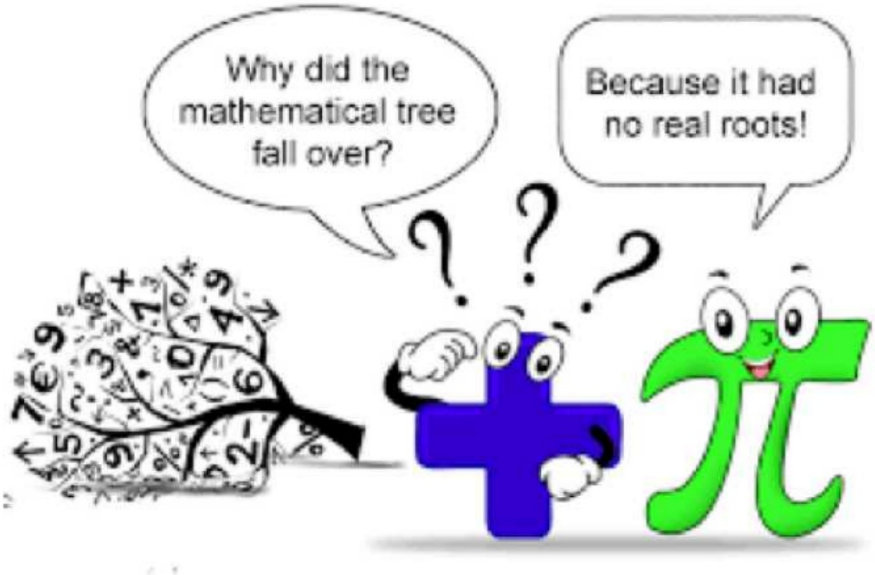
$$
\begin{array}{r|rrrr}
3 & 1 & 3 & -25 & 21 \\
  &   & 3 & 18 & -21 \\
\hline
  & 1 & 6 & -7 & 0
\end{array}
$$

$x^2 + 6x - 7 = 0$

$(x+7)(x-1) = 0$

$\boxed{x = 3, 1, -7}$

2.7 Finding All Zeros Cont.
Analyzing Polynomial Functions



*See printout.

HW:

a.) $-\frac{2}{5}$, $3i$, $-3i$

$$(5x+2)(x-3i)(x+3i)$$

$$(5x+2)(x^2+9)$$

$$f(x) = 5x^3+2x^2+45x+18$$

Sum and Product Rule

$$y = x^2 + bx + c$$

$-2, 5$	$5, -9$
$y = (x+2)(x-5)$ $y = x^2 - 3x - 10$ Sum: $-2+5 = 3$ $-b$ Product: $-2 \cdot 5 = -10$ c	Sum: -4 opposite Product: -45 same $y = x^2 + 4x - 45$

$0, 3+i, 3-i$ $\qquad 9-(-1)$

mult 2

Sum: $3+i+3-i = 6$

product: $(3+i)(3-i) = 9-i^2 = 10$

$$y = x^2(x^2 + \underline{-6}x + \underline{10})$$

sum

product

$$y = x^4 - 6x^3 + 10x^2$$

$4-\sqrt{6},\ 4+\sqrt{6}$

sum $= 8$

product $= 10$

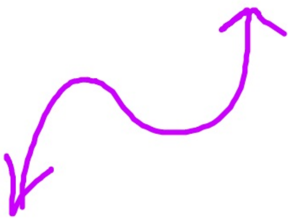
$$y = x^2 + \underline{-8}\,x + \underline{10}$$

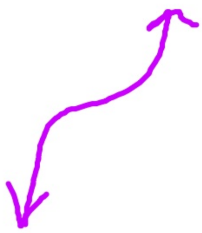
-sum

product

Polynomial Degrees and Number of Turning Points

Polynomial Type	Degree	Maximum Number of Turning Points
Constant	0	0
Linear	1	0
Quadratic	2	1
Cubic	3	2
n^{th} Degree Polynomial	n	$n-1$





Complex Conjugate & Irrational Conjugate Theorem

Imaginary and irrational roots always come in

_____________________________________.

ex: Write a polynomial function in standard form with integral coefficients and the given roots.

a) $-\frac{2}{5}, \quad 3i$

ex: Write a polynomial function in standard form with integral coefficients and the given roots.

b) 0 multiplicity 5, $1-\sqrt{3}$

ex 1) Determine the degree and state the maximum number of turning points.

a) $f(x) = 2x^3 + 5x^2 - 9$

3 ; 2

b) $f(x) = 9 - 6x^2$

2 ; 1

ex 1) Determine the degree and state the maximum number of turning points.

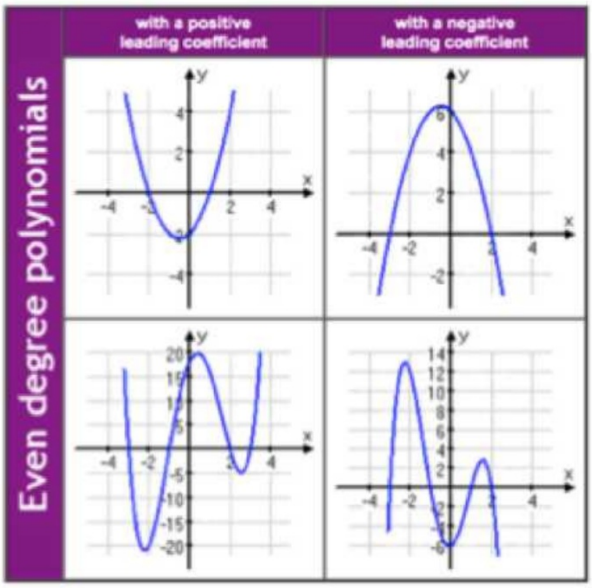
c) $f(x)=(x-2)^5(x+3)^6$

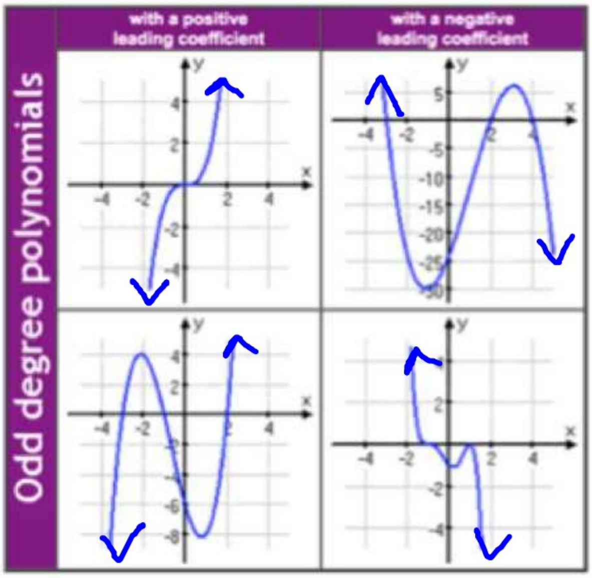
11 ; 10

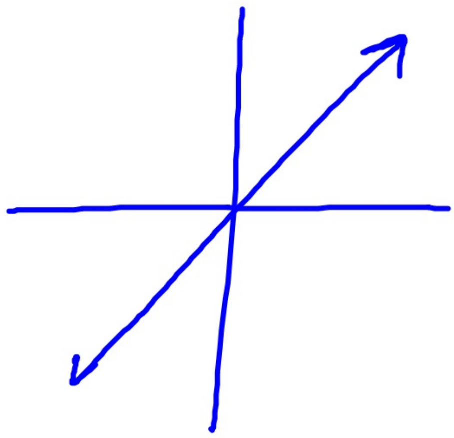
d) $f(x)=5(1-x^2)^7$

14 ; 13

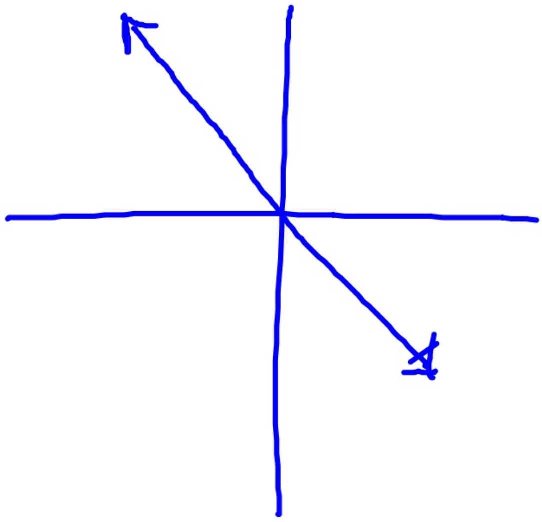
End Behavior



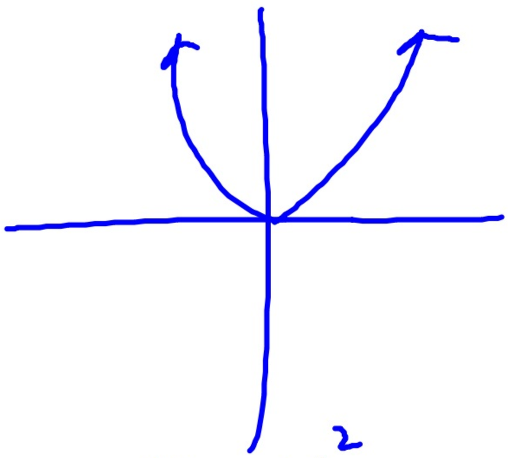




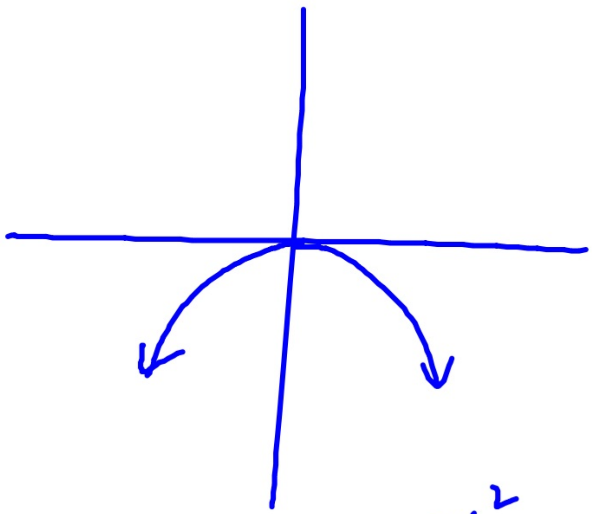
$y = x$



$y = -x$

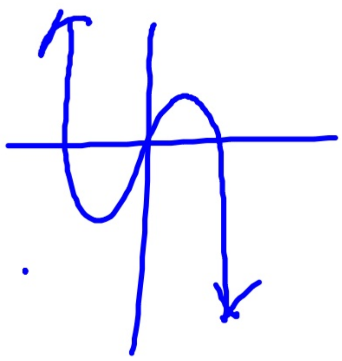


$y = x^2$



$y = -x^2$

Stating End Behavior



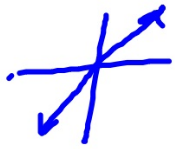
$x \to -\infty, \quad y \to \underline{\ \infty\ }$

$x \to \infty, \quad y \to \underline{\ -\infty\ }$

ex 2) Determine the end behavior of each polynomial.

a) $f(x) = \underline{2x}^3 + 5x^2 - 9$

odd, (+)



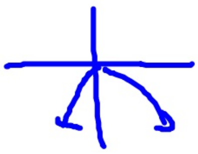
$\downarrow \uparrow$

$x \to -\infty \quad y \to \underline{-\infty}$

$x \to \infty \quad y \to \underline{\infty}$

b) $f(x) = 9 - 6x^2$

even, (−)



$\downarrow \downarrow$

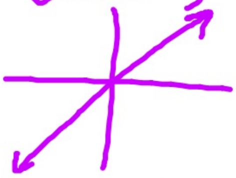
$x \to -\infty \quad y \to \underline{-\infty}$

$x \to \infty \quad y \to \underline{-\infty}$

ex 2) Determine the end behavior of each polynomial.

c) $f(x)=(x-2)^{5}(x+3)^{6}$

odd, (+)

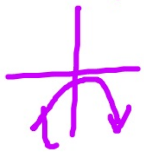


$x \to -\infty$ $y \to$ $\underline{-\infty}$

$x \to \infty$ $y \to$ $\underline{\infty}$

d) $f(x)=5(1-x^{2})^{7}$

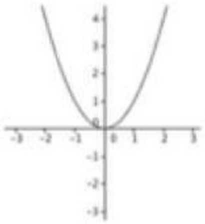
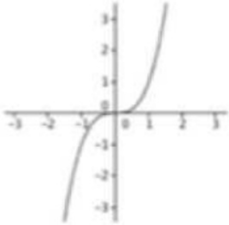
even, (−)



$x \to -\infty$ $y \to$ $\underline{-\infty}$

$x \to \infty$ $y \to$ $\underline{-\infty}$

Bouncing and Crossing Zeros

In the graph below the graph "bounces" off the x-axis at x=0.	In the graph below the graph "crosses" the x-axis at x=0.
	

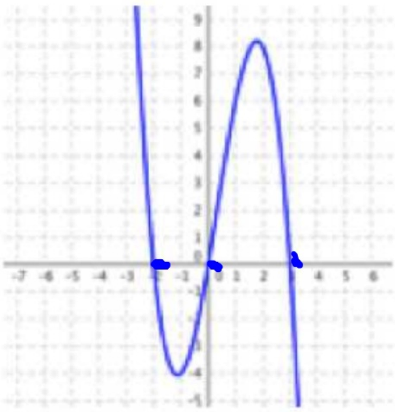
ex 3) Using the graph of the polynomial function,

1. State the degree of the polynomial. 3
2. Find the zeros. State the multiplicity if greater than 1. 0, -2, 3
3. State the end behavior.
4. Determine whether the graph "crosses" the x-axis or "bounces" off the x-axis at each zero.

a) $f(x) = -x(x+2)(x-3)$

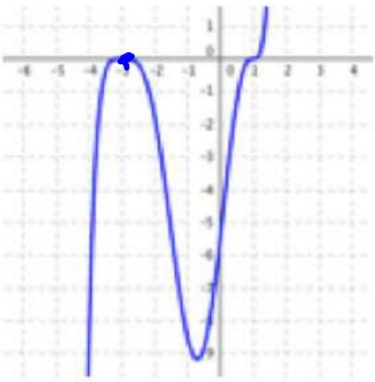
$x \to -\infty \quad y \to \underline{\infty}$

$x \to \infty \quad y \to \underline{-\infty}$



All crossing

b) $f(x)=\frac{1}{15}(x+3)^4(x-1)^3$



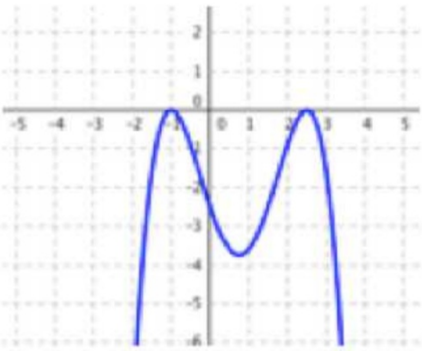
$x=-3$, mult. 4 (bounce)

$x=1$, mult 3 (cross)

$x \to -\infty \quad y \to -\infty$

$x \to \infty \quad y \to \infty$

c) $f(x) = -\frac{1}{10}(2x-5)^2(x+1)^2$



$x = +2.5$, mult 2

$x = -1$, mult 2

$x \to -\infty \quad y \to -\infty$

$x \to \infty \quad y \to -\infty$

- A graph "crosses" the x-axis at a zero if the multiplicity of that zero is odd.

- A graph "bounces" off the x-axis at a zero if the multiplicity of that zero is even.

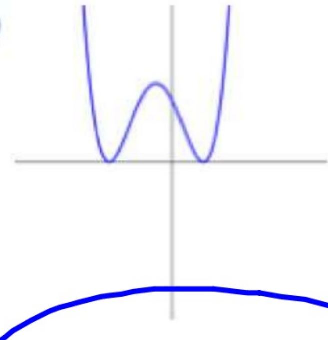
ex 4) Which of the following graphs could represent the polynomial $f(x) = a(x-b)^2(x-c)^3$?

5-odd

$\downarrow \uparrow$

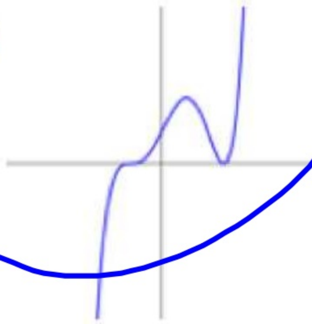
a)

c)



b)

d)



Review

ex: Find the error.

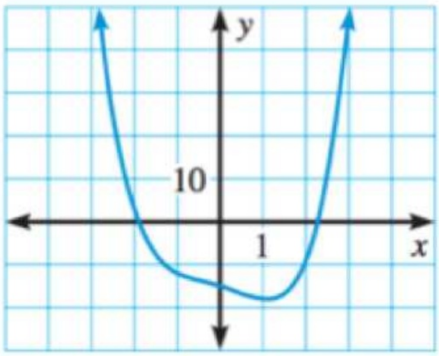
$$\begin{array}{r|rrr} 2 & 1 & -5 & 3 \\ & & 2 & -6 \\ \hline & 1 & -3 & -3 \end{array}$$

$$\frac{x^3 - 5x + 3}{x - 2} = x^2 - 3x - \frac{3}{x - 2}$$

Review

ex: Determine the number of imaginary zeros with the given degree and graph.

Degree: 4



Review

ex: Write a polynomial function in standard form with integral coefficients and the given roots.

b) 0 multiplicity 5, $\sqrt{3}$